DEVELOPMENT OF PUPILS' MATHEMATICAL THINKING

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Abstract. In this paper three types of exercises which develop mathematical reasoning and allow intuitive understanding of mathematical problems are presented. These exercises can be used during the lessons of mathematics.

Introduction

A knowledge of mathematics is recognized as a necessary part of intellectual equipping of every well-educated person. However, at the present time a place of mathematics in education is under the serious threat. Over a long period of time, an interest of pupils and students in this subject is being decreased in Poland. This problem concerns not only this subject, but also those fields of science, where mathematical knowledge is necessary. To prevent aggravating a situation, from the very beginning of education it is necessary to offer pupils an incentive to study mathematics by generating interest in this subject and showing its utility in practice.

Among methods of teaching pointed out by Bruner [1] there is teaching by discovering new phenomena and new truths, new regularities. In this paper the examples of developing mathematical thinking of pupils at various educational levels according to Bruner, i.e. by discovering phenomena and regularities are discussed.

1. Praxis with Möbius strip

An exercise consists in preparation of a traditional band and the Möbius strip and answer to a question: what will be obtained after scission of the traditional band and the Möbius strip along the median line? Verification of answer is carried out by doing scissions.

The next question refers to the result of scission of the Möbius strip into 3 or 4 parts and practical verification whether or not the answers are right [2].

Owing to such experience, a pupil becomes acquainted with the notion of one-side surface and with the Möbius strip and trains an intuition for solving future topological problems.

Fig. 1. The traditional band and the Möbius strip before scission

2. Praxis with mathematical spirals

A sequence of natural numbers 1, 1, 2, 2, 3, 3, 4, 4, … is presented on a checked sheet of paper (Fig. 2) [2]. In such a manner some type of spiral is obtained. The questions to a pupil connected with this exercise are:

- Can you write next numbers in this sequence?
- Can you draw the next part of the spiral?

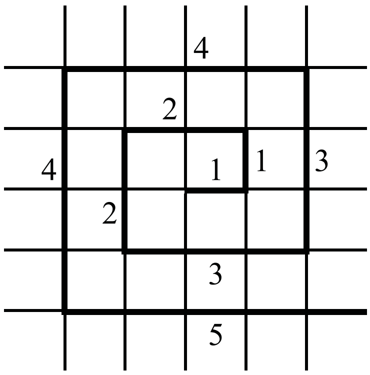

Fig. 2. The spiral obtained from a sequence of numbers 1, 1, 2, 2, 3, 3, 4, 4, …

Now consider another sequence of numbers, for example: 2, 1, 3, 2, 4, 3, 5, 4, 6, 5, … . Pupils without assistance try to realize this sequence just as in the previous exercise. In this way the subsequent spiral is obtained (Fig. 3).

The next exercise is connected with the Fibonacci sequence. This exercise begins from a question: is it possible to draw a sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13… as a spiral? It turns out that in such a manner the Fibonacci spiral is obtained (Fig. 4).

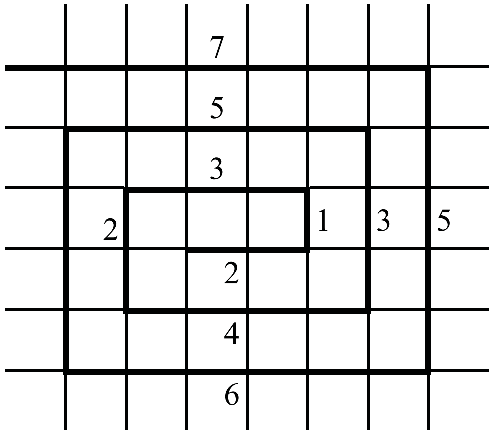

Fig. 3. The spiral obtained from a sequence of numbers 2, 1, 3, 2, 4, 3, 5, 4, 6, 5, …

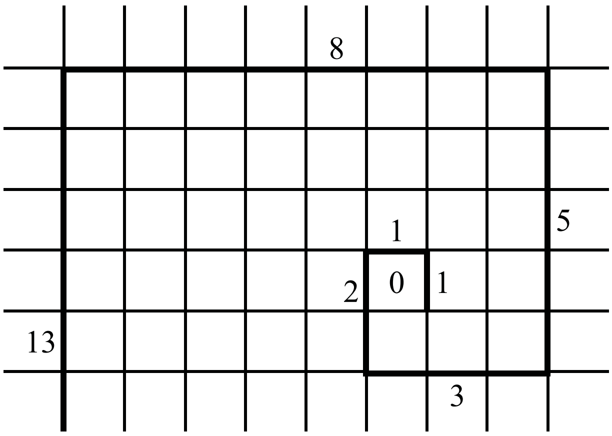

Fig. 4. The Fibonacci spiral

With the help of this exercise a pupil becomes acquainted with the Fibonacci sequence, although the mathematical description of the Fibonacci sequence will be coming to light at a later time. Owing to such exercises a pupil without assistance can find other mathematical spirals described by number sequences.

The next exercise deals with the Ulam spiral. We draw this spiral on a checked sheet of paper by placing successive natural numbers in the manner presented in Figure 5. The number 1 is placed in the central square, the number 2 is placed to the right of 1, the number 3 above 2, the number 4 to the left of 3, and so on.

37	36	35	34	33	32	31	56
38	17	16	15	14	13	30	55
39	18	5	4	3	12	29	54
40	19	6	1	2	11	28	53
41	20	7	8	9	10	27	52
42	21	22	23	24	25	26	51
43	44	45	46	47	48	49	50

Fig. 5. The Ulam spiral

Ulam marked the prime numbers on this spiral and observed that these numbers go into some slanting lines (Fig. 6). Up to the present, the problem is not solved. This brings up the following question: Can these regularities be systematized and described? Pupils can come interested in the problem and can try to describe this phenomenon.

37	36	35	34	33	32	31	56
38	17	16	15	14	13	30	55
39	18	5	4	3	12	29	54
40	19	6	1	2	11	28	53
41	20	7	8	9	10	27	52
42	21	22	23	24	25	26	51
43	44	45	46	47	48	49	50

Fig. 6. The Ulam spiral with marked prime numbers

3. Praxis with fractals

French mathematician B. Mandelbrot (born in Warsaw in 1924) is a discoverer and creator of the theory of fractals. Fractal is a difficult notion arising in the theory of chaos. This notion can be explained to pupils by presenting the models of fractals which occur in surroundings, for example, broccoli (Fig. 7), cauliflower, fiord coastline, a fern leaf, etc.

Fig. 7. Broccoli

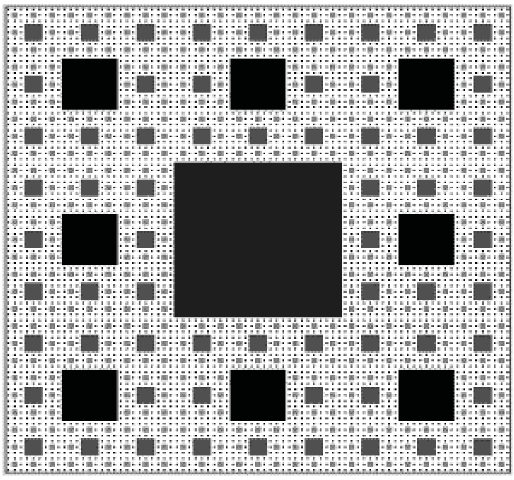

Fig. 8. The Sierpinski carpet

Pupils should become acquainted with the distinguishing features of fractals, for instance, through the construction of the Sierpiński carpet (Fig. 8). A pupil can observe the self-similarity property.

To make easier an understanding of the notion of fractal we can construct its spatial model as shown in Figure 9 [4].

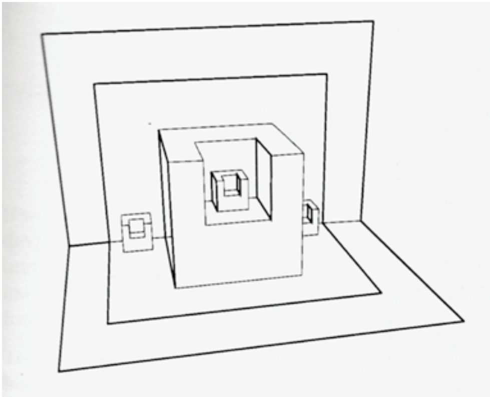

Fig. 9. A spatial model of fractal

By exercises of such a type a pupil observes that mathematical notions can be found in nature and can be inspired by nature. Having familiarized himself with the notion of fractal, a pupils in the future can have taken an interest in the theory of chaos.

Conclusions

If pupils at various levels of education will be guided into mathematical pathways using various exercises, then there is a chance that they develop their logical thinking, extend their interest in mathematics, love mathematics and choose study specialization connected with mathematics.

References

[1] Bruner J.S., *W poszukiwaniu teorii nauczania*, PIW, Warszawa 1966.

[2] Langdon N., Snape C., *Ścieżki matematyki*. Wyd. 2, Gdańskie Wydawnictwo Oświatowe, Gdańsk 1988, 48.

[3] Peitgen H-O., Jürgens H., Savpe D., *Granice chaosu - fraktale cz. 1*, WN PWN, Warszawa 2002.

[4] Uribe D., *Fractal - cuts*, Tarquin Publication, 1998.