APPLICATION OF SIMPLIFIED CURVED BOUNDARY ELEMENTS TO THE PLATE ANALYSIS - PART TWO

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Abstract. Static analysis of Kirchhoff plate by the Boundary Element Method is presented in the paper. The Bettie theorem is used to derive the boundary integral equation. Simplified curved elements are introduced. Modified approach of boundary integral equation formulation is adopted in which there is no need to introduce the equivalent shear forces at the boundary and concentrated forces at the plate corners. Two unknown and independent variables are considered at the boundary element node. The collocation version of boundary element method with singular and non-singular approach is presented.

Introduction

The Boundary Element Method (BEM) was created as a completely independent numerical tool to solve engineering problems [1, 2]. The BEM do not require the all domain discretization but only the boundary of a considered structure. This method reduces the computational dimension by one.

Present paper includes a modified formulation for bending analysis of plates, in which three geometric and three static variables at the plate boundary are considered. Application of curved boundary elements to structural analysis was proposed by Wrobel and Aliabadi [2]. Authors proposed three-node continuous and discontinuous quadratic elements also discussed in the first part of this work. In the paper curved, simplified boundary elements are introduced into the thin plate analysis. This part of elaboration contains numerical examples of static analysis of thin plates using simplified, curved boundary elements.

1. Numerical examples

Circular and elliptic plates are considered. Plates are subjected by concentrated force P acting at the characteristic point or uniformly distributed loading p acting on whole surface. Quasi-diagonal integrals in non-singular approach are evaluated using the method proposed by Okupniak and Sygulski [3] and twenty Gauss point.

Localization of collocation point for simplified, curved element is shown on the Figure 1. Contour of uniformly distributed loading is divided by finite number of linear sections in line with external geometrical nodes of boundary elements.

The results of calculations obtained using boundary element method are compared with the analytical solutions and results obtained by means of finite element method. All results of calculations are presented in non-dimensional parameters.

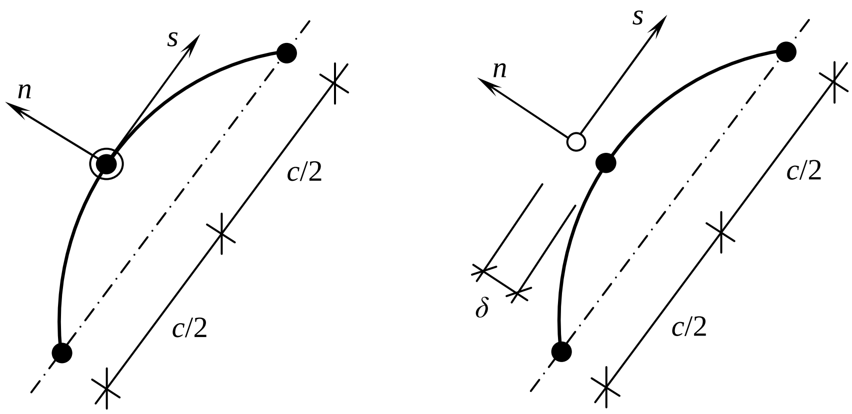

Fig. 1. Boundary element inscribed in curved line. Localization of collocation point for singular and non-singular formulation

2.1. Circular plate, simply-supported on whole boundary and subjected by the uniformly distributed loading

Circular plate of radius r, simply-supported on whole boundary is considered. The plate edge is approximated by the finite number of boundary elements. Plate is loaded by the uniformly distributed loading p. The Poisson ratio ν_p is equal 0.3. Results of calculations are presented in Tables 1, 2, 3 and 4. Value of parameter ε is assumed as 0.1.

Table 1

Deflection at the plate center [7]

Number of boundary elements	$\tilde{w} = \frac{w \cdot D}{p \cdot r^4}$		
	BEM I	**BEM** II	**BEM** III
32	0.0602147	0.0669337	0.0598683
64	0.0623454	0.0642589	0.0631792
128	0.0645226	0.0641676	0.0635861
Analytical solution	0.0637019		

Table 2

Bending moment at the plate center[7]

Number of boundary elements	$\tilde{M}_r = \frac{M_r}{p \cdot r^2}$		
	BEM I	**BEM** II	**BEM** III
32	0.198158	0.214596	0.198221
64	0.209875	0.207696	0.204911
128	0.202980	0.207577	0.205242
Analytical solution	0.206250		

Table 3

Circular plate, simply-supported on whole boundary. Deflection at the plate center. Analysis of influence of parameter $\varepsilon = \delta/d$. 64 boundary elements

$\varepsilon = \delta/d$	0.05	0.1	0.5	1.0
$wD/pr^4 \cdot 10^{-5}$	6601.28	6425.89	6347.73	6364.73

Table 4

Circular plate, simply-supported on whole boundary. Bending moment at the plate center. Analysis of influence of parameter $\varepsilon = \delta/d$. 64 boundary elements

$\varepsilon = \delta/d$	0.05	0.1	0.5	1.0
$M_r/pa^2 \cdot 10^{-4}$	2122.34	2076.96	2056.95	2061.14

2.2. Circular plate, clamped on the half of boundary length with rest of boundary free and subjected by the uniformly distributed loading

Circular plate of radius r, clamped on the half of boundary is considered. The plate edge is approximated by the finite number of boundary elements. Plate is loaded by the uniformly distributed loading p (Fig. 2). The Poisson ratio v_p is equal to 0.167.

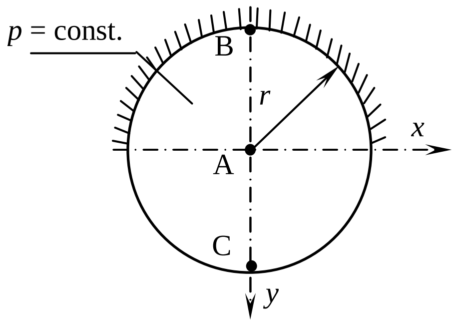

Fig. 2. Circular plate, clamped on the half of boundary length with rest of boundary free and subjected by the uniformly distributed loading p

Results of calculations are shown in Tables 5-8. Value of parameter ε is equal to 0.1.

Table 5

Deflection at the plate center

Number of boundary elements	$\tilde{w}_A = \dfrac{w_A \cdot D}{p \cdot r^4}$	
	BEM I	**BEM** II
32	0.0801344	0.0796683
64	0.0767120	0.0762752
128	0.0714213	0.0700362
FEM	0.0688868	

Table 6

Bending moment at the plate center

Number of boundary elements	$\tilde{M}_{xA} = \dfrac{M_{xA}}{p \cdot r^2}$		$\tilde{M}_{yA} = \dfrac{M_{yA}}{p \cdot r^2}$	
	BEM I	**BEM** II	**BEM** I	**BEM** II
32	0.167795	0.173558	– 0.144354	– 0.132251
64	0.182232	0.183546	– 0.141627	– 0.129376
128	0.184410	0.185174	– 0.130431	– 0.124932
FEM	0.187672		– 0.119842	

Table 7

Deflection on the plate boundary

Number of boundary elements	$\tilde{w}_C = \dfrac{w_C \cdot D}{p \cdot r^4}$	
	BEM I	**BEM** II
32	0.313073	0.301553
64	0.296629	0.284074
128	0.270437	0.264224
FEM	0.252598	

Table 8

Bending moment on the plate boundary

Number of boundary elements	$\tilde{M}_{yB} = \dfrac{M_{yB}}{p \cdot r^2}$	
	BEM I	**BEM** II
32	– 0.177590	– 0.179380
64	– 0.177259	– 0.178179
128	– 0.174915	– 0.173373
FEM	– 0.168545	

BEM I – linear boundary elements of the constant type about the same length in non-singular approach,

BEM II – simplified curved boundary elements of the constant type about the same length in non-singular approach,

BEM III – simplified curved boundary elements of the constant type about the same length in singular approach.

FEM – commercial computational program PL-WIN was used. The plate area was discretized by number of 768 finite triangle elements with three nodes and three degrees of freedom per node.

2.3. Elliptic plate, clamped on whole boundary and subjected by the uniformly distributed loading

Elliptic plate of half-axes a and b, clamped on whole edge is considered. The plate edge is approximated by the finite number of boundary elements. The plate is subjected to the uniformly distributed loading. The Poisson ratio ν_p is equal to 0.3 and a/b = 1.5. Localization of external geometrical nodes for 32 boundary elements is presented in Figure 3. For 64 boundary elements, similar localization is assumed, dividing all of segments: l, $l/2$, $l/3$ and $l/6$ by halves.

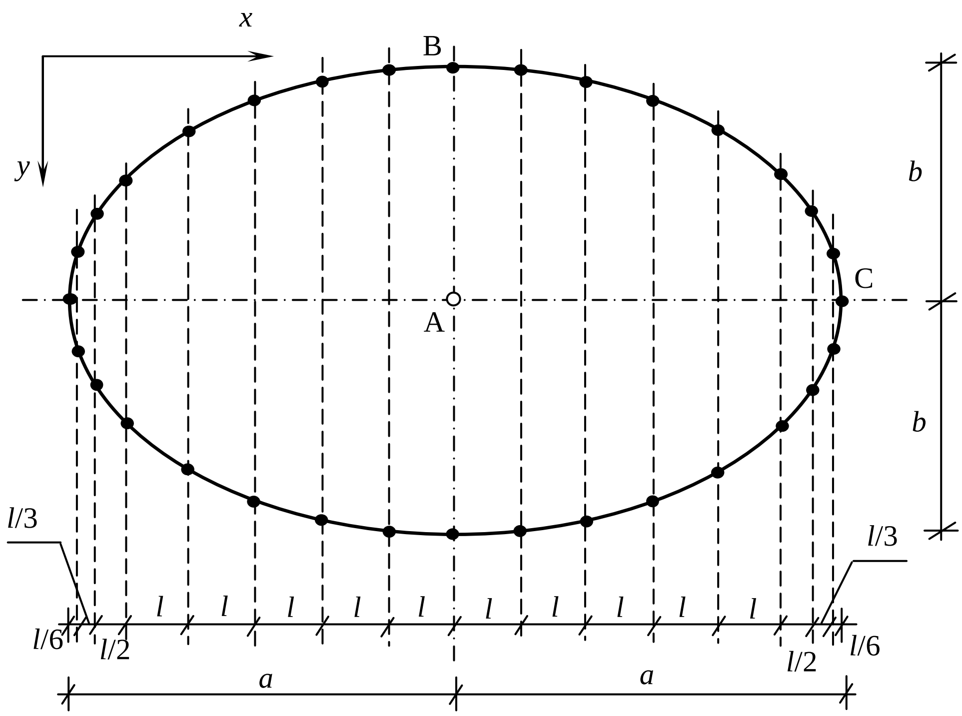

Fig. 3. Localization of boundary elements inscribed in ellipse contour

Results of calculations are presented in Tables 9-11. Value of non dimensional coefficient ε is equal to 0.1.

Table 9

Deflection at the plate center [7]

Number of boundary elements	$\tilde{w}_A = \frac{w_A \cdot D}{p \cdot a^4}$	
	BEM I	**BEM** II
32	0.0274431	0.0279014
64	0.0274481	0.0278955
Analytical solution	0.0278926	

Table 10

Bending moment at the plate center [7]

Number of boundary elements	$\tilde{M}_{xA} = \frac{M_{xA}}{p \cdot a^2}$		$\tilde{M}_{yA} = \frac{M_{yA}}{p \cdot a^2}$	
	BEM I	**BEM** II	**BEM** I	**BEM** II
32	0.0826588	0.0830993	0.125312	0.126464
64	0.0828583	0.0830984	0.125879	0.126463
Analytical solution	0.0830578		0.126446	

Table 11

Bending moment on the plate boundary [7]

Number of boundary elements	$\tilde{M}_{yB} = \frac{M_{yB}}{p \cdot a^2}$		$\tilde{M}_{xC} = \frac{M_{xC}}{p \cdot a^2}$	
	BEM I	**BEM** II	**BEM** I	**BEM** II
32	– 0.220628	– 0.222646	– 0.103246	– 0.102433
64	– 0.221884	– 0.222647	– 0.101109	– 0.101110
Analytical solution	– 0.223140		– 0.0991735	

BEM I – linear boundary elements of the constant type about the same length in non-singular approach,

BEM II – simplified curved boundary elements of the constant type about the same length in non-singular approach.

2.4. Elliptic plate, simply-supported on whole boundary and subjected to the uniformly distributed loading

Elliptic plate of half-axes a and b, simply-supported on whole edge is considered. The plate edge is approximated by the finite number of boundary elements. The plate is subjected to the uniformly distributed loading p. The Poisson ratio ν_p is equal to 0.3 and $a/b = 1.5$. Results of calculations are presented in Tables 12-13 and non-dimensional parameters are introduced. Value of non dimensional coefficient ε is equal to 0.1. The ellipse contour is divided into the boundary elements analogically to example as in 2.4.

Table 12

Deflection at the plate center [7]

Number of boundary elements	$\tilde{w}_A = \dfrac{w_A \cdot D}{p \cdot a^4}$	
	BEM I	**BEM** II
32	0.110148	0.115899
64	0.112546	0.115893
Analytical solution	0.115385	

Table 13

Bending moment at the plate center [7]

Number of boundary elements	$\tilde{M}_{xA} = \dfrac{M_{xA}}{p \cdot a^2}$		$\tilde{M}_{yA} = \dfrac{M_{yA}}{p \cdot a^2}$	
	BEM I	**BEM** II	**BEM** I	**BEM** II
32	0.213114	0.219376	0.356133	0.353365
64	0.213915	0.222126	0.367544	0.368365
Analytical solution	0.222000		0.379000	

BEM I – linear boundary elements of the constant type in non-singular approach,

BEM II – simplified curved boundary elements of the constant type in non-singular approach.

Conclusions

Static analysis of thin plates by the boundary element method was presented in the paper. Physical boundary conditions were introduced. Linear and simplified curved elements of the constant type were applied. The boundary integral equations were formulated in singular and non-singular approach.

In present formulation of plate bending it is no need to introduce equivalent shear forces at the plate edges and concentrated forces at the plate corners. It is element of originality in relation to classic formulation of thin plate bending problem. This element allows one to simplify substantially construction of characteristic matrix **G**. Non-singular formulation of boundary integral equation connected with the linear and simplified, curved boundary elements of the constant type may be applied successfully in static problems of plate bending. It is needed to call attention to selection of value of coefficient ε. This coefficient cannot be arbitrary small, because numerical integration using Gauss method (classic and modified [3]) goes to inaccurate results. The value of parameter ε cannot be arbitrary large, because it influences on conditioning of characteristic matrix **G**, and as a result - on obtained solution. Singular formulation of boundary integral equation eliminates this inconvenience but by the cost of additional procedure which enable calculation of quasi-diagonal elements of matrix **G**. Some inconsistent during creation of numerical notation of boundary integral equation is discretization of loading contour by the linear sections. Discretization of a plate boundary by simplified curved elements goes to more accurate results in relation to linear elements of the constant type.

Present formulation may be treated as a foundations to dynamic analysis of plates with linear and curvilinear edges. May be also useful to static and dynamic of plates resting on internal supports: pillars, linear and curvilinear continuous supports [4-6]. These types of bending problems can be solved using conception of Bèzine [4, 6], in which additional collocation points are applied inside the plate domain. Introduction of simplified, curved boundary element of the constant type and modified formulation of boundary conditions may be an alternative to classic formulation of thin plate bending problem.

Acknowledgement

This work is a part of compilation Exact curved elements in finite and boundary element method, donated by Grant No N N506 1740 33 from the Polish Ministry of Education and Science.

References

[1] Burczyński T., The Boundary Element Method in Mechanics, Scientific, Technical Publishing House, Warszawa 1995 (in Polish).

[2] Wrobel L.C., Aliabadi M.H., The Boundary Element Methods in Engineering, McGraw-Hill College 2002.

[3] Okupniak B., Sygulski R., Non-singular BEM analysis of Reissner plates, CMM-2003 15th International Conference on Computer Methods in Mechanics, Gliwice/Wisła, June 3-6, 2003, Short Papers, s. 265-266, [w:] CD-ROM, Eds.: T. Burczyński, P. Fedeliński, E. Majchrzak.

[4] Guminiak M., Thin plates analysis by the boundary element method using new formulation of a boundary condition (in Polish), PhD Thesis, Poznan University of Technology, Poznan, Poland, 2004.

[5] Rashed Y.F., A coupled BEM-flexibility force method for bending analysis of internally supported plates. International Journal of Numerical Method in Engineering 2002, 54, 1431-1457.

[6] Bèzine G., Gamby D.A., A new integral equations formulation for plate bending problems, Advances in Boundary Element Method, Pentech Press, London 1978.

[7] Timoshenko S., Woinowsky-Krieger S., Theory of plates and schells, Arkady Publishing House, Warszawa 1962.