PERCOLATION WITH THICK SYMMETRIC BARRIER IN FINITE SYSTEMS

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Abstract. In this article site percolation is studied on an $L\times L$ square lattice with a thick, symmetric barrier. Long-range connectivity is the result of the occupancy probability defined on the site. The influence of the thin and thick barrier on the percolation is analysed and the algorithm of control of the effectiveness of the scalability is proposed.

Introduction

The considered model is called percolation theory. It describes connectivity and was proposed by Broadbent and Hammersley [1]. Initially it concerned the flow of fluid by partly blocked system channels. The results[1] obtained and discussed here describe percolation on a finite lattice with a thin or thick barrier and are the continuation of the study presented in [2].

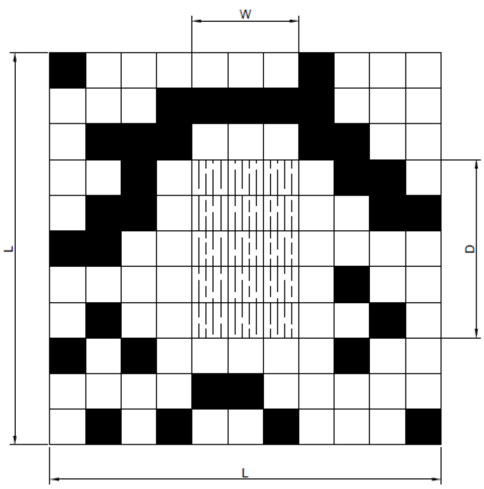

Fig. 1. Square grid $L\times L$ with hatched area denoting barrier of size $W\times D$. Black squares are occupied sites. Path from left to right side is spanning cluster created by randomly chosen sites

[1] Calculations were carried out in Wroclaw Centre for Networking and Supercomputing (http://www.wcss.wroc.pl), grant No. 151.

A grid with sites occupied with probability p is considered (Fig. 1). If p is small, we can see not many occupied sites with a few small groups of them. For larger values of p (when p = 1 every site is occupied), there are more groups of occupied sites. When the value of p is large enough, they can form a spanning cluster (it is a path between the left and right side of a grid). We describe long-range connectivity (L tends to infinity) by the probability of the appearance of spanning cluster P as a function of p.

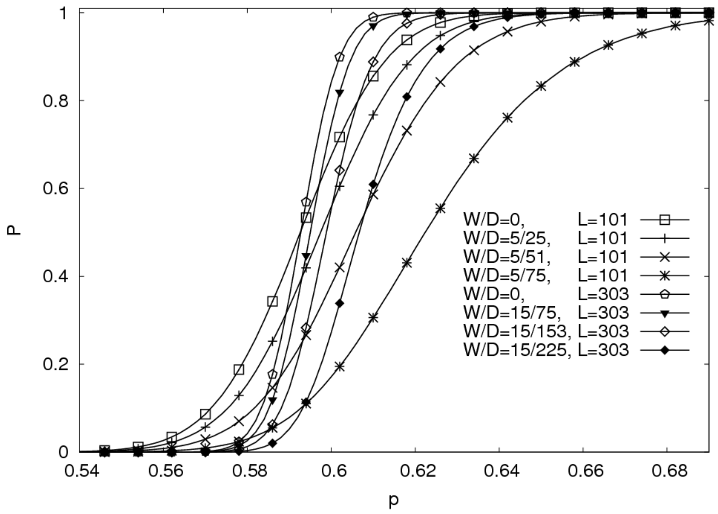

Fig. 2. Probability of appearance of spanning cluster P as function of occupancy probability p for grid size L without barrier and with thin barrier

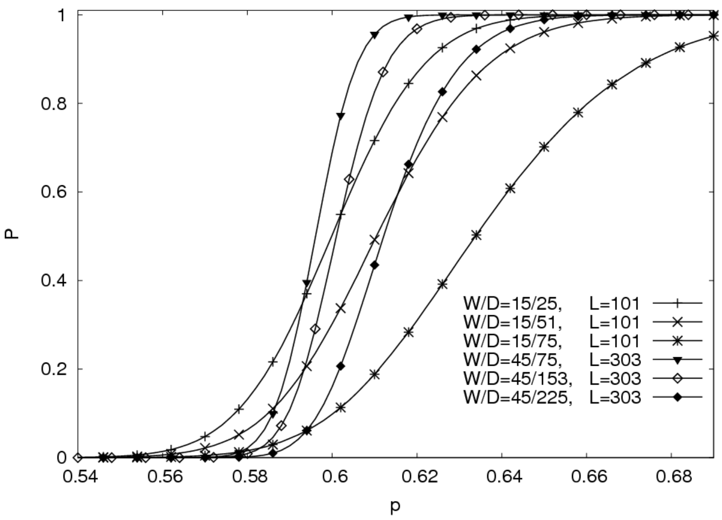

Fig. 3. Probability of appearance of spanning cluster P as function of occupancy probability p for grid size L and with thicker barrier

In Figure 2, we present the results for two cases - without and with a thin barrier (given as a symmetric set of sites with $p = 0$). As we can see for the smaller barrier, the spanning cluster forms faster. Figure 3 shows the percolation on grids with a thicker barrier. In the plots, we show the probability of spanning cluster P as a function of occupancy probability p for grid size L for three cascades with ratio $W*D/L^2$ equal to 0.036, 0.07 and 0.11 respectively.

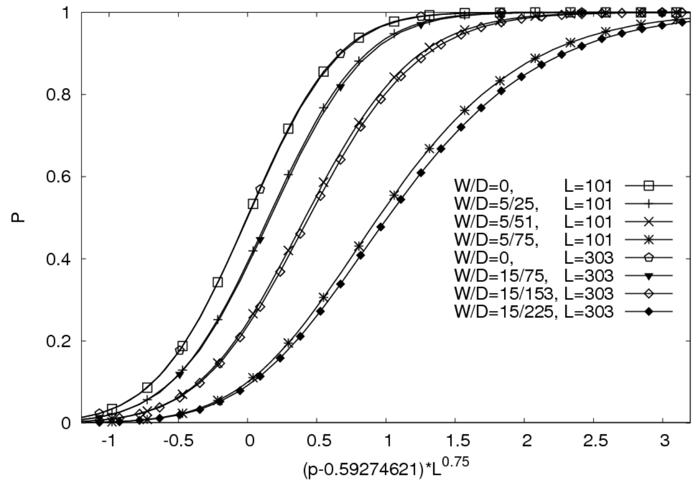

Fig. 4. Scalability of function *P(p)* for grid size *L* without barrier and with thin barrier

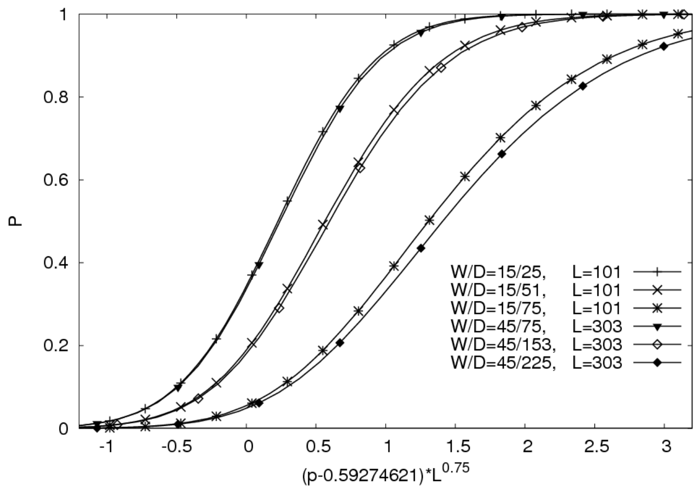

Fig. 5. Scalability of function *P(p)* for grid size *L* with thick barrier

In this study we also consider the behaviour of critical exponent ν by testing the probability of spanning cluster P as a function of a new variable, $(p - p_c)*L^{1/\nu}$. The percolation threshold (p_c) is a value of p for which the spanning cluster occurs for a grid with L tending to ∞. For the two-dimensional square lattice, the percola-percolation threshold is known: $p_c = 0.59274621$ [3]. In Figures 4 and 5 we adopted $1/\nu = 0.75$. Analyzing the scalability in the case without a barrier for two different L sizes, we observe that it agrees with the results known from the literature [4]. In the case when a barrier appears, the scalability with the standard critical exponent $1/\nu = 0.75$ is not so effective.

To improve the scalability for models on a finite grid with a barrier, it is necessary to choose the better critical exponent and to construct an algorithm which allows us to evaluate its effectiveness in scaling the plots.

1. Control algorithm

To improve the scalability for models with barriers and to find the best critical exponent, a special method called a control algorithm is proposed. Its construction is shown in Figure 6. To check the quality of a given ν, we need to estimate the distance between two plots given by two series of data. The first, understood as the base plot, is in our case the plot for a grid with $L = 101$ and the second plot is the plot for $L = 303$. We estimate the distance of two such plots from the same cascade determined by the constant value of ratio $W*D/L^2$. On the first plot we fix the points given by the data; the corresponding points connected to the second plot are understood as points determined using the linear interpolation between the points of the second plot according to Figure 6. Next, for the same arguments of both series of data, the distances between the corresponding points on the base plot and those determined by the interpolation are calculated. The maximal value of such distances is defined as δ. We apply this characteristic to control the effectiveness of scaling the plots of probability of spanning cluster P as a function of occupancy p with given critical exponent ν.

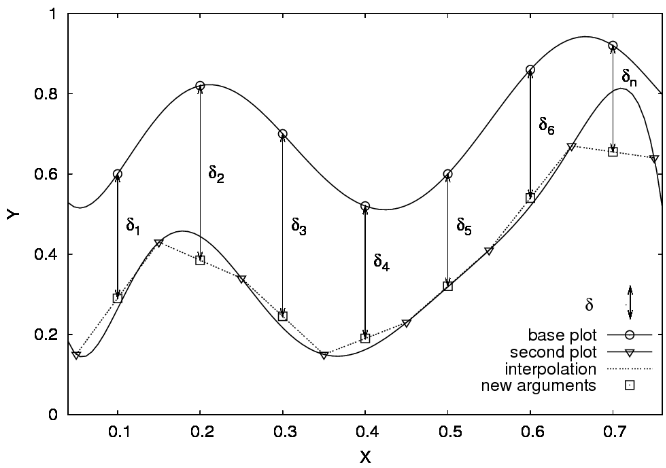

Fig. 6. Construction of control algorithm

In Figures 7 and 8 the scalability of function $P(p)$ for grid size L with barrier size $W*D$ and the best critical exponent is shown. Using the described control algorithm we obtain better scalability than in the case with $1/\nu = 0.75$ for a grid with barriers.

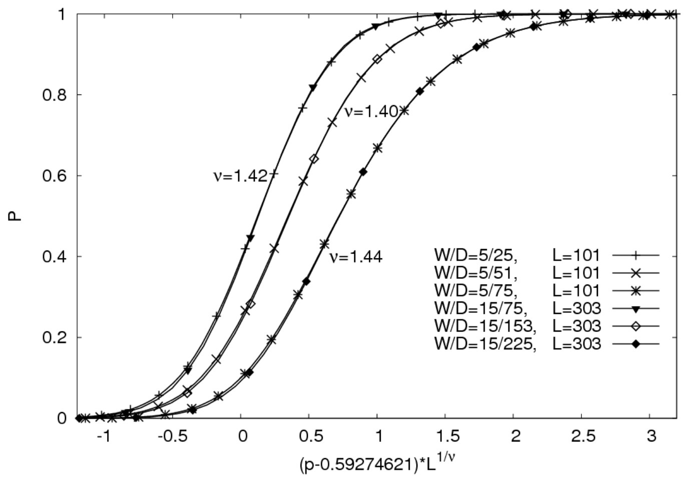

Fig. 7. Scalability of function *P(p)* for grid size *L* with thin barrier and with best critical exponent

In Table 1, we present the best critical exponent ν for every cascade (determined by constant ratio $P_P/P_L = W*D/L^2$) and δ - calculated using the control algorithm.

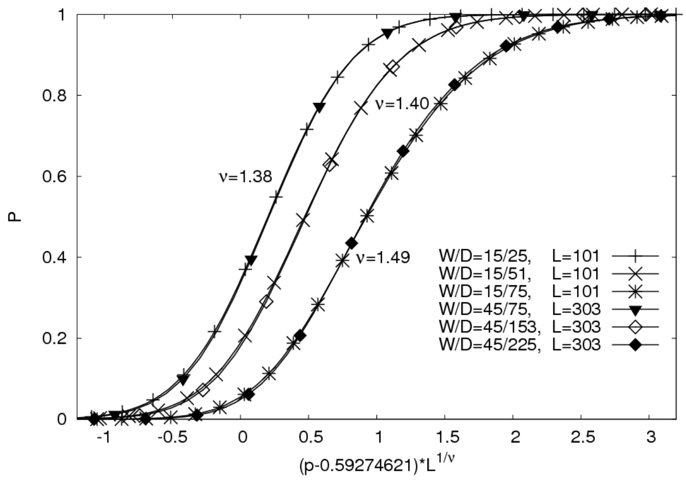

Fig. 8. Scalability of function $P(p)$ for grid size L with thick barrier and with best critical exponent

Table 1

Value of best critical exponent for all cascades and respective δ

$P_P/P_L = W^*D/L^2$	$1/\nu$	δ
0	0.75	0.00493
0.012	0.70	0.01101
0.024	0.71	0.00911
0.036	0.69	0.00929
0.036	0.72	0.00911
0.07	0.71	0.00865
0.11	0.67	0.01279

2. Final remarks

In this study we discussed the influence of the size of the symmetric barrier on the formation of a spanning cluster. The obtained results, scaled according to the standard approach, indicate that plots can be divided into subsets called here cascades and determined by ratio W^*D/L^2. In scaling the plots, we first applied the value of critical exponent known for grids without a barrier. The presented plots show that its effectiveness diminishes when the size of the barrier increases. We proposed to use different values of this exponent for the respective cascades. The constructed control algorithm allows us to estimate the effectiveness of the proposed scaling. Further investigations shall be devoted to the improvement of scalability, its evaluation and to develop a method to calculate effective critical exponents.

References

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