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# EFFECTIVE THERMAL CONDUCTIVITY OF TWO-DIMENSIONAL COMPACTED GRANULATES

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**Abstract**. This paper analyzes the heat transfer through systems formed by square grains with randomly assigned thermal conductivities subjected to different compactions. The efficiency of heat transport related to the density of the system is presented.

## Introduction

In engineering practice and theoretical research, we deal with the flows of different media, frequently realized as mass or energy currents. Such currents are observed in solid and liquid states of matter as well as in mixtures of granular materials. Frequently, they are accompanied by sound propagation, heat conduction or electric charge drifts. In the field of material science, microscopic properties are characterized by measurable intensive quantities. Among them, in the context of heat flows, so-called conductivity and efficient conductivity coefficients play a dominant role. Both these coefficients depend on material and geometrical factors, e.g. surface roughness, chemical composition and external force fields inducing various mechanical pressures.

From the theoretical point of view, battery approaches have been formulated to deal with heat properties evaluation. An efficient approach relies on effective medium approximation (EMA). In the framework of EMA, heat transport through a heterogeneous medium can be expressed analogously to the case of homogeneous media as

$$c(T)\frac{\partial T}{\partial t} = \lambda_{eff}\nabla^2 T \qquad (1)$$

Here, the random nature of the medium is expressed by $\lambda_{eff}$. This coefficient reflects a statistical measure of an ample multitude of possible system microscopic arrangements. The mathematical formulation of an appropriate stochastic problem corresponding to Eq. (1) is presented in [1, 2].

## 1. System compaction protocols

Extensive numerical experiments involving square-shaped grains have been realized according to two compaction protocols presented in Figure 1. For each of these two classes of experiments we constructed macroscopic samples using different grain densities, i.e. a frame with at least 50×50 boxes accessible for grains were filled with grains up to p = 0.1, 0.2, …, 0.9 occupied boxes. Two values of λ were randomly assigned to grains according to given two-point distributions and then the corresponding $\lambda_{eff}$ were estimated. Thus, for the two above-mentioned compaction geometries we have obtained statistical ensembles, each with about $10^5$ samples.

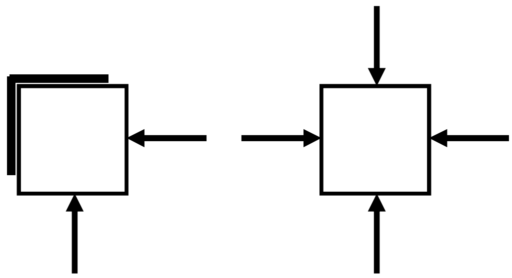

Fig. 1. Two protocols of compaction of system, arrows show direction of compaction of entire system

Figure 2 shows one of the members of the statistical ensemble for square-shaped grains and the distribution of the temperature within the macroscopic sample when the steady state is attained.

Fig. 2. Sample test system with λ = 10 and λ = 1 form macroscopic sample. Temperature distributions correspond to $p(\lambda = 10) = 0.5$. Number of square-shaped grains is 50%

The system is structurally heterogeneous with respect to the thermal conductivity coefficients. The detailed numerical procedures concerning the heat conductivity can be found in [2]. Compaction of the system (see Fig. 1), eliminates empty spaces between the neighbouring grains and thus this process yields a growing heat conduction.

### 1.1. Protocol I

According to the Protocol two neighbouring sides of the square are pressed sequentially in two steps: first vertically then horizontally. After each step, a small fraction of grains (typically 1%) randomly changes positions in order to eliminate mechanically unstable configurations. These position changes are allowed only in the close vicinity of the original temporal grain positions. The algorithm stops when no further compaction is possible.

Figure 3 shows the example of a system compacted under Protocol I. The resulting steady-state-temperature field distribution is also presented.

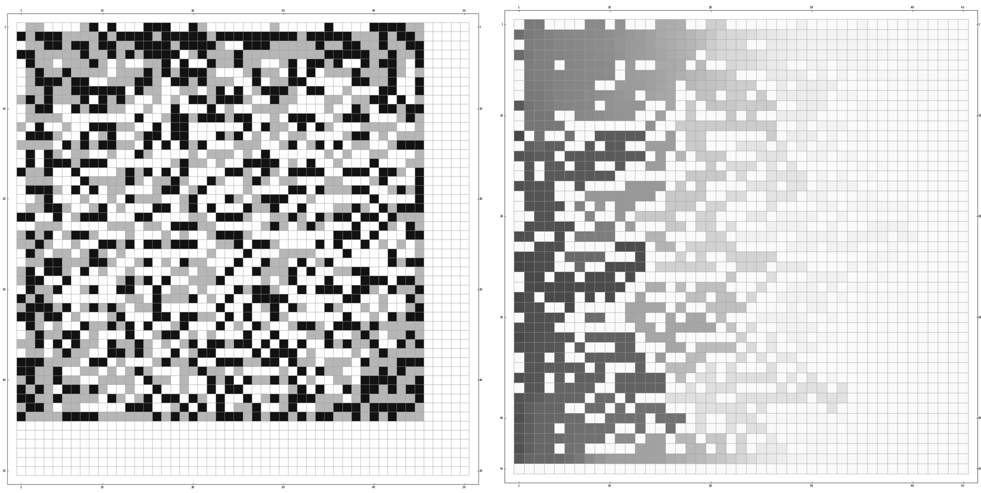

Fig. 3. Example system using concentrated Protocol I.
Temperature distributions correspond to: $p(\lambda = 10) = 0.5$
Number of square-shaped grains is 50%

### 1.2. Protocol II

By using Protocol two opposite sides of the square are pressed sequentially in two steps: first vertically then horizontally. As in Protocol I, after each step a small fraction of grains (typically 1%) randomly changes. The algorithm stops when no further compaction is possible.

Figure 4 shows the example of a system compacted under Protocol II. The resulting steady-state-temperature field distribution is also presented.

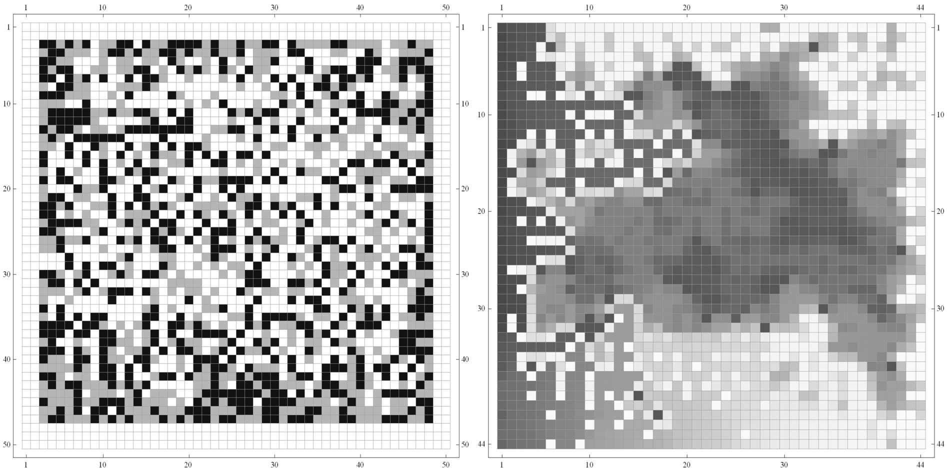

Fig. 4. Example system using concentrated Protocol II. Temperature distributions correspond to: $p(\lambda = 10) = 0.5$. Number of square-shaped grains is 50%

## 1.3. Comparison of results for different protocols

The presented compaction protocols used for the experiment are on sam- ples. The results of the density of the system are shown in Figure 5.

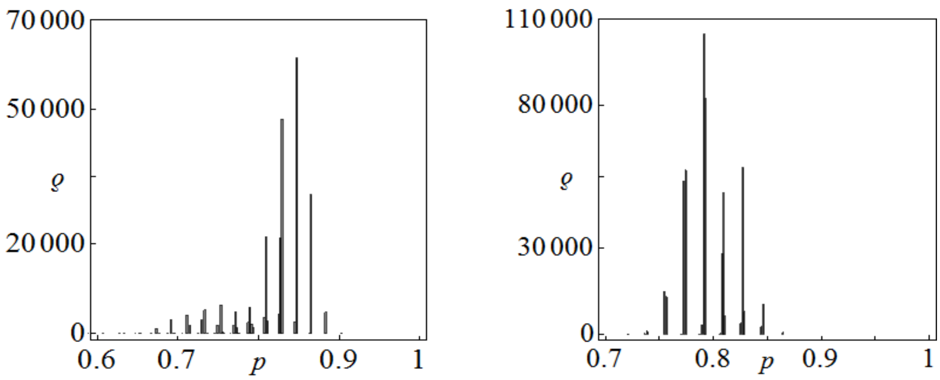


Fig. 5. Density of system for an array of 50×50 square grains with random values of $\lambda_1 = 10$ and $\lambda_2 = 1$, and $p(\lambda_2) = 0.5$ is computed with $5 \cdot 10^5$ samples. Number of square-shaped grains is 50%

As shown in the graphs, a much greater density of the system occurs after the application of Protocol II. Protocol II is therefore more efficient than Protocol I.

Figure 6 shows the results of the density of the system that are obtained by filling the system with the amount of square-shaped grains at 70 and 80%.

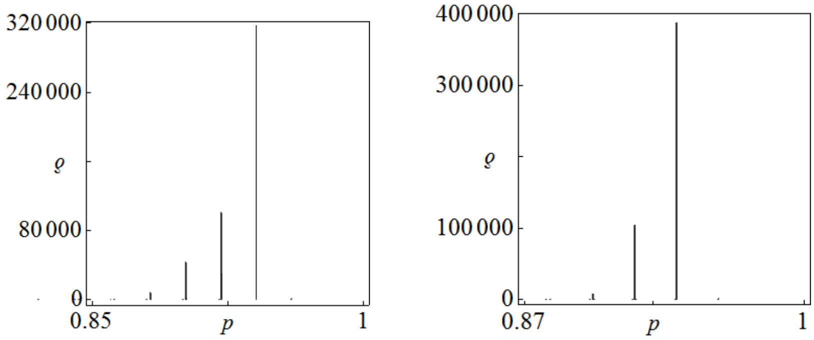


Fig. 6. Density of system for array of 50×50 square grains with random values of $\lambda_1 = 10$ and $\lambda_2 = 1$, and $p(\lambda_2) = 0.5\,\delta$ is computed with $5 \cdot 10^5$ samples. Number of square-shaped grains is 70 and 80%

## 2. Heat transfer

An ensemble of geometrically identical grains with random transport capabilities yields effective transport properties of the whole grain. The Thermal Particle Dynamics (TPD) simulation techniques we used allows us to compute transport properties of a granular system under a static condition [3-5]. The main advantage of TPD is that by incorporating contact conductance theories, simultaneous two-body interactions may be used to model the heat transfer in a system composed of many particles.

The heat transport system was calculated after the compaction algorithm. The coefficient of thermal conductivity for a system of empty seats was equal to $10^{-6}$. The results for both protocols allowed us to evaluate their performance.

The corresponding probability density $P(\lambda_{eff})$ is presented in Figure 7. On the basis of the graphs, it can be concluded that the application of Protocol II increases the efficiency of heat transfer.

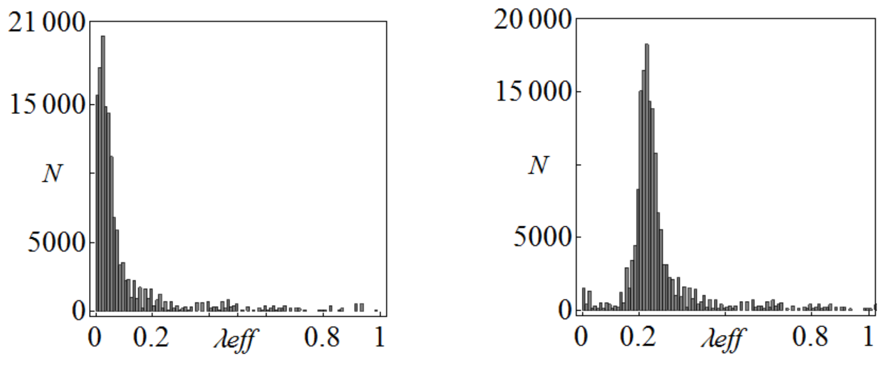


Fig. 7. Probability density $P(\lambda_{eff})$ for array of 10×10 square grains with random values of $\lambda_1 = 1$ and $\lambda_2 = 10$, and $p(\lambda_2) = 0.5\lambda_{eff}$ is computed with $5 \cdot 10^5$ samples. Number of square-shaped grains is 50%

## Conclusion

In this paper we have reported the results of the EMA approach to heat transfer through interfaces formed by square-shaped grains. The number of grains filling the system was different, therefore free spaces between grains appeared.

Two protocols of system compaction were examined numerically: the compaction of two sides and by simultaneous shrinking of four sides. The resulting distributions of the density of the system show that both protocols represent efficient methods of compaction.

Heat transport through heterogeneous granular systems is strongly dependent on the history of compaction and its efficiency increases when density of the system grows.

## References

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