THE MOTION OF SLAB IN CONTINUOUS CASTING OF STEEL TAKING INTO ACCOUNT SUPPORTING ROLLERS

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Abstract. An analysis of influence of supporting rollers on the vibration of slab in continuous casting of steel was carried out in the paper. The equation of motion for the slab taking into account effect of supporting rollers was formulated. An analysis of results obtained numerically was also made. The results were presented graphically.

Introduction

The analysed motion of the slab has been caused by the vibration of a caster. The caster has a given harmonic motion $v = \delta_0 \sin(\omega t)$, which effects the slab with a specified axial load $s(x,t)$, per unit length. This load is the result of the total normal interactions between the solidifying slab layer and the caster, and the friction conditions between the slab and the caster [1, 2]. This system consist of a supporting roller section below the below the caster which creates the homing channel. The rollers has an influence on the slab through a friction force $T(x,t)$ caused by moment of inertia of the rollers J_0 and local acceleration u_k of the slab. The mentioned above acceleration is caused by the slab vibrations. The withdrawal force of the slab is realised by a pulling roller. The rollers are pressed down to the slab with normal force and they produce the determined withdrawal force via the coefficient of rolling friction. The angular velocity of the rollers is chosen purposely to obtain the required velocity of the slab withdrawal w_0 (casting speed). It is assumed that solidified part of the slab is viscoelastic with Young's modulus E and coefficient of dynamic viscosity μ.
The axial force of the slab takes the form

$$N(x) = \sigma_x A = EA\left(\varepsilon_x + \frac{\mu}{E}\dot{\varepsilon}_x\right) = EA\left(\frac{\partial u}{\partial x} + \frac{\mu}{E}\left(\frac{\partial^2 u}{\partial t \partial x} + w_0 \frac{\partial^2 u}{\partial x^2}\right)\right) \tag{1}$$

Taking into account the above assumptions, one can obtain the equation of motion for forced vibrations in the form of differential heterogeneous equation:

$$-\frac{\partial}{\partial x}\left(EA\left(\frac{\partial u}{\partial x}+\frac{\mu}{E}\left(\frac{\partial^2 u}{\partial x \partial t}+w_0\frac{\partial^2 u}{\partial x^2}\right)\right)\right)+\rho A\left(\frac{\partial^2 u}{\partial t^2}+2w_0\frac{\partial^2 u}{\partial t \partial x}+w_0^2\frac{\partial^2 u}{\partial x^2}\right)=s(x,t)+T(x,t) \tag{2}$$

where:
ρ - density of the slab,
A - cross-section area of the slab.
The friction force, present at the contact of supporting roller and the slab, is given by relationship:

$$T(x,t)=\sum_k\left(Q_k\frac{f_k}{r_k}-J_0\frac{\ddot{u}_k}{r_k^2}\right) \tag{3}$$

where:
J_0 - moment of inertia of the roller,
f_k - coefficient of a rolling friction,
r_k - a radius of supporting roller,
k - a number of supporting rollers.

where: $\ddot{u}_k=\frac{\partial^2 u}{\partial t^2}\delta(x-x_k)$ and $\delta(x-x_k)$ - is Dirac delta function.

The boundary conditions of the equation are specified by the free end of the slab and its restrain by the pulling roller action, what gives:

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0}=0; \qquad \left.u(x,t)\right|_{x=L}=0 \tag{4}$$

The initial conditions are equal to:

$$\left.u(x,t)\right|_{t=0}=0, \qquad \left.\frac{du_0(x,t)}{dt}\right|_{t=0}=0 \tag{5}$$

The considered equation of motion of the slab - caster system is a partial differential equation of second order with appropriate unique conditions (geometrical, boundary and initial). An effective solution of the above equation, due to its complication, can be only obtained with the use of approximation methods [1, 3]. The commonly used Galerkin's method was applied in this paper. According to this method, the solution of equation (2) takes the shape of the following series:

$$u(x,t)=\sum_{i=1}^{\infty}S_i(t)U_i(x) \tag{6}$$

where: $U_i(x) = \cos\left(\left(i - \frac{1}{2}\right)\frac{\pi}{L}x\right)$ are basic functions chosen to fulfil given boundary conditions. The predicted solution is substituted into the motion equation (2), then consecutively multiplied by all the basic functions and finally integrated in the interval (0,L). Thus the equation of motion is transformed into a system of ordinary differential equations in relation to unknown time functions $S_i(t)$, in the following form:

$$\sum_{i=1}^{n} a_{ij}\ddot{S}_i = \sum_{i=1}^{n}\left(b_{ij}\dot{S}_i + c_{ij}S_i\right) + d_j \tag{7}$$

where $j = 1,2,..., n$
and stated coefficients are:

$$a_{ij} = \frac{L}{2} + \sum_k \frac{J_0}{r_k^2} U_j(x_k)U_i(x_k)$$

$$b_{ij} = \begin{cases} -\frac{\mu}{\rho}(2i-1)^2\frac{\pi^2}{8L} & i = j \\ 0 & i \neq j \end{cases}$$

$$c_{ij} = \begin{cases} \frac{E}{\rho}(2i-1)^2\frac{\pi^2}{8L} & i = j \\ 0 & i \neq j \end{cases}$$

$$d_j = \sum_k \frac{Q_k f_k}{r_k}\int_0^L U_j(x)dx + \int_0^L s(x,t)U_j(x)dx \tag{8}$$

1. Numerical solution of the system of ordinary differential equations

It follows from the conducted tests that only the two first terms of the series should be taken into account. The summing up of the other terms of the series (7) is practically meaningless, because those quantities are very small. The time functions are solved in relation to the second time derivatives and for the two first terms of series one can obtain:

$$\begin{aligned} \ddot{S}_1 &= G_{11}\dot{S}_1 + G_{12}\dot{S}_2 + H_{11}S_1 + H_{12}S_2 + K_1 \\ \ddot{S}_2 &= G_{21}\dot{S}_1 + G_{22}\dot{S}_2 + H_{21}S_1 + H_{22}S_2 + K_2 \end{aligned} \tag{9}$$

The coefficients, present in the above equation for the two first terms of the series, can be written as:

$$G_{11} = \frac{b_{11}a_{22}}{a_{11}a_{22} - a_{12}a_{21}} \qquad G_{12} = \frac{-b_{22}a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$G_{21} = \frac{-b_{11}a_{12}}{a_{11}a_{22} - a_{12}a_{21}} \qquad G_{22} = \frac{b_{22}a_{11}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$H_{11} = \frac{c_{11}a_{22}}{a_{11}a_{22} - a_{12}a_{21}} \qquad H_{12} = \frac{-c_{22}a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

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$$K_1 = \frac{d_1a_{22} - d_2a_{21}}{a_{11}a_{22} - a_{12}a_{21}} \qquad K_2 = \frac{d_2a_{11} - d_1a_{12}}{a_{11}a_{22} - a_{12}a_{21}} \tag{10}$$

Substituting the dependences: $S_1 = y_1;\ S_2 = y_2;\ \dot{S}_1 = \dot{y}_1 = y_3;\ \dot{S}_2 = \dot{y}_2 = y_4$, into the system (9) one can transform the system of second order differential equations into the system of the first order differential equations:

$$\begin{cases} \dot{y}_1 = y_3 \\ \dot{y}_2 = y_4 \\ \dot{y}_3 = G_{11}y_3 + G_{12}y_4 + H_{11}y_1 + H_{12}y_2 + K_1 \\ \dot{y}_4 = G_{21}y_3 + G_{22}y_4 + H_{21}y_1 + H_{22}y_2 + K_2 \end{cases} \tag{11}$$

Such a system is solved numerically with the use of the Runghe - Kutha - Merson method. Unknown time functions and their derivatives are determined, and the displacement of the slab in the arbitrary cross-section is calculated according to the equation (6).

2. Examples of computations

The system of slab - caster is considered in the paper. The casting mould has given oscillatory motion $v = \delta_0 \sin(\omega t)$ while the force from caster in the form of concentrated force, applied in a ξ distance from the free end of the slab, is defined by the equation

$$s(x,t) = F(t) \cdot \delta(x - \xi) \tag{12}$$

The course of the concentrated force with respect to time is determined on the basis of analysis of slab and caster interaction. The above force is dependent on caster velocity towards the slab (relative velocity $v - w_0$). Position ξ equals $\frac{2}{3}L_{kr}$.

The value of an exciting force is represented by an expression

$$F(t) = \frac{1}{2} f(t) g \rho L_{kr}^2 \frac{a}{2} \tag{13}$$

where: $f(t) = f_s \operatorname{sgn}(\dot{\delta} - w_0)$, ρ - is the density of the slab material, g - gravitational acceleration, f_s - coefficient of a dry friction, α - cross dimension of the slab. Young's modulus and coefficient of viscosity, occurred in the above equation are constant and different from zero. Numerical computations were made for given values of system parameters: ρ = 7850 kg/m^3, δ_0 = 0.01 m, ω = 1.2 rad/s, μ = = $5 \cdot 10^7$ Ns/m^2, L = 8.0 m, L_{kr} = 0.7 m, f_s = 0.1, f_k = 0.0025, r_k = 0.1, J_0 = 0.3083 kg · m, w_0 = 0.008 m/s, α = 0.14 m. The displacements for different cross-sections of the slab in the system without rollers and with three additional supporting rollers placed in 1, 2 and 3 m from metal surface, were determined. The displacements, defined in x = 0.7 m cross-section at outlet of the slab from caster, are presented in Figures 1 and 2.

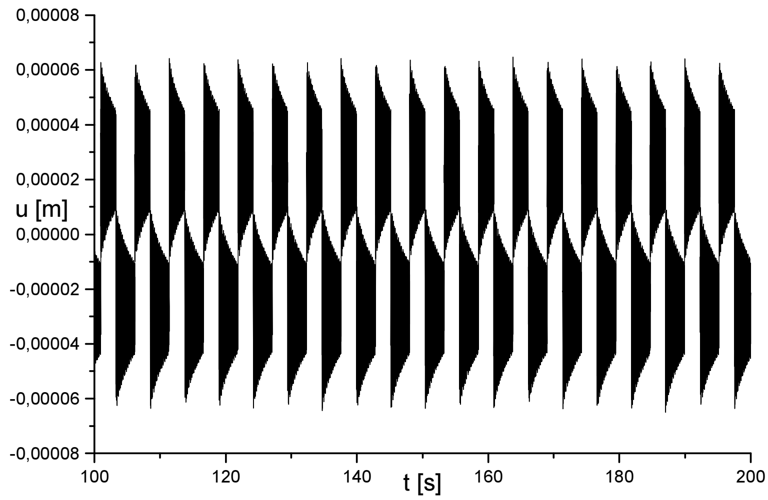

Fig. 1. The displacement of the slab for the system without supporting rollers

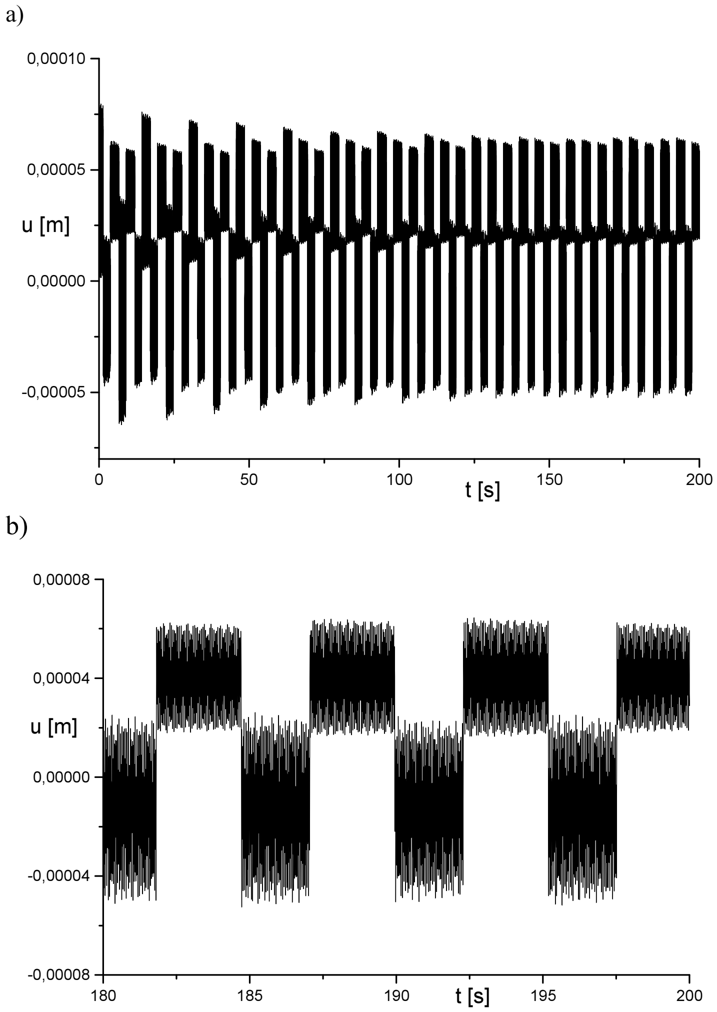

Fig. 2. The displacement of the slab for the system with three supporting rollers:
a) for the first 200 s, b) for steady motion

References

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