POWER LAWS IN FINITE ISING SYSTEMS

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Abstract. The density-matrix renormalization-group method has been applied to infinite Ising strips of finite widths. In the presence of the small external magnetic field the infinite system critical power laws can be observed. The single power law describes the field dependence of the magnetization or the longitudinal correlation length only on the infinite system critical isotherm.

Introduction

Continuous phase transitions and associated critical phenomena are understood at the thermodynamic limit of infinite systems as a singularity in the free energy [1]. Distinctive features of critical phenomena are power laws which can be ascribed to diverging length scales. The free energy of a finite system is an analytical function. Thus, a phase transition cannot take place in a finite system, and one cannot expect in such a system strict power laws in consequence. The question is, if one can define in a reasonable way the critical region for a trully finite system or system which exhibits a quasi-one-dimensional behaviour and does not exhibit finite temperature phase transition

$$\mathrm{H} = -J \sum_{<i,j>} S_i S_j - H \sum_i S_i \tag{1}$$

The properties of this model depend on three variables: temperature (T), external field (H), and strip width L or in the reduced variables on $\tau = (T_c - T)/T_c$, $h = H/J$ and L. The standard scaling form for the magnetization is

$$m \approx h^{1/\delta} f(\tau / h^{1/\beta\delta}, \tau / L^{-1/\nu}) \tag{2}$$

where the scaling function f has the limiting behaviour $f \to$ const as $\tau \to 0$ and $\tau L^{1/\nu} \to$ const. The last condition ensures that the strict power law is recovered when $L \to \infty$. However, in practise, the „correct power laws" can be observed in a very good approximation also for a finite system, for example for an infinite strip of width L, if the longitudinal correlation length $\xi_{\|}$ is large enough but still smaller than the width of a strip ($\xi_{\|} < \mathrm{L}$). Thus, one can try to define the critical region as a range over which the correct infinite system power laws are observed in a finite system with a desirable accuracy.

We use the density-matrix renormalization group (DMRG) based on the transfer matrix approach [2] to find the shape of the critical region of the Ising strip of width L up to 400 lattice spacing with free boundaries in the external magnetic field at several temperatures. We have found the magnetization profiles, specific heat, and longitudinal correlation length [3].

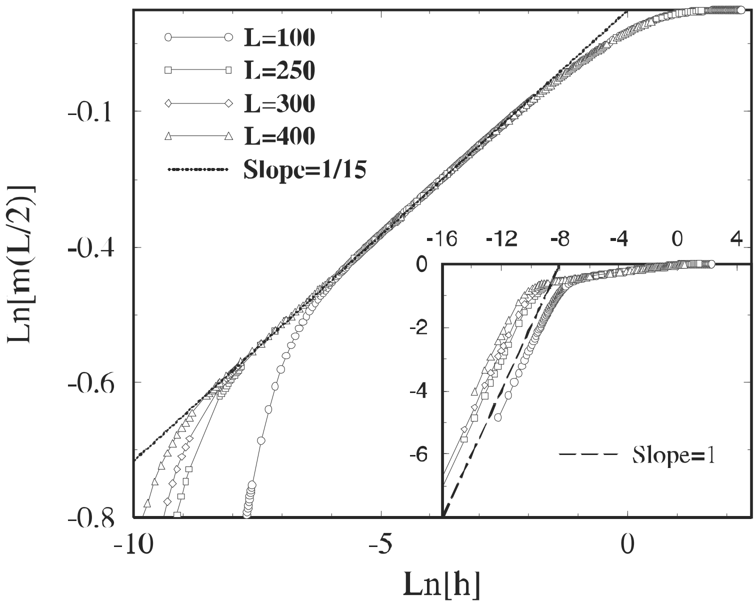

Fig. 1. Log-log plot of the magnetization in the center of a strip as a function of the magnetic field at the infinite system critical temperature $T = T_c$ for several strip widths. Inset: the linear response of the magnetization for the weakest magnetic fields

1. Magnetization at $T = T_c$

In Figure 1 the log-log plots of the magnetization in the center of a strip $m_{L/2}$ at the bulk critical temperature $T = T_c \approx 2.269$ as a function of the magnetic field for the strips of different width are presented. It is seen that even for the relatively small L there is a range of the field $h_{fs} < h < h_{cl}$ for which the infinite system power law $m \sim h^{1/\delta}$ with $\delta = 15$ is valid in the reasonable approximation. The lower field h_{fs} is connected with finite-size effects, and for $h < h_{fs}$ the magnetization $m_{L/2}$ varies linearly with h (see inset) [4]. The upper field h_{cl} is connected with the region where the single power law loses its validity for an infinite system. Of course, the upper field is L-independent, whereas the lower field decreases with L.

2. Longitudinal correlation length

We have found the inverse longitudinal correlation length from the formula [1]

$$\xi_{\parallel}^{-1} = \ln(\lambda_0 / \lambda_1) \tag{4}$$

where λ_0 and λ_1 are the largest and the second largest eigenvalues of the transfer matrix. Figure 2 shows that similarly as for the magnetization also for the correlation length there is a range in the field for which the infinite system power law $\xi_{\parallel} \sim h^{\upsilon h}$ with $\upsilon_h = 8/15$ is observed even for $L = 100$ at $T = T_c$. This range of the field is in agreement with the corresponding range of the field found for the magnetization.

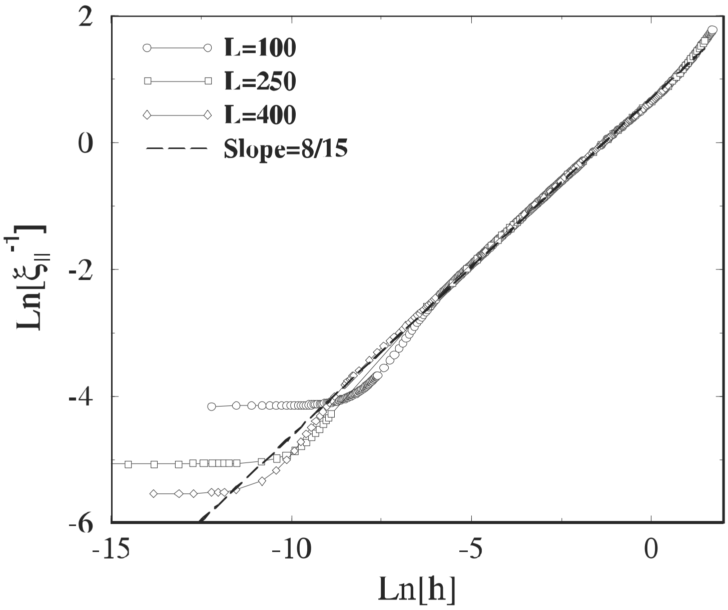

Fig. 2. Log-log plot of the inverse correlation length as a function of the field for several strip width at $T = T_c$. The dashed line corresponds to the infinite system critical exponent $\nu_h = 8/15$

3. Magnetization at $T \neq T_c$

In the thermodynamic limit of the infinite system the specific heat, susceptibility, and correlation length in the absence of the symmetry breaking field diverge at the well-defined critical temperature $T = T_c$. In a finite system and/or in the presence of the field all those values instead of singularities exhibit more or less pronounced maxima at different temperatures. These characteristic temperatures depend on L and h. However, the power law behaviour attributed to the infinite system criticality is visible in the reasonable approximation at $T = T_c$ for $h_{fs} < h < h_{cl}$ also for finite systems. One may ask if some power law behaviour in the similar approximation should be observed at the characteristic temperatures of the several thermodynamic values maxima.

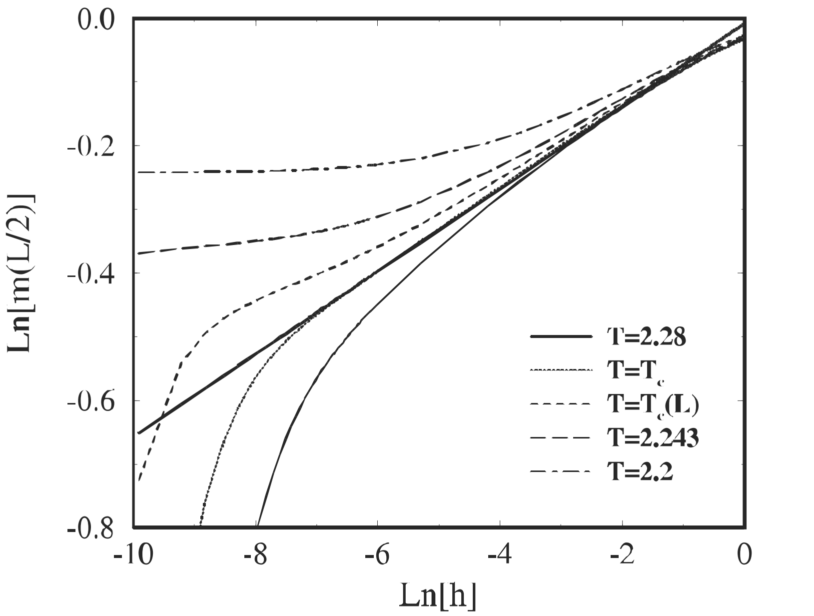

Fig. 3. Field dependence of the magnetization in the center of the strip of width $L = 200$ at several temperatures around the infinite system critical temperature T_c. The thick solid line is a guide for the eye

To be specific we shall concentrate on the field dependence of the magnetization at the temperatures in the vicinity of the infinite system critical temperature T_c, especially at the characteristic temperature of the specific heat maximum. The point is: if for the finite system the variation of the magnetization with field can be satisfactory fitted to a single power law beyond infinite system critical isotherm. In Figure 3 the field dependence of the magnetization for $L = 200$ at several temperatures around T_c is presented. $T_c(L) \approx 2.259$ denotes the characteristic temperature of the specific heat maximum for given L and $h = 0$. It is seen that even in a rather poor approximation it would be quite difficult to fit the results to a single power law at any temperature except for $T = T_c$.

Conclusions

From theoretical point of view the continuous phase transition, critical phenomena and associated strict power laws can be observed only in the thermodynamic limit of the infinite system. We have presented DMRG calculations which point out that the correct infinite system power laws can be observed in the reasonable approximation also in the infinite strip of width L even smaller than 100 lattice spacing provided an appropriate symmetry breaking field is applied. Although in such a case the power-law divergences of the infinite system are replaced by maxima, there is a range in the field for which the thermodynamic values vary according to the critical system power laws.

It is worth stressing that in the finite systems the power law behaviour can be observed in the reasonable approximation only on one isotherm - critical isotherm of the infinite system $T = T_c$.

References

[1] Yeomans J.M. (in:) Statistical Mechanics of Phase Transitions, Clarendon Press, Oxford 1992.

[2] Lectures Notes in Physics, ed. Peschel I., Wang X., Kaulke M., Hallberg K., Springer, Berlin 1999, Vol. 528; Schollwoeck U., Rev. Mod. Phys. 2005, 77, 259.

[3] Drzewiński A., Sznajd J., Szota K., The extension of these results can be found, Phys. Rev. 2005, B 72, 014441.

[4] Drzewiński A., Maciołek A., Ciach A., Phys. Rev. 2000, E 61, 5009.