SENSITIVITY ANALYSIS OF BIOHEAT TRANSFER IN HUMAN CORNEA SUBJECTED TO LASER IRRADIATION. PART 2: SHAPE VARIATION

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Abstract. The paper is continuation of part 1, in which the sensitivity analysis with regard to the thermophysical and optic parameters of human cornea have been discussed. Numerical analysis of thermal processes proceeding in the domain of human cornea subjected to laser irradiation is presented. Heat transfer in the tissue domain was assumed to be transient and one-dimensional. The internal heat sources resulting from laser irradiation based on the Beer law are taken into account. The paper deals with sensitivity analysis of temperature field with respect to the geometrical parameters of cornea. At the stage of numerical computations the boundary element method has been used. In the final part of paper the results of computations are presented.

1. Material derivative

In order to discuss shape sensitivity analysis the concept of material derivative has been applied. Using this concept we can write [1-3]

$$\frac{DT}{Db_s} = \frac{\partial T}{\partial b_s} + T_{,i} v^s \tag{1}$$

where $v^s = v^s(x, b_s)$ is the velocity associated with design parameter b_s. In the current analysis is assumed that $b_0 = L_0$ and $b_1 = L_1$, where L_0 and L_1 correspond to the external (anterior) and internal (posterior) surface of cornea.

Because (c.f. equation (1))

$$\frac{DT_{,i}}{Db_s} = \frac{\partial T_{,i}}{\partial b_s} + T_{,ii} v^s = \left(\frac{\partial T}{\partial b_s}\right)_{,i} + T_{,ii} v^s \tag{2}$$

and

$$\left(\frac{DT}{Db_s}\right)_{,i} = \left(\frac{\partial T}{\partial b_s} + T_{,i} v^s\right)_{,i} = \left(\frac{\partial T}{\partial b_s}\right)_{,i} + T_{,ii} v^s + T_{,i} v^s_{,ii} \tag{3}$$

therefore

$$\left(\frac{DT_{,i}}{Db_s}\right)=\left(\frac{\partial T}{\partial b_s}\right)_{,i}+T_{,i}v^s_{,i} \tag{4}$$

In similar way one obtains

$$\frac{D\dot{T}}{Db_s}=\frac{\partial}{\partial t}\left(\frac{DT}{Db_s}\right) \tag{5}$$

Using formula (4) we have

$$\frac{DT_{,ii}}{Db_s}=\frac{D\left[(T_{,i})_{,i}\right]}{Db_s}=\left(\frac{DT_{,i}}{Db_s}\right)_{,i}-T_{,ii}v^s_{,i} \tag{6}$$

and next

$$\frac{DT_{,ii}}{Db_s}=\left[\left(\frac{DT}{Db_s}\right)_{,i}-T_{,i}v^s_{,i}\right]_{,i}-T_{,ii}v^s_{,i}=\left(\frac{DT}{Db_s}\right)_{,ii}-2T_{,ii}v^s_{,i}-T_{,i}v^s_{,ii} \tag{7}$$

2. Shape sensitivity analysis - direct approach

If the direct approach of sensitivity method is applied then the Pennes equation [4-9] and the boundary initial conditions (see chapter: Governing equations in Part 1) are differentiated with respect to parameter b_s [8, 9].

Taking into account the bioheat transfer equation one has

$$x\in\Omega:\quad c\frac{D\dot{T}}{Db_s}=\lambda\frac{DT_{,ii}}{Db_s}+\frac{DQ_{las}}{Db_s} \tag{8}$$

Let assume that

$$U^s=\frac{DT}{Db_s},\quad \dot{U}^s=\frac{\partial U^s}{\partial t},\quad U^s_{,ii}=\frac{DT_{,ii}}{Db_s} \tag{9}$$

Taking into account the formulas (4), (5), (7) one has

$$c\dot{U}^s=\lambda\left[U^s_{,ii}-2T_{,ii}v^s_{,i}-T_{,i}v^s_{,ii}\right]+\frac{DQ_{las}}{Db_s} \tag{10}$$

From the Pennes equation (see equation (1) in Part 1) results that

$$\lambda T_{,ii} = c\dot{T} - Q_{las} \tag{11}$$

So the equation (10) takes a form

$$c\dot{U}^s = \lambda U^s_{,ii} - 2\left[c\dot{T} - Q_{las}\right]v^s_{,i} - \lambda T_{,i} v^s_{,ii} + \frac{DQ_{las}}{Db_s} \tag{12}$$

The last component of the equation (12) can be expressed as

$$\frac{DQ_{las}}{Db_s} = \frac{\partial Q_{las}}{\partial b_s} + Q_{las_{,i}} v^s \tag{13}$$

Taking into account a form of laser heat source (see equation (3) in Part 1) one has

$$Q_{las_{,i}} = \left[\mu_a I_0 \exp(-\mu_a x)\right]_{,i} = -\mu_a^2 I_0 \exp(-\mu_a x) \tag{14}$$

and

$$\frac{\partial Q_{las}}{\partial b_s} = \frac{\partial}{\partial b_s}\left[\mu_a I_0 \exp(-\mu_a x)\right] = -\mu_a^2 I_0 \exp(-\mu_a x)\frac{\partial x}{\partial b_s} = Q_{las_{,i}} \frac{\partial x}{\partial b_s} \tag{15}$$

Using equations (14) and (15) one obtains

$$\frac{DQ_{las}}{Db_s} = Q_{las_{,i}} \left(\frac{\partial x}{\partial b_s} + v^s\right) \tag{16}$$

We can write the final form of equation (12) as [5, 6]

$$c\dot{U}^s = \lambda U^s_{,ii} + Q^s_V \tag{17}$$

where a sensitivity source function Q^s_V is defined as

$$Q^s_V = -2\left[c\dot{T} - \mu_a I_0 \exp(-\mu_a x)\right]v^s_{,i} - \lambda T_{,i} v^s_{,ii} - \mu_a^2 I_0 \exp(-\mu_a x)\left(\frac{\partial x}{\partial b_s} + v^s\right) \tag{18}$$

Differentiating the boundary conditions on the anterior (external) surface of cornea we obtain (see equation (4) in Part 1)

$$x = L_0: \quad -\lambda \frac{DT_{,i}}{Db_s} = \alpha \frac{DT}{Db_s} + \varepsilon\sigma \frac{D\left(T^4\right)}{Db_s} \tag{19}$$

or (c. f. equation (4))

$$x = L_0: \quad -\lambda\left[U^s_{,i} + T_{,i}v^s_{,i}\right] = \alpha U^s + 4\varepsilon\sigma T^3 U^s \tag{20}$$

and finally

$$x = L_0: \quad Q^s = \alpha U^s + 4\varepsilon\sigma T^3 U^s + q v^s_{,i} \tag{21}$$

In similar way, the boundary condition on the posterior (internal) surface of cornea is differentiated

$$x = L_1: \quad Q^s(x,t) = Q^s_1 = q_1 v^s_{,i} \tag{22}$$

It should be pointed out that in equations (21) and (22) Q^s is defined as [8, 9]

$$Q^s = -\lambda U^s_{,i} n_i \tag{23}$$

To obtain form of initial condition once more the material derivative is used (c.f. equation (1)) [8, 9]

$$\frac{DT_p}{Db_s} = \frac{\partial T_p}{\partial b_s} + T_{p,i} v^s \tag{24}$$

and next

$$t = 0: \quad U^s_p = T_{p,i} v^s \tag{25}$$

It should be pointed out that velocities associated with design parameters are defined as [1, 2, 8, 9]

$$x \in \Omega: \quad \begin{cases} v^0(x, b_s) = \dfrac{L_1 - x}{L_1 - b_s}, & b_s = L_0 \\ v^1(x, b_s) = \dfrac{x - L_0}{b_s - L_0}, & b_s = L_1 \end{cases} \tag{26}$$

so their derivatives are

$$v^0_{,i} = \frac{-1}{L_1 - b_0}, \quad v^0_{,ii} = 0 \tag{27}$$

and

$$v^1_{,i} = \frac{1}{b_1 - L_0}, \quad v^1_{,ii} = 0 \tag{28}$$

3. Results of computations

As in Part 1, the 1D task has been taken into account, and at the stage of numerical computations the 1st scheme of boundary element method has been applied [10]. The BEM has been used both to calculate the temperatures field and sensitivities functions of design parameters L_0 and L_1. All data has been assumed the same as previously in Part 1.

In Figures 1 and 2 the sensitivities functions of investigated parameters are presented. In both cases sensitivity values are multiplied by $\Delta L_0 = \Delta L_1 = 0.6$ μm.

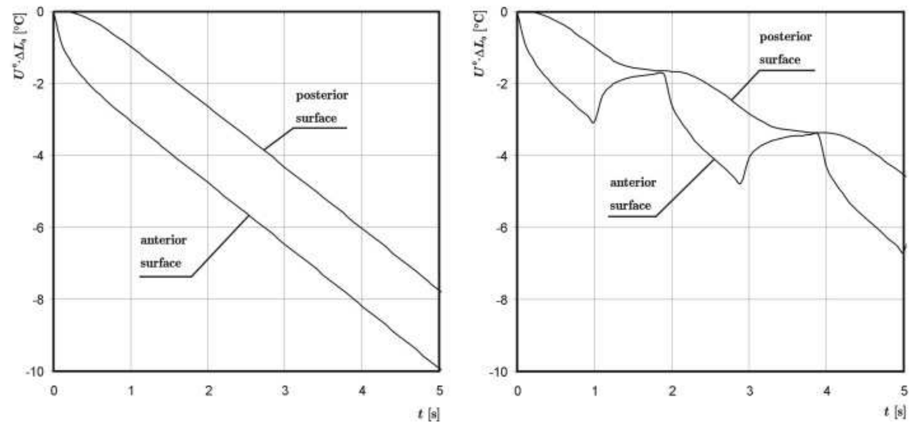

Fig. 1. Course of sensitivity function for design parameter L_0

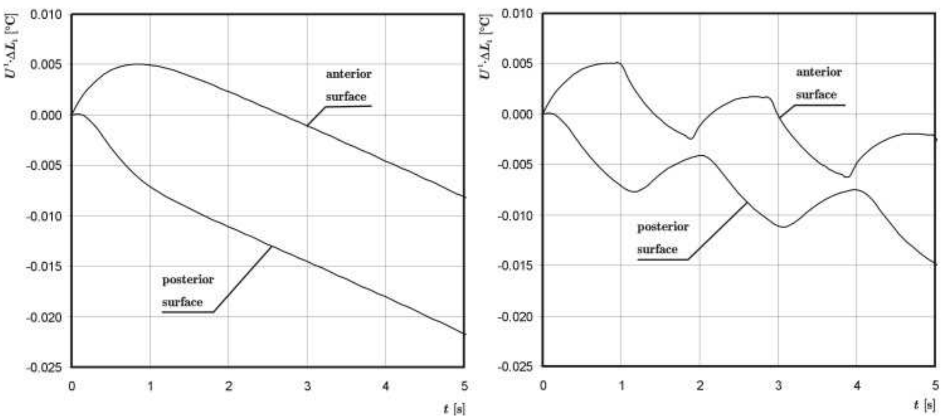

Fig. 2. Course of sensitivity function for design parameter L_1

Final remarks

The sensitivity studies show that the greatest influence on temperature has the change of L_0 (Fig. 1). After 5 seconds the changes are up to −10°C on the anterior (external) surface of cornea and about −7°C on the posterior (internal) surface (for constant irradiation). For time-varying irradiation the changes of temperature are about −6°C and −4°C, respectively.

In the case of variation of L_1 (Fig. 2) corresponding to internal surface of cornea the differences are much smaller, but still visible. For constant irradiation they are from the scope from −0.055 to −0.02°C while for modulated irradiation from −0.015 to −0.002°C.

Similarly to conclusion from Part 1, one can find that also changes of geometrical parameters of cornea have visible effect in temperature level during laser irradiation.

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