NATURAL CONVECTION NUMERICAL MODEL BASED ON GFD METHOD

Adam Kulawik

Institute of Computer and Information Science, Czestochowa University of Technology, Poland
adam.kulawik@icis.pcz.pl

Abstract. Classical finite difference method (FDM) is typically used for solving Navier-Stokes (N-S) equation. However to obtain a solution on irregular grid of points other methods have to be applied. Generalized finite difference method (GDFM) is one of the methods that my be used to solve mentioned problem. In this paper N-S and heat transfer equation have been solved using the GFDM. Results of numerical solutions for cooling processes with convection move in two-dimensional region are presented.

Introduction

Cooling process modeling for a number of technological processes (heat treatment, casting) requires the consideration of convective motion of the coolant. The presented model considers an area filled with liquid-metal medium and the convective motions caused by temperature changes. In most articles using numerical methods, in particular the finite differences method, the authors consider a simple geometry, discretized by rectangular grids. In this paper the numerical model, which does not require regular grids, based on a generalized finite element method is presented [1-3].

1. Mathematical model

Considered region contains two regions - steel element and liquid coolant. Heat from the coolant is transferred outside the boundary with using of appropriate boundary condition. Fluid motion in the coolant caused by vertical gradient of the temperature is taken into account. Initial liquid movement caused by putting the element into coolant is neglected.

The base of mathematical model consists of partial differential equations of Navier-Stokes, continuity and heat transport with convective term [3, 4-6]

$$\nabla \cdot (\lambda \nabla T) - \rho C \frac{\partial T}{\partial t} - \rho C \nabla T \cdot V = q_v \qquad (1)$$

where T [K] is the temperature, t [s] is the time, λ [W/mK] is the thermal conductivity, ρ [kg/m^3] is the density, C [J/kgK] is the specific heat, V [m/s] is the velocity, and q_v [W/m^3] is the volumetric heat source.

$$\left(\frac{\mu}{\rho}V_{\alpha},_{\beta}\right),_{l} - V_{\beta}V_{\alpha},_{\beta} - \frac{1}{\rho}p,_{\alpha} - g_{\alpha}\varepsilon\left(T - T_{ref}\right) = \dot{V}_{\alpha} \tag{2}$$

$$V_{\alpha},_{\alpha} = 0 \tag{3}$$

where V_α [m/s] is the velocity component in the α-direction, μ [kg/ms] is the dynamic viscosity, g_α [m/s^2] is the acceleration component in the α -direction, ε [K^{-1}] is the volumetric thermal expansion coefficient, T_{ref} [K] is the reference temperature, p [Pa] is the pressure.

The Equations (1), (2) are supplemented by appropriate boundary and initial conditions.

2. Numerical model

Heat transfer equation solved by generalized finite difference method using explicit time scheme, is written as

$$\begin{gathered} T_i^s = \left(1 - \frac{\lambda\Delta t}{\rho C}\sum_{j=1}^{n} z_j^{(3,4)}\right)T_i^{s-1} + \frac{\lambda\Delta t}{\rho C}\sum_{j=1}^{n} z_j^{(3,4)}\,T_j^{s-1} + \frac{\Delta t}{\rho C}q_{Vi} + \\ -\Delta t\left(\left(\sum_{j=1}^{n} z_j^{(1)}T_j^{s-1} - \sum_{j=1}^{n} z_j^{(1)}T_i^{s-1}\right)\left(V_x\right)_i^{s-1} + \left(\sum_{j=1}^{n} z_j^{(2)}T_j^{s-1} - \sum_{j=1}^{n} z_j^{(2)}T_i^{s-1}\right)\left(V_y\right)_i^{s-1}\right) \end{gathered} \tag{4}$$

where z_j are coefficients of approximation of derivatives for GDFM [3].

Above scheme is stabilized, if Δt is limited by [3]

$$\Delta t \le \left(\frac{\rho C}{\lambda \sum z_j^{(3,4)}}\right) \tag{5}$$

To stabilize heat transfer equation the Streamline Upwind Petrov Galerkin method (SUPG) was used

$$z_{\alpha} = \left(1 - sign\left(z_{\alpha}\cdot v_{\alpha}\right)\zeta_{\alpha}\right)z_{\alpha} \quad \zeta_{\alpha} = \left|\frac{1}{\tanh\left(Pe_{\alpha}\right)} - \frac{1}{Pe_{\alpha}}\right| \tag{6}$$

where Pe is a local Peclet number defined as $Pe_\alpha = v_\alpha r_\alpha C/\lambda$, r_α is a characteristic size of element of grid in the α direction.

Navier-Stokes equation (2) is solved only in region filled with coolant with using characteristic based split (CBS) scheme. CBS is based on the projection method developed by Chorin [7] and described by Zienkiewicz and Codina [6]. In this method an auxiliary velocity field V^* is introduced [3, 7] to uncouple equations (2) and (3)

$$V_\alpha^* = \Delta t\left(\frac{\mu}{\rho}V_{\alpha},_{\beta\beta} - V_\beta V_{\alpha},_{\beta} - g_\alpha \varepsilon\left(T - T_{ref}\right)\right) + V_\alpha^{s-1} \tag{7}$$

Momentum equation was solved by GDFM using implicit time scheme for *i*-th node of grid

$$\begin{gathered}\left(1+\frac{\mu\Delta t}{\rho}\right)\sum_{j=1}^{n} z_j^{(3,4)} V_{i\alpha}^{*(s)} - \frac{\mu\Delta t}{\rho}\sum_{j=1}^{n} z_j^{(3,4)} V_{j\alpha}^{*(s)} = \\ \Delta t\left(-V_\beta^{(s-1)}\left(\sum_{j=1}^{n} z_j^{(\beta)} V_{\alpha j}^{s-1} - \sum_{j=1}^{n} z_j^{(\beta)} V_{\alpha i}^{s-1}\right) - g_\alpha \varepsilon\left(T - T_{ref}\right)\right) + V_\alpha^{s-1}\end{gathered} \tag{8}$$

Pressure by solution of following Poisson equation is obtained

$$p,_{\alpha\alpha} = \frac{\rho}{\Delta t} V_\alpha^*,_\alpha \tag{9}$$

Above equation for *i*-th node in GDFM convention takes following form

$$\begin{gathered}\sum_{j=1}^{n} z_j^{(3,4)} p_j - p_i \sum_{j=1}^{n} z_j^{(3,4)} = \\ \frac{\rho}{\Delta t}\left(\left(\sum_{j=1}^{n} z_j^{(1)} V_{xj}^* - \sum_{j=1}^{n} z_j^{(1)} V_{xi}^*\right) + \left(\sum_{j=1}^{n} z_j^{(2)} V_{yj}^* - \sum_{j=1}^{n} z_j^{(2)} V_{yi}^*\right)\right)\end{gathered} \tag{10}$$

The final velocity field is corrected by the pressure increment

$$\Delta V_\alpha^* = -\frac{\Delta t}{\rho}\left(p,_\alpha\right) \tag{11}$$

The solution of equation (11) in GDFM for *i*-th node is as follows

$$\Delta V_{\alpha i}^* = -\frac{\Delta t}{\rho}\left(\left(\sum_{j=1}^{n} z_j^{(\alpha)} p_{\alpha j} - \sum_{j=1}^{n} z_j^{(\alpha)} p_{\alpha i}\right)\right) \tag{11}$$

3. Examples of calculation

Temperature distribution and movement of coolant were analyzed for the following parameters: area of coolant 0.1 x 0.1 m, centrally located steel piece of circular cross section R = 0.01 m. Following boundary and initial conditions were introduced for heat transport equation: the Newton boundary condition on external boundary with α = 1000 W/m^{2}K, T_∞ = 375 K, the ideal contact between steel element and coolant on internal boundary, (IV boundary condition on internal boundary); the initial temperature of steel element was equal to T_H = 1500 K, the initial temperature of coolant T_C = 300 K. Material properties of steel C45 and liquid sodium for cooling element and coolant have been taken into account respectively.

Navier-Stokes equation was completed by the following boundary and initial conditions: Dirichlet boundary condition on boundaries V_α = 0 m/s, initial velocities in coolant V_α = 0 m/s.

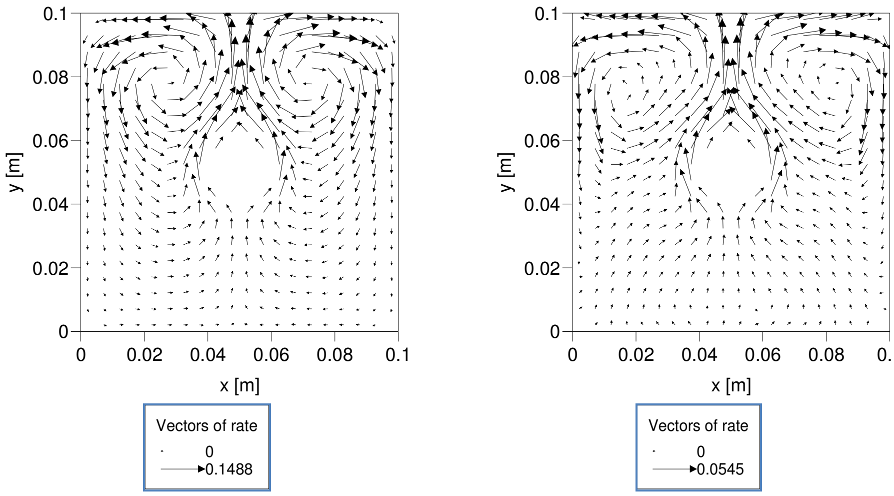

Fig. 1. Velocity fields of the coolant: a) t = 1 s, b) t = 5 s

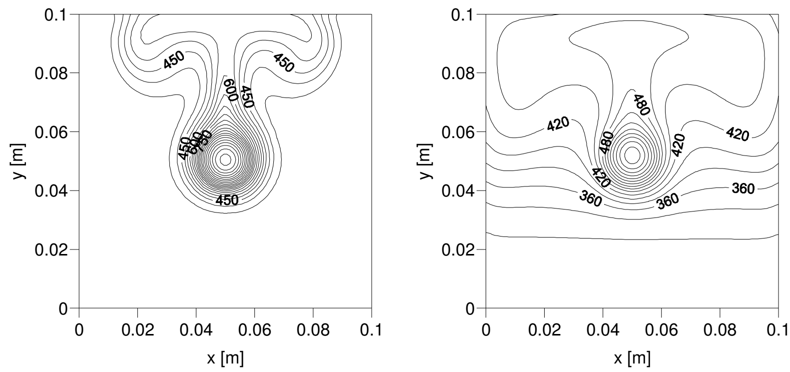

Fig. 2. Temperature distribution in the coolant: a) t = 1 s, b) t = 5 s

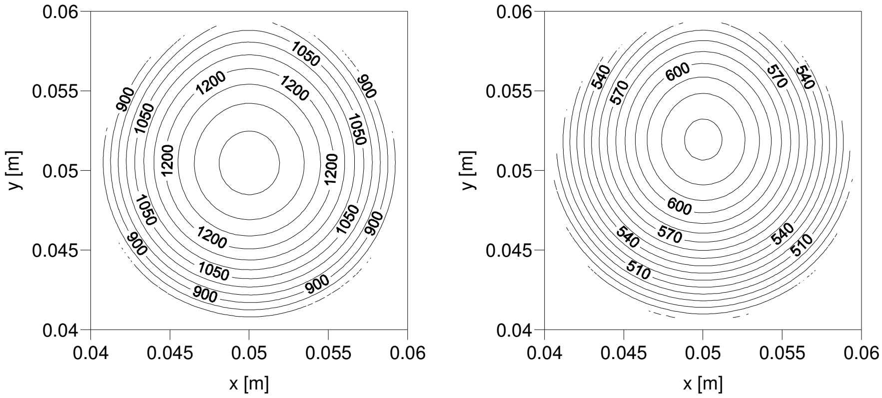

Fig. 3. Temperature distribution in the cooled element: a) t = 1 s, b) t = 5 s

Conclusions

Generalized finite difference is a method that can be successfully applied to irregular grids. It gives great opportunities when grid adaptation during calculations is performed. Because of those qualities GFDM is effective in the modeling processes for complex geometries. Presented model may be used to estimate of temperature fields during cooling process for tools of steel after solidification process. It may be adopted to optimization of heat treatment processes.

References

[1] Benitoa J.J., Ureńaa F., Gaveteb L., Solving parabolic and hyperbolic equations by the generalized finite difference method, Journal of Computational and Applied Mathematics 2007, 209, 208-233.

[2] Liszka T., An interpolation method for an irregular net of nodes, Internat. J. Numer. Methods Eng. 1984, 20, 1599-1612.

[3] Pawlak E., Application of GFDM to model of linear and nonlinear heat diffusion problems, PhD Thesis, Częstochowa 1999 (in Polish).

[4] Węgrzyn-Skrzypczak E., Modelling of solidification with motion of the fluid in liquid and mushy zone, PhD Thesis, Częstochowa 2005 (in Polish).

[5] Węgrzyn-Skrzypczak E., Skrzypczak T., Numerical Model of the Solidification of Alloys with Natural Convection of the Liquid, Arch. Foundry Eng 2008, 8(1), 109-112

[6] Zienkiewicz O.C., Codina R., A general algorithm for compressible and incompressible flow, Part I. The split characteristic based scheme, International Journal for Numerical Metods in Fluids 1995, 20:869-885.

[7] Chorin A.J., Numerical solution of the Navier-Stokes equation, Math. Comput. 1968, 23, 745--762.