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A NEW APPROACH TO COMPARING INTUITIONISTIC FUZZY VALUES

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Abstract. This paper presents a new approach to comparing intuitionistic fuzzy values. Score and accuracy functions are used to build the "net profit" and "risk" local criteria, which are aggregated in a generalized criterion taking into account the weights of the considered local criteria depending on the risk aversion of a decision maker. As opposed to known methods, the new approach makes it possible to estimate the strength of the relations between real-valued intuitionistic fuzzy values. Using some numerical examples, it is shown that the proposed approach provides intuitively clear results.

Introduction

The intuitionistic fuzzy set proposed by Atanassov [1], abbreviated here as $A-IFS$ (the reasons for this are presented in [2]), is one of the possible generalizations of fuzzy sets theory and appears to be relevant and useful in some applications. The concept of the $A-IFS$ is based on the simultaneous consideration of membership μ and non-membership ν of an element of a set to the set itself [1]. By definition $0 \le \mu + \nu \le 1$, notation $< \mu, \nu >$ is usually used for the presentation of intuitionistic fuzzy values.

An important characteristic of the $A-IFS$ is the so-called hesitation degree (or degree of uncertainty) which is defined as follows: $\pi = 1 - \mu - \nu$. Therefore $\pi + \mu + \nu = 1$.

It is clear that if $\pi = 0$ then the $A-IFS$ is reduced to ordinary fuzzy set $< \mu, 1 - \mu >$.

A similar approach, the so-called vague sets, proposed by Gau and Buehrer in [3] is proved to be equivalent to the $A-IFS$ in a formal mathematical sense (see [4]). Since vague sets were proposed later than the $A-IFS$, in the current paper, we shall always write about $A-IFS$.

There are many papers devoted to the theoretical problems of the $A-IFS$ in scientific literature (see [5] for an overview).

The most important applications of the $A-IFS$ are the decision making problem [6-12] and group decision making problem [13-20], when the values of the local criteria (attributes) of alternatives and/or their weights are intuitionistic fuzzy values (IFV). It seems quite natural that if the local criteria used in the formulation of a decision making problem are $IFVs$, then the resulting alternative evaluation should be an IFV as well. Therefore, there are many methods for aggregating local criteria in the $A-IFS$ setting proposed in the literature (see, e.g., [21-23]), which provide final scores in the form of $IFVs$. The most recent and comprehensive review of such methods is presented in [24].

If the final scores of alternatives are presented by $IFVs$, the problem of comparing of such values arises. Bustince and Burillo [25] analyzed the general properties of intuitionistic fuzzy relations and showed that the definition of these properties does not always coincide with the definition of the properties of fuzzy relations. Therefore, specific methods were developed to compare $IFVs$. For this purpose, Chen and Tan [6] proposed to use the so-called score function $S(x)=\mu(x)-\nu(x)$, where x is an IFV. Let a and b be $IFVs$. It is intuitively assumed that if $S(a)>S(b)$, then a should be greater (better) than b, but if $S(a)=S(b)$ this does not always mean that a is equal to b. Therefore, Hong and Choi [7] in addition to the above score function introduced the so-called accuracy function, $H(x)=\mu(x)+\nu(x)$, and showed that the relation between functions S and H is similar to the relation between mean and variance in statistics. Xu [26] used functions S and H to construct order relations between any pair of intuitionistic fuzzy values as follows:

$$\begin{aligned}
&If\,(S(a)>S(b)),\ then\ b\ is\ smaller\ than\ a;\\
&If\,(S(a)=S(b)),\ then\\
&(1)\ If\,(H(a)=H(b)),\ then\ a=b;\\
&(2)\ If\,(H(a)<H(b)),\ then\ a\ is\ smaller\ than\ b.
\end{aligned} \tag{1}$$

Based on these relations, Xu [26] introduced the concepts of an intuitionistic preference relation, consistent intuitionistic preference relation, incomplete intuitionistic preference relation and acceptable intuitionistic preference relation. The method for $IFVs$ comparison based on functions S and H seems to be intuitively obvious and this is its undeniable merit. On the other hand, as two different functions S and H are needed to compare $IFVs$, this method generally does not provide an appropriate technique for the estimation of an extent to which an IFV is greater/lesser than another one, whereas such information is usually important for a decision maker. This problem was discussed in [7, 11], where the heuristic methods for aggregating functions S and H were developed.

Therefore, in this paper, we propose a new two-criteria approach based on the real-valued score and accuracy functions which is free of the above-mentioned limitations of known methods for $IFVs$ comparison.

For these reasons, the rest of the paper is set out as follows. In the first Section, we analyze the limitations of known approaches to *IFVs* comparison based on method (1) and propose a new two-criteria method for comparing *IFVs*, which is free of these limitations. Illustrative numerical examples are presented as well. Finally, the concluding section summarizes the paper.

1. Two-criteria method for comparing intuitionistic fuzzy values

Let us start by analyzing the limitations of the known methods for *IFVs* comparison based on reasoning (1).

Let $A=\langle \mu_A,\nu_A\rangle$, $B=\langle \mu_B,\nu_B\rangle$ be *IFVs*. Then the score and accuracy functions for A and B are calculated as follows: $S_A=\mu_A-\nu_A$, $H_A=\mu_A+\nu_A$, $S_B=\mu_B-\nu_B$, $H_B=\mu_B+\nu_B$.

A score function is usually treated as a "net membership". Therefore if A is a local criterion in a decision making problem, then S_A may be treated as the "net profit" provided by A.

Accuracy function $H_A=\mu_A+\nu_A$ may be presented in its equivalent form $H_A=1-\pi_A$, where π_A is the hesitation degree or degree of uncertainty. Hence π_A may be treated as the degree of risk associated with "net profit" S_A. Therefore the following thinking may be justified: the smaller H_A is, the greater hesitation π_A is and, as a consequence, the smaller A is. There are three important limitations of method (1):

1) This method generally does not provide a technique for the estimation of a degree to which an *IFV* is greater/lesser than another one, whereas such information is usually important for a decision maker.
2) The lack of continuity in the comparison of *IFVs* by this method.

Let us consider the following critical example. For two *IFVs*, $A=\langle 0.5,0.3\rangle$ and $B=\langle 0.4,0.2\rangle$, we obtain: $S_A=0.2$, $S_B=0.2$, $H_A=0.8$, $H_B=0.6$. Since $S_A = S_B$ and $H_A>H_B$, using (1) we get $A>B$.

Let us introduce a slight modification of B in this example: $B'=\langle 0.4,0.1999\rangle$. Then we obtain: $S_A=0.2$, $S_{B'}=0.2001$. Since $S_A<S_{B'}$, taking into account (1), we are forced to conclude that $A<B'$, although the difference $S_{B'}-S_A=0.0001$ which can serve as an argument in favor of $A<B'$ is negligible in comparison to the difference $H_A-H_{B'}=0.2001$, which is the evidence for $A>B'$. Obviously, in the last case, it should be acknowledged that $A>B'$ if the accuracy function is not completely negligible in local criterion for the comparison of *IFVs*. In our opinion, the shown problems with method (1) are caused by the fact that when comparing $IFVs$, we deal with two local criteria: the “net profit”' represented by score function S and the “risk” criterion represented by accuracy function H. From this point of view, we can see that in method (1), the “risk” criterion is implicitly assumed to be of negligible importance, whereas the weight of this criterion depends on the risk aversion of the decision maker.

3) In method (1), the implicitly introduced local “net profit”' and “risk” criteria are not taken into account simultaneously.

Therefore, to avoid the above mentioned limitations of known methods, we propose to formulate the problem of $IFVs$ comparison directly as a two-criteria task. In the new method, possibilities $P(A > B)$ and $P(A < B)$ are calculated to indicate when the IFV is greater and to obtain the strength of inequality.

For two $IFVs$ A and B, we denote $\Delta S = S_A - S_B$ and $\Delta H = H_A - H_B$ and introduce two functions, $\mu_{\Delta S}(\Delta S)$ and $\mu_{\Delta H}(\Delta H)$, representing the local “net profit” and “risk” criteria respectively.

These functions

$$\mu_{\Delta S}(\Delta S) = \frac{\Delta S + 2}{4}, \mu_{\Delta H}(\Delta H) = \frac{\Delta H + 2}{4} \quad (2)$$

defined in intervals $-2 \le \Delta S \le 2$ and $-2 \le \Delta H \le 2$ are shown in Figure 1.

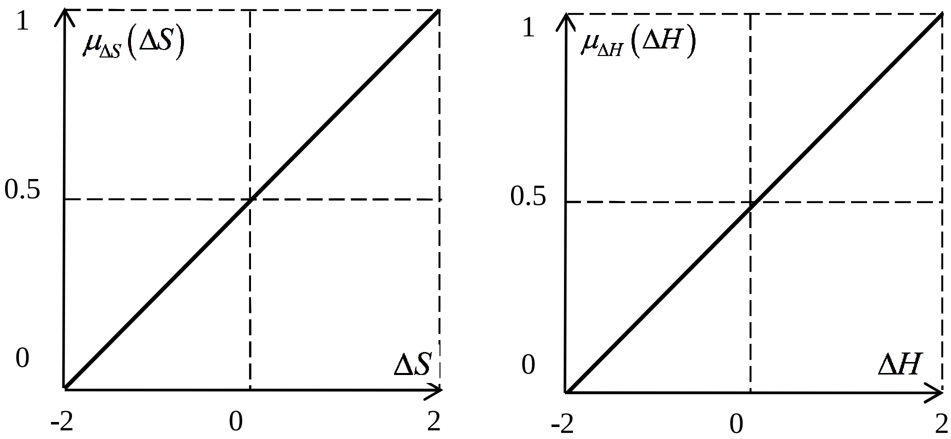

Fig. 1. Local criteria

Functions $\mu_{\Delta S}(\Delta S)$ and $\mu_{\Delta H}(\Delta H)$ can be naturally treated as the local criteria and should be aggregated (taking into account their weights) to obtain the final evaluation of the possibility that an IFV is greater/lesser than another one.

There are many approaches to the aggregation of local criteria proposed in the literature. Generally, the choice of an appropriate method for aggregation is a context dependent problem. Since in our case we assume that a small value of local criterion based on $\Delta S = S_A - S_B$ may be partially compensated by a large value of criterion based on $\Delta H = H_A - H_B$, the weighted sum seems to be the most suitable aggregating mode.

Then possibilities $P(A > B)$ and $P(A < B)$ can be presented as aggregations of the introduced local criteria:

$$P(A > B) = \alpha\mu_{\Delta S}(S_A - S_B) + (1-\alpha)\mu_{\Delta H}(H_A - H_B)$$

$$P(B > A) = \alpha\mu_{\Delta S}(S_B - S_A) + (1-\alpha)\mu_{\Delta H}(H_B - H_A) \qquad (3)$$

where $0 \le \alpha \le 1$ is the weight, which depends on the risk aversion of the decision maker.

Functions (2) and possibilities (3) are constructed in such a way that if $P(A > B) > P(B > A)$, then $A > B$ and

$$ST(A > B) = P(A > B) - P(B > A) \qquad (4)$$

is the strength of this inequality.

It is easy to see that in the case of equality ($S_A = S_B$, $H_A = H_B$) from (3), we get $P(A > B) = P(B < A) = 0.5$. We shall expose the features of the proposed method using the examples presented below. To make the obtained results comparable (at least on the qualitative level) to those obtained using method (1), in all examples we shall use α=0.98, i.e., we suppose that the “net profit” criterion is much more important that the “risk” criterion. It is easy to see that method (1) is implicitly based on this assumption.

Example 1. Consider the above critical example. Let $A = \langle 0.5, 0.3\rangle$ and $B = \langle 0.4, 0.2\rangle$. Then from (3) and (4) we get $P(A > B) = 0.501$, $P(B > A) = 0.499$ and $ST(A > B) = 0.002$. Therefore, $A > B$ with a strength equal to 0.002. After a slight modification of B in this example: $B' = \langle 0.4, 0.1999\rangle$, we get $P(A > B') =$ $= 0.506775$, $P(B' > A) = 0.499024$ and $ST(A > B') = 0.00775$. Therefore, in this case we have $A > B'$ with a small strength equal to 0.00775. As noted above, this is a more justified result than the one obtained using (1), i.e., $A < B'$.

Example 2. Consider $A = \langle 0.4, 0.1\rangle$ and $B = \langle 0.3, 0.6\rangle$. Then from (1) we get $A > B$. Using our approach we obtain $P(A > B) = 0.645$, $P(B > A) = 0.355$ and $ST(A > B) = 0.29$. Therefore, $A > B$ with a strength equal to 0.29.

In this example, the great strength is caused by the great difference between S_A and S_B.

Using the following two examples, we show that the proposed approach to $IFVs$ comparison is transitive on the quantitative level.

Example 3. For $A = \langle 0.5, 0.3 \rangle$ and $B = \langle 0.4, 0.2 \rangle$ from (1), we get $A > B$ and using our approach, we obtain the same result with $ST(A > B) = 0.02$.

For $A = \langle 0.5, 0.3 \rangle$ and $B' = \langle 0.4, 0.15 \rangle$ from (1), we get $B' > A$ and using our approach, we obtain $P(A > B') = 0.489$, $P(B' > A) = 0.511$ and $ST(B' > A) = 0.022$.

For $A = \langle 0.5, 0.3 \rangle$ and $B'' = \langle 0.4, 0.1 \rangle$ from (1), we get $B'' > A$ and using our approach, we obtain $P(A > B'') =$ 0.477, $P(B'' > A) =$ 0.523 and $ST(B'' > A) =$ $= 0.046$.

For $B' = \langle 0.4, 0.15 \rangle$ and $B'' = \langle 0.4, 0.1 \rangle$ from (1), we get $B'' > B'$ and using our approach, we obtain $P(B' > B'') =$ 0.488, $P(B'' > B') =$ 0.512 and $ST(B'' > B') =$ $= 0.024$.

If our approach to *IFVs* comparison is transitive, then the strength of $B'' > B'$ should be close to the difference of $ST(B'' > A) - ST(B' > A) = 0.24$. Since we have obtained $ST(B'' > B') = 0.024$, we can say that the proposed method is a transitive one.

Example 4. Consider $A = \langle 0.6, 0.4 \rangle$, $B = \langle 0.4, 0.1 \rangle$ and $C = \langle 0.3, 0.2 \rangle$. Then from (1) we get $B > A > C$ and using our approach we obtain

$$P(B > A) = 0.522,\ P(A > B) = 0.478 \text{ and } ST(B > A) = 0.044,$$

$$P(A > C) = 0.527,\ P(C > A) = 0.473 \text{ and } ST(A > C) = 0.054,$$

$$P(B > C) = 0.549,\ P(C > B) = 0.451 \text{ and } ST(B > C) = 0.098.$$

Since $ST(B > A) =$ 0.044 and $ST(A > C) =$ 0.054 we can expect that the strength of $B > C$ should be close to $ST(B > A) + ST(A > C) =$ 0.098. As in the considered case we have obtained $ST(B > C) =$ 0.098, we can conclude that our approach to the *IFVs* comparison is practically transitive on the quantitative level.

Summarizing we can say that the proposed approach to *IFVs* comparison is free of limitations of known method (1) and provides transitive quantitative assessments of a degree to which an *IFV* is greater/lesser than another one.

Conclusion

The two-criteria approach to comparing real-valued intuitionistic fuzzy values is developed. The first local criterion termed “net profit” is based on the real-valued score function in the case of real-valued intuitionistic fuzzy values. The second local criterion called “risk” is based on the real-valued accuracy function. These local criteria are aggregated into a generalized one taking into account the weights of the considered local criteria dependent on the risk aversion of the decision maker. As opposed to the known methods, the developed approach makes it possible to estimate the strength of the relations between the compared intuitionistic fuzzy values. The proposed approach to *IFVs* comparison is free of limitations of the known method and provides transitive quantitative assessments of a degree to which an *IFV* is greater/lesser than another one.

With the use of some illustrative examples, it is shown that the proposed approach provides intuitively clear results.

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