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TWO TYPES OF PARAMETERIZATION OF MASS DENSITY DISTRIBUTION DURING IMMUNE OPTIMIZATION OF THE 3D STRUCTURE

Arkadiusz Poteralski

Institute of Computational Mechanics and Engineering, Silesian University of Technology
Gliwice, Poland
arkadiusz.poteralski@polsl.pl

Abstract. The paper is devoted to an application of the immune algorithm and the finite element method for generation of 3D structures using two different types of parameterization and comparing the final results. The shape, topology and material of the structure are generated for optimization criterion like minimum of the mass of the structure. Two different interpolation functions like: multinomial interpolation and interpolation based on the neighbourhood of elements are used. The purpose of these procedures is appropriate selection of mass densities.

Keywords: *immune algorithm, optimization, finite element method, 3D structures, interpolation function, smooth procedure*

Introduction

Immune methods have been applied in mechanics, especially in structural optimization [1]. The principle of operation of these methods is based on the mechanism discovered in biological immune systems.

The artificial immune systems (AIS) are developed on the basis of a mechanism discovered in biological immune systems [2]. An immune system is a complex system which contains distributed groups of specialized cells and organs. The main purpose of the immune system is to recognize and destroy pathogens - fungi, viruses, bacteria and improperly functioning cells.

The artificial immune systems take only a few elements from the biological immune systems [3-5]. The most frequently used are the mutation of the B-cells, proliferation, memory cells, and recognition by using the B- and T-cells. The presented approach is based on the Wierzchon's algorithm [6], but the mutation operator is changed. The Gaussian mutation is used instead of the nonuniform mutation in the presented approach. At the beginning of the AIS the memory cells are created randomly. They proliferate and mutate creating B-cells. The number of clones created for each memory cell is determined on the basis of the value of objective function of memory cells. The objective functions for B-cells are evaluated.

The selection process exchanges some memory cells for better B-cells. The selection is performed on the basis of the geometrical distance between each memory cell and B-cells (measured by using design variables). The crowding mechanism removes similar memory cells. The similarity is also determined as the geometrical distance between memory cells. The process is iteratively repeated until a termination criterion is fulfilled.

The main advantage of the immune algorithm is the fact that this approach does not need any information about the gradient of the fitness function and gives a strong probability of finding the global optimum. The main drawback of this approach is a lengthy process of the calculations. In order to eliminate this disadvantage the hybrid immune algorithm can be used to speed up the computations [7]. The fitness function concerns the minimization of mass of the structure with constraints imposed on equivalent stresses and resultant displacements of the structure.

More recently, other bio-inspired approaches, alternative to AIS [8], as the Particle Swarm Optimizers (PSO) [9, 10] or the Evolutionary Algorithms (EA) have gained popularity [11, 12].

Two types of interpolations are used. The purpose of these procedures is appropriate selection of mass densities. The first of them is the multinomial interpolation, the other the interpolation based on the neighbourhood of elements.

After optimization the procedure which smoothes an external and internal boundary of a three-dimensional structure is used.

In the present work the original concept [13-16] of a immune generation of shape, topology and material properties of 3D structures is developed.

As a tool for solving the direct problems concerning displacement and stress analysis problems of 3D elastic structures the finite element method (FEM) [17] is chosen.

1. The interpolation procedures

Parameterization is the key stage in the structural immune optimization. The great number of design variables causes the optimization process to be ineffective. A connection between design variables (parameters of B-cell receptor) and number of finite element leads to poor results. Better results can be obtained when the hyper surface of mass density distribution is interpolated by suitable number of values given in control points $(X)_j$. This number, on the one hand, should provide the good interpolation, and on the other hand, the number of design variables should be small.

Two different types of the interpolation procedures for the function of three variables $f(x,y,z)$ were applied. First the multinomial interpolation described below for a 3D structure was introduced. The second interpolation procedure based on some nodes overlapping selected FEM nodes has been introduced (interpolation bases on the neighbourhood of elements).

1.1. The procedure for the interpolation function of three variables - multinomial interpolation

In the case of optimization of 3D structures, a procedure of interpolation function of three variables $f(x, y, z)$ [18] is expressed as an approximation of a set of values of the function at the nodes

$$(x_i, y_j, z_k) \quad \begin{matrix} i = 1,...,n \\ j = 1,...,n \\ k = 1,...,n \end{matrix} \tag{1}$$

applying a three-dimensional domain, with a grid using steps $\Delta x, \Delta y, \Delta z$ (Fig. 1).

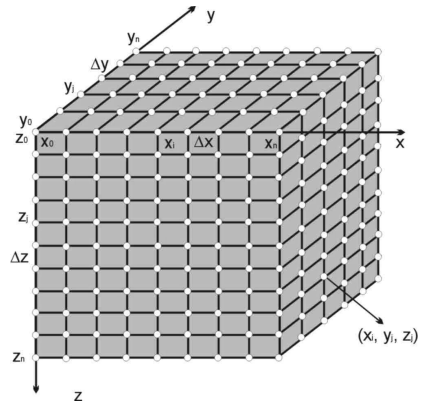


Fig. 1. Interpolation area of function of three variables $f(x, y, z)$

Additionally, one assumes that

$$\begin{aligned} x_0 &= 0,\ x_1 = 1,\ \ldots,\ x_n = n, \\ y_0 &= 0,\ y_1 = 1,\ \ldots,\ y_n = n, \\ z_0 &= 0,\ z_1 = 1,\ \ldots,\ z_n = n \end{aligned} \tag{2}$$

For a given coordinate $y = y_j$ and $z = z_k$ hyper surface $F(x, y_j, z_k)$ is approximated by multinomial $W\left(x, y_j, z_k\right)$:

$$W(x, y_j, z_k) = [1,\ x,\ x^2,\ldots,x^n]\mathbf{X}^{-1}\begin{bmatrix} F(x_0, y_j, z_k) \\ F(x_1, y_j, z_k) \\ \vdots \\ F(x_n, y_j, z_k) \end{bmatrix} \tag{3}$$

similarly

$$W(x_i, y, z_k) = [1,\ y,\ y^2, ..., y^n]\mathbf{Y}^{-1}\begin{bmatrix} F(x_i, y_0, z_k) \\ F(x_i, y_1, z_k) \\ \vdots \\ F(x_i, y_n, z_k) \end{bmatrix} \tag{4}$$

and next

$$W(x_i, y_j, z) = [1,\ z,\ z^2, ..., z^n]\mathbf{Z}^{-1}\begin{bmatrix} F(x_i, y_j, z_0) \\ F(x_i, y_j, z_1) \\ \vdots \\ F(x_i, y_j, z_n) \end{bmatrix} \tag{5}$$

where $\mathbf{X}^{-1}$, $\mathbf{Y}^{-1}$, $\mathbf{Z}^{-1}$ - regular Lagrange matrices for nodes $0,1,...,n$.

It can be proved that

$$W(x, y, z) = ([1, x, ..., x^n] \otimes [1, y, ..., y^n] \otimes [1, z, ..., z^n])\ (\mathbf{X^{-1}} \otimes \mathbf{Y^{-1}} \otimes \mathbf{Z^{-1}})\mathbf{F} \tag{6}$$

where

$$\mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} = \begin{bmatrix} F(x_0, y_0, z_0) \\ F(x_1, y_1, z_1) \\ \vdots \\ F(x_n, y_n, z_n) \end{bmatrix} \tag{7}$$

and expression $(\mathbf{X^{-1}} \otimes \mathbf{Y^{-1}} \otimes \mathbf{Z^{-1}})$ is the tensor product (Kronecker) of matrix $\mathbf{X}^{-1}$, $\mathbf{Y}^{-1}$, $\mathbf{Z}^{-1}$.

The interpolation procedure for optimization of 3D structures for 27 control points of hyper surface (interpolation nodes) is used

$$\begin{aligned} F(x_0, y_0, z_0) &= h_1^j \\ F(x_1, y_1, z_1) &= h_2^j \\ &\vdots \\ F(x_{27}, y_{27}, z_{27}) &= h_{27}^j \end{aligned} \tag{8}$$

where $h_1^j, h_2^j, ..., h_{27}^j$ are parameters of j-th lymphocyte of population of B-cells t:

$$limf_t^j = \left[h_1^j, h_2^j, ..., h_{27}^j \right] \tag{9}$$

In that case the base is assumed

$$\begin{aligned}\mathbf{\Phi} = [1,x,x^2]\otimes[1,y,y^2]\otimes[1,z,z^2] = [1,z,z^2,y,yz,yz^2,y^2,y^2z,y^2z^2,x,xz,xz^2,\\ xy,xyz,xyz^2,xy^2,xy^2z,xy^2z^2,x^2,x^2z,x^2z^2,x^2y,x^2yz,x^2yz^2,x^2y^2,x^2y^2z,x^2y^2z^2]\end{aligned} \tag{10}$$

Matrices $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$ take the form

$$\mathbf{X} = \mathbf{Y} = \mathbf{Z} = \begin{matrix}1 & x & x^2\\ \end{matrix}\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 1 & 2 & 4\end{bmatrix}\begin{matrix}0\\ 1\\ 2\end{matrix} \tag{11}$$

The multinomial interpolation after inversion of the matrices $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ and determination of a matrix which is the tensor product (Kronecker) of matrix $\mathbf{X}^{-1}$, $\mathbf{Y}^{-1}$, $\mathbf{Z}^{-1}$ takes the following form

$$W_E(x,y,z) = \mathbf{\Phi}\ (\mathbf{X^{-1}}\otimes\mathbf{Y^{-1}}\otimes\mathbf{Z^{-1}})\begin{bmatrix}h_1^j\\ h_2^j\\ \vdots\\ h_{27}^j\end{bmatrix} \tag{12}$$

After multiplication of the expression, the final expression is obtained

$$\begin{aligned}W(x,y,z) = a_1 + a_2z + a_3z^2 + a_4y + a_5yz + a_6yz^2 + a_7y^2 + a_8y^2z + a_9y^2z^2 +\\ a_{10}x + a_{11}xz + a_{12}xz^2 + a_{13}xy + a_{14}xyz + a_{15}xyz^2 + a_{16}xy^2 + a_{17}xy^2z + a_{18}xy^2z^2 +\\ a_{19}x^2 + a_{20}x^2z + a_{21}x^2z^2 + a_{22}x^2y + a_{23}x^2yz + a_{24}x^2yz^2 + a_{25}x^2y^2 + a_{26}x^2y^2z +\\ a_{27}x^2y^2z^2\end{aligned} \tag{13}$$

where the coefficients of the multinomial $a_1, a_2, ..., a_{27}$ are determined on the basis of an expression $(\mathbf{X^{-1}}\otimes\mathbf{Y^{-1}}\otimes\mathbf{Z^{-1}})[h_1^j, h_2^j, ..., h_{27}^j]^T$.

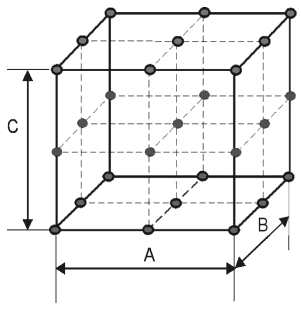


Fig. 2. Arrangement of control points

The structure which is under the optimization process is inserted into a cube H^3 whose edges have length A = 2, B = 2, C = 2, and 27 control points are arranged regularly (Fig. 2). In this case the number of control points is fixed. In the case when the body has complex geometry whose overall dimensions are considerably different from the space H^3, this approach can lead to the lower accuracy of the interpolation process. Then the domain Ω does not cover the working space (Fig. 3).

1.2. The procedure for the interpolation function of three variables - interpolation bases on the neighbourhood of elements

In order to overcome these difficulties, the second interpolation procedure based on some nodes overlapping selected FEM nodes has been introduced. This procedure (Tab. 1) is based on the analysis of the neighbourhoods of the individual nodes and enables the introduction of an optional number of the control points in any nodes of the finite element mesh.

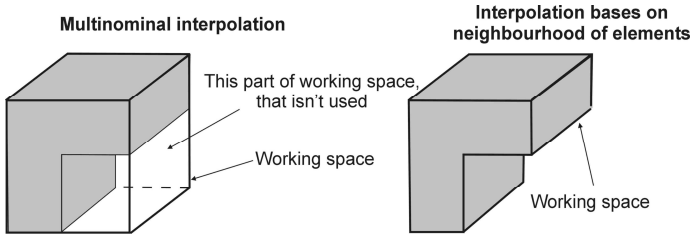


Fig. 3. Comparison of workspace for two interpolation procedures

Table 1

Interpolation procedure in the optimization of 2D and 3D structures

Load nodes $i = 1,2,...,W$ and elements $e = 1,2,...,R$
For $e = 1,2,...,E$ load the initial vector of interpolation parameters $p_1^0, p_2^0, ..., p_E^0$
For $k = 0,1,2,...,K$ "k - step of iteration"
{
  For $i = 1,2,...,E$ "for all the elements"
  {
    If T[i]=0 "i-th element does not contain a control point"
    {
    For $j = 1,2,...,M$ "for all neighbouring elements of i-th node"
    calculate $max(p_j)$
    calculate $min(p_j)$
    calculate $p_i^{k+1} = 1/2[max(p_j^k) + min(p_j^k)]$
    }
    If T[i] = 1 $p_i^{k+1} = p_i^k$ "i-th element contains a control point"
  }
}

This interpolation procedure works in an iterative way

$$\mathbf{W^{k+1}} = f(\mathbf{W^k}),\ k = 0,1,2,...,K \tag{14}$$

where the approximations of the interpolation vector in the following steps $k = 0$ are given by the expression

$$\mathbf{W^0} = [p_1^0, p_2^0, ..., p_i^0, ..., p_E^0],\ i = 1,2,...,R \tag{15}$$

where $p_i^0,\ i = 1,2,...,R$ are the initial values of the optimal parameters for particular elements of the mesh, R is the total number of elements. The values of optimization parameters in control points are the values of parameters $h_i^j,\ i = 1,2,...,n;\ j = 1,2,...,N$ j-th lymphocyte $limf_t^j = [h_1^j, h_2^j, ..., h_n^j]$ population of B-cells t. For this element value of T[i] is equal to one. The others values of initial vector are equal to

$$p_{av} = \frac{1}{2}([h_i^j]_{\min} + [h_i^j]_{\max}) \tag{16}$$

For these elements value of T[i] is equal to zero.

The next approximations of the vector of optimal parameters

$$\mathbf{W^{k+1}} = [p_1^{k+1}, p_2^{k+1}, ..., p_W^{k+1}],\ k = 0,1,2,...,K \tag{17}$$

are calculated by equation

$$p_i^{k+1} = \frac{1}{2}[\max(p_j^k) + \min(p_j^k)],\ j = 1,2,...,M \tag{18}$$

where:

T[i] - vector deciding about position of control points (if T[i] is equal to one - i-th element contains control point, if T[i] is equal to zero - i-th element doesn't contain a control point),

M - the number of neighbours $S_j,\ j = 1,2,...,M$ for i-th element $P_i,\ i = 1,2,...,R$ (Fig. 4),

p_i^{k+1} - the value of the interpolation parameter for i-th element, in step $k+1$,

p_j^k - the value of the interpolation parameter for j-th element which is a neighbour for element i-th, in step k-th,

$\max(p_j^k)$ - the maximal value of the interpolation parameter for elements which are neighbours for element i-th, in step k-th,

$\min(p_j^k)$ - the minimal value of the interpolation parameter for elements which are neighbours for element i-th, in step k-th.

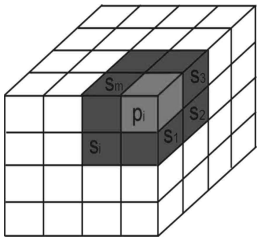


Fig. 4. Elements S_j neighbouring with element P_i

Expression (18) provides convergence of the iterative process, and the efficiency of the interpolation method. In addition the following expressions are also checked

$$p_i^{k+1} = \frac{1}{M}\sum_{j=1}^{M} p_j^k \tag{19}$$

$$p_i^{k+1} = \frac{1}{M}\sum_{j=1}^{M} w(P_i, S_j)(p_j^k) \tag{20}$$

$$p_i^{k+1} = \sqrt[M]{\prod_{j=1}^{M} p_j^k} \tag{21}$$

$$p_i^{k+1} = \sqrt[M]{\prod_{j=1}^{M} w(P_i, S_j)(p_j^k)} \tag{22}$$

$$1 - p_i^{k+1} = \sqrt[M]{\prod_{j=1}^{M} \left(1 - p_j^k\right)} \tag{23}$$

$$1 - p_i^{k+1} = \sqrt[M]{\prod_{j=1}^{M} w(P_i, S_j)\left(1 - p_j^k\right)} \tag{24}$$

where:

M - number of neighbours S_j, $j = 1,2,...,M$ for i-th element P_i, $i = 1,2,...,R$ (Fig. 4),

$w(P_i, S_j)$ - weight function of influence of neighbour parameter S_j, $j = 1,2,...,M$ for the value of the parameter optimization for the element P_i, $i = 1,2,...,R$,

p_i^{k+1} - the value of the interpolation parameter for i-th node, in step $k+1$,

p_j^k - the value of the interpolation parameter for j-th node which is a neighbour for node i-th, in step k-th.

These equations like the expression (18) provide the convergence of the iterative process, but their use in the optimization process is less preferred than the use of the expression (18).

2. Numerical examples

The optimized structure (Fig. 5) is discretized by cubic finite elements and subjected to the stress constraint in the case of the minimization of the mass of the structure. The dimensions and loading of 3D structures are included in Table 2 and the parameters of the immune algorithm are included in Table 3. Table 4 includes information about two types of constraints for the immune algorithm.

In the first of three examples a 3D structure is optimized by means of multinomial interpolation with constant number of control points. Next two examples are optimized by means of interpolation based on neighborhood of elements with eleven control points and the last example with twenty-two control points (Fig. 6).

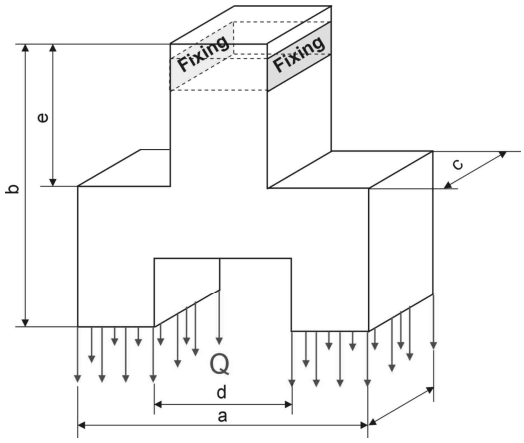


Fig. 5. The geometry and scheme of loading

Table 2

The dimensions and loading of 3D structure

Dimensions [mm]	
a	260
b	320
c	80
d	100
e	160
Loading [kN]	
Q	1.5

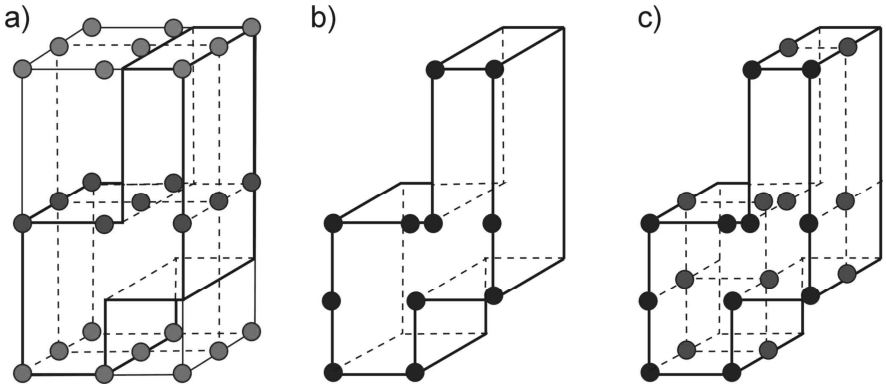


Fig. 6. Spacing of control points: a) multinomial interpolation, b) interpolation based on the neighbourhood of elements with 11 control points, c) interpolation based on the neighbourhood of elements with 22 control points

The results of the computations are presented in Table 5. Material properties of finite elements are changed and some of them are eliminated by means of the proposed method [13, 15]. As the result, the optimal shape, topology and material of the structures are obtained.

The main aim of these examples is to compare two different types of parametrization. In order to compare these procedures the number and distribution of control points are changed. After immune optimization process different 3D structure are obtained (Tab. 5).

Table 3

Parameters of the AIS

Numbers of design variables	The number of memory cells	The number of the clones	Crowding factor	Gaussian mutation
27, 11, 22	6	30	0.25	0.25

Table 4

Constraints

Case 1	Case 2
Maximal stress	
2 MPa	2.5 MPa
Maximal displacement	
0.013 mm	0.018 mm

Table 6 includes information about fitness functions for optimized 3D structure for three different interpolations for two different constraints.

Table 5

Results of optimization of 3D structure for two types of parameterizations

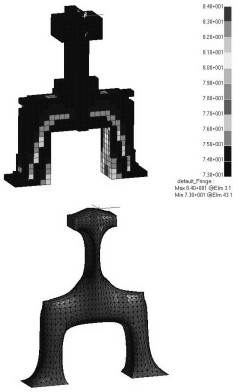
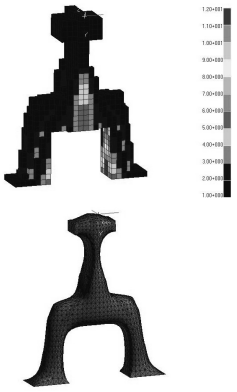
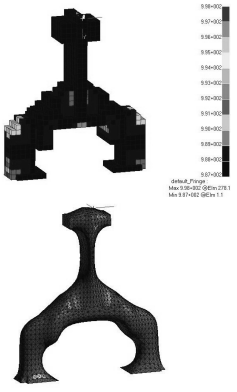
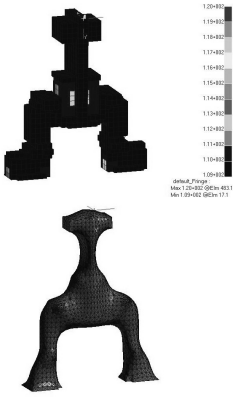
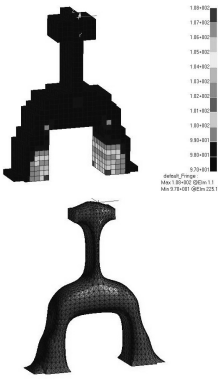
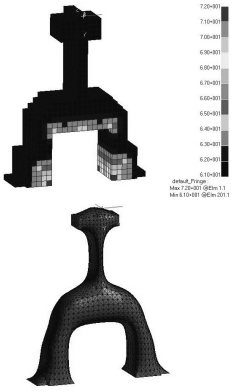
	Case 1 - map of mass densities and structure after smooth procedure	Case 2 - map of mass densities and structure after smooth procedure
multinomial interpolation		
interpolation bases on the neighbourhood of elements with 11 control points		
interpolation bases on the neighbourhood of elements with 22 control points		

Table 6

Comparison of the final results of optimization

	Case 1 of constraints	Case 2 of constraints
	Value of fitness functions [mm^3]	
multinomial interpolation	$1.763 \cdot 10^6$	$1.695 \cdot 10^6$
interpolation based on the neighbourhood of elements with 11 control points	$1.791 \cdot 10^6$	$1.722 \cdot 10^6$
interpolation based on the neighbourhood of elements with 22 control points	$1.748 \cdot 10^6$	$1.684 \cdot 10^6$

Conclusions

An effective tool of immune optimization of 3D structures has been presented. Using this approach, the shape, topology and material optimization are performed simultaneously. The important feature of this approach is its universality for 3D problems. The implementation of the AIS to this approach gives a great probability of finding the global optimal solution; however, the proper choice of the interpolation method is very important. On the basis of the obtained results the interpolation based on the neighbourhood of elements is better than multinomial interpolation.

In addition to second interpolation based on the neighbourhood of elements the optional number of control points can be loaded and all working space is always used. It is the following advantage of this method.

In addition, the increase in the number of control points permit to obtain more accurate distribution of hyper surface but for higher number of control points, the number of individuals in each generation must be increased.

The described approach is free from limitations connected with classic gradient optimization methods referring to the continuity of the objective function, the gradient or hessian of the objective function and the substantial probability of getting a local optimum. Besides in the case of using gradient methods finding the global solution depends on the starting point. The immune algorithm performs multidirectional optimum searching by exchanging information between lymphocytes and allows best B-cell receptors to survive.

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