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APPLICATION OF HM-NETWORKS WITH INPATIENT CLAIMS IN FINDING THE MEMORY CAPACITY IN INFORMATION SYSTEMS

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Abstract. To solve the problem of determining the memory capacity of the information systems (IS), the use of the stochastic model is proposed, based on the use of HM (Howard--Matalytski) - queueing networks with revenues. This model allows one to take into account time dependencies of the message processing from their capacities, the possibility changes of the messages capacities over time and also the possibility of leaving messages from queues in nodes of IS, without getting into them appropriate processing. The expressions for the mean (expected) values of total message capacities in the IS nodes have been obtained.

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1. Introduction

In the IS, the total amount of memory capacity is bounded by some value, which is usually called memory capacity [1]. In the IS, designing the main task is the determination of the mean memory capacity so as to take into account the conditions that limit the proportion of the lost information. One of the methods for solving problems in IS design is the use of HM-queueing networks [2]. Furthermore, under the IS we will mean systems, objects of transformation in which is information, incoming portions as messages [1]. HM-networks can be used to determine the capacity of a buffer storage of systems that are representing processing IS nodes and transferring messages. Note that the considered problem is one of the main ones, for example, in the design of communication centers or hubs in data communication networks. Neglecting time dependence of the message processing from their capacity can lead to errors in determining the buffer capacity memory in the IS and in the calculation of the probability of message loss.

The model expounded below can also be used in solving the actual problem that has emerged recently in the IS, namely, excessive buffering problems (i.e. the definition of the required memory capacity) [3]. Bufferbloat - a phenomenon that occurs in packet communication networks, when the buffering causes excessive increase in the time of the packets and packet delay variation, and the resulting decrease in throughput capacity of IS.

Neglect time dependence of messages processing of their capacities can lead to serious errors in finding the buffer memory in IS. In the general case solving the problems discussed above can be based on the use of HM-networks with revenues. In such networks, the claim during the transition from one queueing system (QS) to another brings some revenue last (which is equal to the capacity of this claim), and revenue (capacity) of the first QS is reduced by this amount.

It should be noted that the method for finding the stationary state probabilities and the mean characteristics of the queueing network with bounded time of the same type claims, operating under a heavy traffic regime, using the apparatus of multivariate generating functions, has been described in the monograph [4] and to the network with heterogeneous claims - in [5], HM-networks with bounded waiting time of claims have been studied in [6, 7].

For the first time application of the HM-networks for estimating the memory capacity in the IS has been described in [8]. In [9] a method of finding the mean total capacity of the same type claims in open systems of HM-network with a bounded their numbers of queues and claims bypassing of queueing systems.

2. Finding the expected capacities of claims in the case, when is known only the first moments of the revenues from the transitions between the network states

Let an independent Poisson flow arrive to the network with rate λ. The intensity of service of claims at time t has rate $\mu_i(k_i(t))$ in the system S_i and depends on the count of claims in this system, $i=\overline{1,n}$. The length of stay of claims in the queue of the *i*-th QS is a random variable, distributed exponentially with parameter $\theta_i(k_i(t))$, and does not depend on other factors, for example, the residence time in the queue of other claims.

Consider the dynamics of income changes of a network system S_i. Denote by the $V_i(t)$ its income at moment time t. Let the initial moment time revenue of the system equal $V_i(0)=v_{i0}$. The revenue of its QS at moment time $t+\Delta t$ can be represented in the form

$$V_i(t+\Delta t)=V_i(t)+\Delta V_i(t,\Delta t), \tag{1}$$

where $\Delta V_i(t,\Delta t)$ - revenue changes of the system S_i at the time interval $[t,t+\Delta t)$, $i=\overline{1,n}$. To find its value, we write down the value of the conditional probabilities of events that may occur during Δt and the revenue changes of its QS, associated with these events:

1) With probability $\lambda p_{0i}\Delta t + o(\Delta t)$ to the system S_i will arrive a claim from the external environment, which will increase its total capacity of a claim by a value r_{0i}, where r_{0i} - a random variable (RV) with the expectation $E\{r_{0i}\}=a_{0i}$, $i=\overline{1,n}$.
2) With probability $\mu_i(k_i(t))u(k_i(t))p_{i0}\Delta t + o(\Delta t)$ a claim after it has been serviced in QS S_i is headed for the external environment, wherein total capacity of claims in the system S_i reduced by the amount of R_{i0}, where R_{i0} RV with $E\{R_{i0}\}=b_{i0}$.
3) With probability $\mu_j(k_j(t))u(k_j(t))p_{ji}\Delta t + o(\Delta t)$ a claim, after servicing in the QS S_j, heads for the system S_i, in such a transition claim capacity in the system S_i increases by a value r_{ji}, and claims capacity in the system S_j reduced by this value, where r_{ji} RV with $E\{r_{ji}\}=a_{ji}$, $i,j=\overline{1,n}$, $i\neq j$.
4) With probability $\mu_i(k_i(t))u(k_i(t))p_{ij}\Delta t + o(\Delta t)$ a claim from the system S_i heads for the system S_j, in such a transition a claim capacity of the system S_i reduced by the value R_{ij}, and claims capacity in the system S_j will increase by this value, where R_{ij} - RV with $E\{R_{ij}\}=b_{ij}$, $i,j=\overline{1,n}$, $i\neq j$. It's obvious that $r_{ji}=R_{ji}$ with probability 1, i.e.

$$a_{ji}=b_{ji},\ i,j=\overline{1,n}. \qquad (2)$$

5) With probability $\theta_i(k_i(t))u(k_i(t))q_{i0}\Delta t + o(\Delta t)$ a claim without waiting for service in the system S_i, will moves from the queue of this QS to the external environment, and claims capacity in it reduced by the value H_{i0}, where H_{i0}- RV with $E\{H_{i0}\}=\overline{H}_{i0}$, $i=\overline{1,n}$.
6) With probability $\theta_j(k_j(t))u(k_j(t))q_{ji}\Delta t + o(\Delta t)$ a claim without waiting for service in the system S_j, moves from the queue of this QS to the QS S_i, in such a transition claims capacity in it will increase by a value h_{ji}, and claims capacity in the QS S_j reduced by this value, where h_{ji}- RV with $\overline{h}_{ji}$, $i,j=\overline{1,n}$, $i\neq j$.

7) With probability $\theta_i(k_i(t))u(k_i(t))q_{ij}\Delta t + o(\Delta t)$ a claim without waiting for service in the system S_i, moves from this QS to the system S_j in such a transition a claim capacity of the system will increase by a value H_{ij}, and claims capacity in the QS S_i reduced by this value, where H_{ij} - RV with $\overline{H}_{ij}$; it's clear that $h_{ji} = H_{ji}$ with probability 1, i.e.

$$\overline{h}_{ji} = \overline{H}_{ij}, \ i, j = \overline{1, n}. \tag{3}$$

8) With probability

$$1 - [\lambda p_{0i} + (\mu_i(k_i(t)) + \theta_i(k_i(t)))u(k_i(t)) + \\ + \sum_{\substack{j=1 \\ j \neq i}}^{n} (\mu_j(k_j(t))p_{ji} + \theta_j(k_j(t))q_{ji})u(k_j(t))]\Delta t + o(\Delta t)$$

on time interval $[t, t + \Delta t)$ there will be no change of system S_i nothing is going to happen, $i = \overline{1, n}$.

In addition, for each small time interval Δt claim capacity in the system S_i increased by a value $r_i \Delta t$, where r_i – RV with $E\{r_i\} = c_i$, $i = \overline{1, n}$. We shall also assume, that RV r_{0i}, R_{0i}, r_i, r_{ji}, R_{ij}, h_{ji}, H_{ij} pairwise independent, $i, j = \overline{1, n}$. Then from the aforesaid follows:

$$\Delta V_i(t, \Delta t) = \begin{cases} r_{0i} + r_i \Delta t & \text{with probability } \lambda p_{0i}\Delta t + o(\Delta t), \\ -R_{i0} + r_i \Delta t & \text{with probability } \mu_i(k_i(t))u(k_i(t))p_{i0}\Delta t + o(\Delta t), \\ r_{ji} + r_i \Delta t & \text{with probability } \mu_j(k_j(t))u(k_j(t))p_{ij}\Delta t + o(\Delta t), \\ -R_{ij} + r_i \Delta t & \text{with probability } \mu_i(k_i(t))u(k_i(t))p_{ij}\Delta t + o(\Delta t), \\ -H_{i0} + r_i \Delta t & \text{with probability } \theta_i(k_i(t))u(k_i(t))q_{i0}\Delta t + o(\Delta t), \\ h_{ji} + r_i \Delta t & \text{with probability } \theta_j(k_j(t))u(k_j(t))q_{ji}\Delta t + o(\Delta t), \\ -H_{ij} + r_i \Delta t & \text{with probability } \theta_i(k_i(t))u(k_i(t))q_{ij}\Delta t + o(\Delta t), \\ r_i \Delta t & \text{with probability} \\ & 1 - [\lambda p_{0i} + (\mu_i(k_i(t)) + \theta_i(k_i(t)))u(k_i(t)) + \\ & + \sum_{\substack{j=1 \\ j \neq i}}^{n} (\mu_j(k_j(t))p_{ji} + \theta_j(k_j(t))q_{ji})u(k_j(t))]\Delta t + o(\Delta t)]. \end{cases} \tag{4}$$

At a fixed implementation process $k(t)$ and considering (4) we can write

$$E\{\Delta V_i(t,\Delta t)/k(t)\}=\left[\lambda p_{0i}a_{0i}-\mu_i(k_i(t))u(k_i(t))\left(p_{i0}b_{i0}+\sum_{\substack{j=1\\ j\neq i}}^{n}p_{ij}b_{ij}\right)-\right.$$

$$-\theta_i(k_i(t))u(k_i(t))\left(q_{i0}\overline{H}_{i0}+\sum_{\substack{j=1\\ j\neq i}}^{n}q_{ij}\overline{H}_{ij}\right)+\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\mu_j(k_j(t))u(k_j(t))p_{ji}a_{ji}+\right.$$

$$\left.\left.+\theta_j(k_j(t))u(k_j(t))q_{ji}\overline{h}_{ji}\right)\right]\Delta t+o(\Delta t).$$

Averaging over $k(t)$ and taking into account the normalization condition $\sum_k P(k(t)=k)=1$, to change the expected revenue of the system we obtain

$$E\{\Delta V_i(t,\Delta t)\}=\sum_k P(k(t)=k)E\{\Delta V_i(t,\Delta t)/k(t)\}=$$

$$=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\ldots\sum_{k_n=0}^{\infty}P(k(t)=(k_1(t),k_2(t),\ldots,k_n(t)))E\{\Delta V_i(t,\Delta t)/k(t)=(k_1(t),k_2(t),\ldots,k_n(t))\}=$$

$$=\left[\lambda p_{0i}a_{0i}+c_i-\left(p_{i0}b_{i0}+\sum_{\substack{j=1\\ j\neq i}}^{n}p_{ij}b_{ij}\right)\sum_k P(k(t)=k)\mu_i(k_i(t))u(k_i(t))-\right.$$

$$-\left(q_{i0}\overline{H}_{i0}+\sum_{\substack{j=1\\ j\neq i}}^{n}q_{ij}\overline{H}_{ij}\right)\sum_k P(k(t)=k)\theta_i(k_i(t))u(k_i(t))+$$

$$\left.+\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_k P(k(t)=k)\left(\mu_j(k_j(t))u(k_j(t))p_{ji}a_{ji}+\theta_j(k_j(t))u(k_j(t))q_{ji}\overline{h}_{ji}\right)\right]\Delta t+o(\Delta t).$$

Let the QS S_i contains m_i identical service lines, in each of which service times of claims and leaving it from the queue distributed exponentially according to the parameters μ_i and θ_i, $i=\overline{1,n}$. In this case we have the expressions

$$\mu_i(k_i(t))u(k_i(t))=\begin{cases}\mu_i k_i(t), & k_i(t)\le m_i,\\ \mu_i m_i, & k_i(t)>m_i,\end{cases}=\mu_i\min(k_i(t),m_i),\ i=\overline{1,n}, \quad (5)$$

$$\theta_i(k_i(t))u(k_i(t))=\begin{cases}0, & k_i(t)\le m_i,\\ \theta_i(k_i(t)-m_i), & k_i(t)>m_i,\end{cases}=\theta_i(k_i(t)-m_i)u(k_i(t)-m_i),\ i=\overline{1,n}, \quad (6)$$

Also we assume that the relations are valid

$$E\min\left(k_i(t), m_i\right) = \min\left(N_i(t), m_i\right),\ i = \overline{1,n}, \tag{7}$$

$$E\left[\left(k_i(t) - m_i\right)u\left(k_i(t) - m_i\right)\right] = \left(N_i(t) - m_i\right)u\left(N_i(t) - m_i\right),\ i = \overline{1,n}, \tag{8}$$

where $N_i(t) = E\{k_i(t)\}-$ the mean number of claims (waiting and servicing) in the system S_i on the time interval $[0,t]$, $i = \overline{1,n}$.

Considering these assumptions, we obtain the following approximate relation

$$E\{\Delta V_i(t,\Delta t)\} = \left[\lambda p_{0i}a_{0i} + c_i - \mu_i\left(p_{i0}b_{i0} + \sum_{\substack{j=1\\ j\neq i}}^{n} p_{ij}b_{ij}\right)\min\left(N_i(t), m_i\right) + \right.$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{n}\mu_j p_{ji}a_{ji}\min\left(N_j(t), m_j\right) - \theta_i\left(q_{i0}\overline{H}_{i0} + \sum_{\substack{j=1\\ j\neq i}}^{n} q_{ij}\overline{H}_{ij}\right)\left(N_i(t) - m_i\right)u\left(N_i(t) - m_i\right) +$$

$$\left. + \sum_{\substack{j=1\\ j\neq i}}^{n}\theta_j q_{ji}\overline{h}_{ji}\left(N_j(t) - m_j\right)u\left(N_j(t) - m_j\right)\right]\Delta t + o(\Delta t). \tag{9}$$

According to the Poisson process of rate λ, then the mean number of claims arrived from outside to the system S_i during time Δt equals $\lambda p_{0i}\Delta t$. Denote by $\rho(t)$ - the mean number of busy service lines in the system S_i at time t, $i = \overline{1,n}$. Then $\mu_i\rho_i(t)\Delta t$ and $\theta_i\left(N_i(t) - m_i\right)u\left(N_i(t) - m_i\right)\Delta t$ - the mean number of claims, leaving the system S_i during time Δt respectively, after serving in it and without waiting for the service in it; $\sum_{\substack{j=1\\ j\neq i}}^{n}\mu_j\rho_j(t)p_{ji}\Delta t$ and $\sum_{\substack{j=1\\ j\neq i}}^{n}\theta_j\left(N_j(t) - m_j\right)u\left(N_j(t) - m_j\right)q_{ji}\Delta t$ the mean number of claims, arriving to the system S_i during time Δt from other QS respectively, after serving in it and without waiting the service in it. Therefore

$$N_i(t+\Delta t) - N_i(t) = \lambda p_{0i}\Delta t + \sum_{\substack{j=1\\ j\neq i}}^{n}\left[\mu_j\rho_j(t)p_{ji} + \theta_j\left(N_j(t) - m_j\right)u\left(N_j(t) - m_j\right)q_{ji}\right]\Delta t -$$

$$-\left[\mu_i\rho_i(t) + \theta_i\left(N_i(t) - m_i\right)u\left(N_i(t) - m_i\right)\right]\Delta t,\ i = \overline{1,n},$$

where at $\Delta t \to 0$. It follows the system of ODE for $N_i(t)$:

$$\frac{dN_i(t)}{dt} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \left[\mu_j \rho_j(t) p_{ji} + \theta_j \left(N_j(t) - m_j\right) u\left(N_j(t) - m_j\right) q_{ji} \right] -$$

$$-\mu_i \rho_i(t) - \theta_i \left(N_i(t) - m_i\right) u\left(N_i(t) - m_i\right) + \lambda p_{0i}, \; i = \overline{1,n}. \tag{10}$$

To find the value $\rho_i(t)$ exactly is impossible, therefore, as we have done previously, we approximate its expression

$$\rho_i(t) = \begin{cases} N_i(t), & N_i(t) \le m_i, \\ m_i, & N_i(t) > m_i \end{cases} = \min\left(N_i(t), m_i\right).$$

Then the system of equations (10) takes the form

$$\frac{dN_i(t)}{dt} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \left[\mu_j \min\left(N_j(t), m_j\right) p_{ji} + \theta_j \left(N_j(t) - m_j\right) u\left(N_j(t) - m_j\right) q_{ji} \right] -$$

$$-\mu_i \min\left(N_i(t), m_i\right) - \theta_i \left(N_i(t) - m_i\right) u\left(N_i(t) - m_i\right) + \lambda p_{0i}, \; i = \overline{1,n}. \tag{11}$$

That is a system of inhomogeneous linear ODE with discontinuous right-hand sides. It should be solved by dividing the phase space into a number of areas and finding solutions to each of them.

We introduce the notation $v_i(t) = M\{V_i(t)\}$. From (1) follows $v_i(t+\Delta t) = v_i(t) + M\{\Delta V_i(t, \Delta t)\}$. Considering (9), (2), (3), and passing to the limit $\Delta t \to 0$, we have linear inhomogeneous first order ODE

$$\frac{dv_i(t)}{dt} = -\left[\mu_i \left(p_{i0} b_{i0} + \sum_{\substack{j=1 \\ j\neq i}}^{n} p_{ij} a_{ij} \right) \min\left(N_i(t), m_i\right) + \right.$$

$$+ \theta_i \left(q_{i0} \overline{H}_{i0} + \sum_{\substack{j=1 \\ j\neq i}}^{n} q_{ij} \overline{h}_{ji} \right) \left(N_i(t) - m_i\right) u\left(N_i(t) - m_i\right) +$$

$$+ \sum_{\substack{j=1 \\ j\neq i}}^{n} \left[\mu_j p_{ji} a_{ji} \min\left(N_j(t), m_j\right) + \theta_j q_{ji} \overline{h}_{ji} \left(N_j(t) - m_j\right)\left(N_j(t) - m_j\right) \right] + \lambda p_{0i} a_{0i} + c_i. \tag{12}$$

By setting the initial conditions $v_i(0) = v_{i0}, \; i = \overline{1,n}$, we can find the total expected capacity of claims in network systems.

If the network operates as there are no observed queues in the average (low-traffic regime), i.e. $\min(N_i(t), m_i) = N_i(t)$, $u(N_i(t) - m_i) = 0$, $i = \overline{1, n}$, then relations (11) and (12) will have the form:

$$\frac{dN_i(t)}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \mu_j p_{ji} N_j(t) - \mu_i N_i(t) + \lambda p_{0i}, \ i = \overline{1, n}, \tag{13}$$

$$\begin{cases} \dfrac{dv_i(t)}{dt} = -\mu_i \left(p_{i0} b_{i0} + \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} p_{ij} b_{ij} \right) N_i(t) + \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \mu_j p_{ji} a_{ji} N_j(t) + \lambda p_{0i} a_{0i} + c_i, \\ v_i(0) = v_{i0}, \ i = \overline{1, n}. \end{cases} \tag{14}$$

If QS operates under a heavy-traffic regime, then $\min(N_i(t), m_i) = m_i$, $u(N_i(t) - m_i) = 1$, $i = \overline{1, n}$ and relations (11), (12) take the form

$$\frac{dN_i(t)}{dt} = -\theta_i N_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{n} \theta_j q_{ji} N_j(t) +$$

$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} (\mu_j p_{ji} - \theta_j q_{ji}) m_j - (\mu_i - \theta_i) m_i + \lambda p_{0i}, \ i = \overline{1, n}, \tag{15}$$

$$\begin{cases} \dfrac{dv_i(t)}{dt} = -\left[\mu_i \left(p_{i0} b_{i0} + \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} p_{ij} a_{ij} \right) m_i + \theta_i \left(q_{i0} \overline{H}_{i0} + \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} q_{ij} \overline{h}_{ji} \right) (N_i(t) - m_i) \right] + \\ \qquad + \sum\limits_{\substack{j=1 \\ j \neq i}}^{n} \left[\mu_j p_{ji} a_{ji} m_j + \theta_j q_{ji} \overline{h}_{ji} (N_j(t) - m_j) \right] + \lambda p_{0i} a_{0i} + c_i, \\ v_i(0) = v_{i0}, \ i = \overline{1, n}. \end{cases} \tag{16}$$

The system (15) can be rewritten in matrix form $\frac{dN(t)}{dt} = DN(t) + f$, where: $N^T(t) = (N_1(t), N_2(t), \ldots N_n(t))$, D - square matrix consisting of elements $d_{ij} = \theta_j q_{ji}$, $i, j = \overline{1, n}$, f - a column vector whose elements are the values

$f_i = \lambda p_{0i} + \sum_{j=1}^{n}(\mu_j p_{ji} - \theta_j q_{ji})m_j$, $p_{ii} = q_{ii} = -1$, $i = \overline{1,n}$. The solution of the last system has the form $N(t) = N(0)e^{Dt} + f\int_0^t e^{D(t-\tau)}d\tau$, where $N(0)$ - given initial conditions.

3. Expected revenues of the systems in the closed network with central QS

Consider a closed network with a central QS (Fig. 1). All queueing systems operate under a heavy-traffic regime, i.e. $\forall t > 0 \quad k_i(t) > 0$, $i = \overline{1,n}$. In this case $\min(N_i(t), m_i) = m_i$, $i = \overline{1,n}$. Claims without waiting for service can only move between the peripheral QS and leave the central QS.

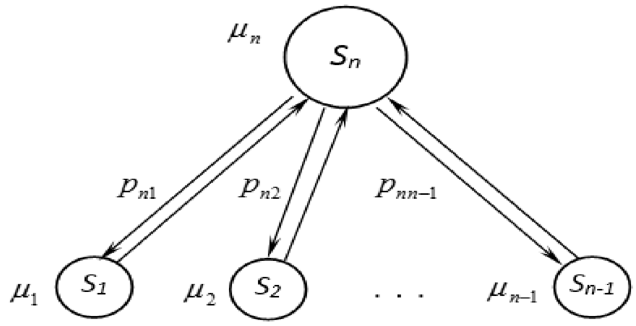

Fig. 1. The closed network with a central QS

Thus $p_{ij} = \begin{cases} p_{nj}, & i = n,\ j = \overline{1,n-1}, \\ 0, & i \neq n,\ j = \overline{1,n-1}, \\ 1, & i = \overline{1,n-1},\ j = n, \end{cases}$ $\quad q_{ij} = \begin{cases} q_{nj}, & i = n,\ j = \overline{1,n-1}, \\ q_{ij}, & i, j = \overline{1,n-1},\ i \neq j, \\ 0, & i = \overline{1,n-1},\ j = n. \end{cases}$

The system (11) then has the form

$$\begin{cases} \dfrac{dN_i(t)}{dt} = -\theta_i N_i(t) + \sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \theta_j q_{ji} N_j(t) - \sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \theta_j q_{ji} m_j - (\mu_i - \theta_i)m_i + \mu_n p_{ni} m_n, \\ \dfrac{dN_n(t)}{dt} = -\theta_n N_n(t) - (\mu_n - \theta_n)m_n + \sum\limits_{j=1}^{n-1} \mu_j m_j, i = \overline{1,n-1}, \end{cases} \tag{17}$$

and $\sum_{i=1}^{n} N_i(t) = K$, where K - claims count in the network. The system (16) for the expected revenues can be written as:

$$\begin{cases} \dfrac{dv_i(t)}{dt} = -\theta_i \sum\limits_{\substack{j=1 \\ j\neq i}}^{n-1} q_{ij}\overline{h}_{ji} N_i(t) + \sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \theta_j q_{ji}\overline{h}_{ji} N_j(t) + \theta_i \sum\limits_{\substack{j=1 \\ j\neq i}}^{n-1} q_{ij}\overline{h}_{ji} m_i - \\ \qquad - \sum\limits_{\substack{j=1 \\ j\neq i}}^{n} \theta_j q_{ji}\overline{h}_{ji} m_j + \mu_n p_{ni} a_{ni} m_n - \mu_i a_{in} m_i + c_i, \quad i = \overline{1, n-1}, \\ \dfrac{dv_n(t)}{dt} = \theta_n \sum\limits_{j=1}^{n-1} q_{nj}\overline{h}_{jn} N_n(t) + \sum\limits_{j=1}^{n-1} \left(\theta_n q_{nj}\overline{h}_{jn} m_n + \mu_j a_{nj} m_j - \mu_n p_{nj} a_{jn} m_n\right) + c_n. \end{cases} \tag{18}$$

Then, for example, the mean number of claims in the central QS on initial conditions $N_n(0)$, equal

$$N_n(t) = -\frac{\mu_n m_n}{\theta_n} + m_n + \frac{1}{\theta_n}\sum_{j=1}^{n-1} \mu_j m_j + e^{\theta_n t}\left(N_n(0) + (\mu_n - \theta_n)\frac{m_n}{\theta_n} - \frac{1}{\theta_n}\sum_{j=1}^{n-1} \mu_j m_j \right),$$

and expected revenue on initial conditions $v_n(0) = v_{n0}$, takes the form

$$\begin{aligned} v_n(t) = v_{n0} + \sum_{j=1}^{n-1} q_{nj}\overline{h}_{jn} \Bigg[\left(\theta_n m_n + \sum_{j=1}^{n-1} \mu_j m_j \right) t - \\ + \mathrm{e}^{-\theta_n t} \left(N_n(0) - (\mu_n - \theta_n)\frac{m_n}{\theta_n} - \frac{1}{\theta_n}\sum_{j=1}^{n-1} \mu_j m_j \right) \Bigg] + \\ + \left[\sum_{j=1}^{n-1} \left(\theta_n m_n q_{nj}\overline{h}_{jn} + \mu_j a_{nj} m_j - \mu_n m_n p_{nj} a_{jn} \right) + c_n \right] t + \\ + \sum_{j=1}^{n-1} q_{nj}\overline{h}_{jn} \left(N_n(0) - \frac{m_n}{\theta_n}(\mu_n - \theta_n) - \frac{1}{\theta_n}\sum_{j=1}^{n-1} \mu_j m_j \right). \end{aligned} \tag{19}$$

4. Model example

Consider a closed network with a central QS consisting of $n = 6$ QS, $m_1 = 3$, $m_2 = m_5 = 2$, $m_3 = m_4 = 5$, $m_6 = 20$, $K = 100$. Service rates of claims equal: $\mu_1 = 5$, $\mu_2 = \mu_3 = 4$, $\mu_4 = 2$, $\mu_5 = 3$, $\mu_6 = 15$. The mean duration of waiting claims in the queue QS: $\theta_1^{-1}=0.167$, $\theta_2^{-1}=0.125$, $\theta_3^{-1}=0.222$, $\theta_4^{-1}=0.385$, $\theta_5^{-1}=0.182$, $\theta_6^{-1}=0.167$. Transition probabilities of claims between network QS - $p_{6i}=1/5$, $p_{i6}=1$, $i=\overline{1,5}$; let also $p_{ii}=-1$, $i=\overline{1,6}$; other $p_{ij}=0$, $i,j=\overline{1,6}$. Transition probabilities of claims without waiting for service between network QS: $q_{12}=q_{25}=q_{31}=q_{45}=q_{52}=2/3$, $q_{14}=q_{23}=q_{34}=q_{41}=q_{52}=1/3$, $q_{6i}=1/5$, $i=\overline{1,5}$, other $q_{ij}=0$, a $q_{ii}=-1$ $i,j=\overline{1,6}$. Let also $N_i(0)=16$, $i=\overline{1,5}$, $N_6(0)=20$. We define the values for the required expectations:

$$a_{6i}=a_{i6}=(0.3,0.4,1,0.9,1.6),\ i=\overline{1,5},\ \overline{h}_{6i}=\overline{h}_{i6}=(0.3,0.5,0.6,0.8,0.7),\ i=\overline{1,5},$$

$$c=(11,20,20,14,12,17).$$

Solving the system (17) by the direct method, we get the expression for the mean number of claims in each of the network systems

$$N_1(t)=10.024-8.224e^{-6.522t}+e^{-4.501t}(4.809\sin 0.766t+5.933\cos 0.766t)+ +e^{-8.538t}(0.267\cos 1.139t+1.114\sin 1.139t),$$

$$N_2(t)=11.234-5.151e^{-6.522t}+e^{-4.501t}(1.575\sin 0.766t+3.084\cos 0.766t)+ +e^{-8.538t}(3.551\sin 1.139t-1.176\cos 1.139t),$$

$$N_3(t)=11.087-9.185e^{-6.522t}+e^{-4.501t}(4.112\sin 0.766t+8.746\cos 0.766t)- -e^{-8.538t}(3.768\sin 1.139t+2.647\cos 1.139t),$$

$$N_4(t)=19.584-0.531e^{-6.522t}+e^{-4.501t}(1.418\sin 0.766t-9.897\cos 0.766t)- -e^{-8.538t}(0.677\sin 1.139t+1.157\cos 1.139t),$$

$$N_5(t)=18.694-3.831e^{-6.522t}-e^{-4.501t}(2.829\sin 0.766t+3.629\cos 0.766t)- -e^{-8.538t}(7.52\sin 1.139t+3.233\cos 1.139t),$$

$$N_6(t)=29.167-9.167e^{-6t}.$$

In solving the system (18) with initial conditions $v_i(0)=25$, $i=\overline{1,5}$, $v_6(0)=50$, we obtained the expressions for the expected claim capacities:

$$v_1(t)=0.55e^{-6t}+33.92t+24.45,\ v_2(t)=0.917e^{-6t}+57.1t+24.083,$$

$$v_3(t)=1.1e^{-6t}+82.12t+23.9,\ v_4(t)=1.467e^{-6t}+82.2t+23.533,$$

$$v_5(t) = 1.283e^{-6t} + 121.22t + 23.717,$$

$$v_6(t) = \begin{cases} 5.317e^{-6t} - 21.6t + 44.683, 21.6t - 5.317e^{-6t} \le 44.683, \\ 0, 21.6t - 5.317e^{-6t} > 44.683. \end{cases}.$$

Charts of change in the expected capacity of claims in the network systems are shown in Figures 2 and 3.

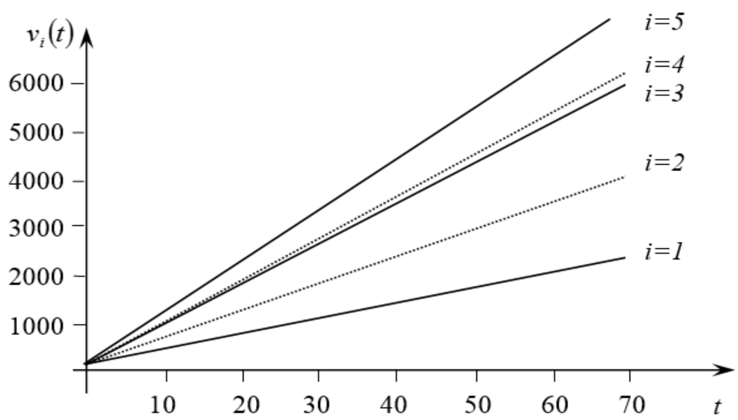

Fig. 2. Expected claims capacity in the systems S_i, $i = \overline{1, 5}$, of the network

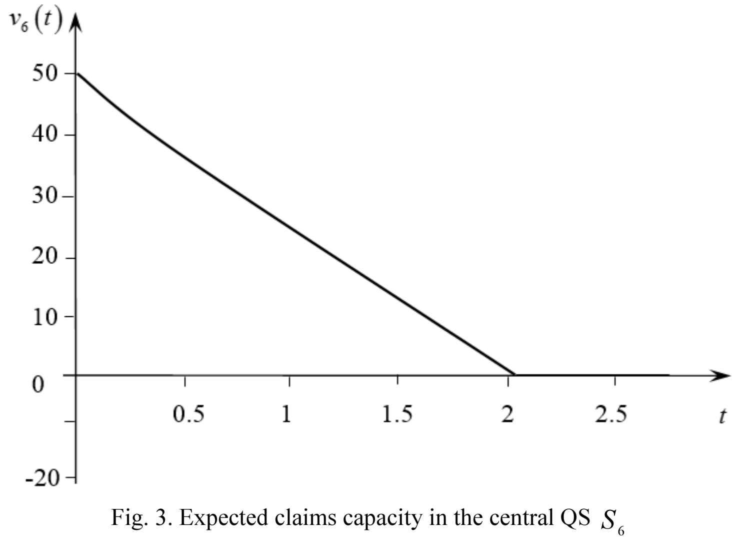

Fig. 3. Expected claims capacity in the central QS S_6

5. Conclusions

Further investigations in this area may be associated with the analysis of arbitrary (non-Markov) networks with claims of random capacity and Markov networks with different features, for example, with unreliable service systems, etc.

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