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A NUMERICAL METHOD FOR VISCOUS FLOW IN A DRIVEN CAVITY WITH HEAT AND CONCENTRATION SOURCES PLACED ON ITS SIDE WALL

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Abstract. This paper proposes a method to numerically study viscous incompressible two-dimensional steady flow in a driven square cavity with heat and concentration sources placed on its side wall. The method proposed here is based on streamfunction-vorticity $(\psi - \xi)$ formulation. We have modified this formulation in such a way that it suits to solve the continuity, x and y-momentum, energy and mass transfer equations which are the governing equations of the problem under investigation in this study. No-slip and slip wall boundary conditions for velocity, temperature and concentration are defined on walls of a driven square cavity. In order to numerically compute the streamfunction ψ, vorticityfunction ξ, temperature θ, concentration C and pressure P at different low, moderate and high Reynolds numbers, a general algorithm was proposed. The sequence of steps involved in this general algorithm are executed in a computer code, developed and run in a C compiler. We propose that, with the help of this code, one can easily compute the numerical solutions of the flow variables such as velocity, pressure, temperature, concentration, streamfunction, vorticityfunction and thereby depict and analyze streamlines, vortex lines, isotherms and isobars, in the driven square cavity for low, moderate and high Reynolds numbers. We have chosen suitable Prandtl and Schmidt numbers that enables us to define the average Nusselt and Sherwood numbers to study the heat ad mass transfer rates from the left wall of the cavity. The stability criterion of the numerical method used for solving the Poisson, vorticity transportation, energy and mass transfer has been given. Based on this criterion, we ought to choose appropriate time and space steps in numerical computations and thereby, we may obtain the desired accurate numerical solutions. The nature of the steady state solutions of the flow variables along the horizontal and vertical lines through the geometric center of the square cavity has been discussed and analyzed. To check the validity of the computer code used and corresponding numerical solutions of the flow variables obtained from this study, we have to compare these with established steady state solutions existing in the literature and they have to be found in good agreement.

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1. Introduction

Viscous flow in a driven square cavity with heat and concentration sources on its side wall is an important phenomenon in the engineering system. In the recent past, this problem has drawn wide attention due to its extensive engineering applications such as solar collectors, cooling of containment buildings, room ventilations, lubrications and drying technologies, electronic packaging and ignition of solid fuels. Since people spent most of their time indoors, the indoor air environment is receiving increasing concern as it is closely related to health comfort and productivity of the occupants. Some applications of heat and concentration sources are cooking, smoking, burning furnace, blazing window, office automation equipment, which continuously produce allergens, heat, gas components and volatile organic compounds.

Computations of unsteady, natural convection in an enclosure by using finite-differences was presented by Torrance [1]. Natural convection in an enclosure with localized heating from below was numerically studied by Torrance and Rockett [2]. Axisymmetric eddies in a rotating stream have been investigated by Kopecky and Torrance [3]. Viscous flow in a square cavity was numerically studied by Bozeman and Dalton [4]. The multigrid method to determine high-resolutions for 2-D incompressible Navier-Stokes equations was presented by Ghia et al. [5]. A Bench mark numerical solution for the problem of natural convection of air in a square cavity has been proposed by Davis [6]. Li et al. [7] have proposed a compact fourth order finite difference scheme for the steady incompressible Navier-Stokes equations. Tian and Ge [9] have proposed a fourth-order compact finite difference scheme on the nine-point 2-D stencil for solving the steady-state Navier-Stokes/Boussinesq equations. Zhang [10] has numerically simulated the 2-D square driven cavity flow at low and high Reynolds numbers.

Mixed convection in a differentially heated square cavity was investigated by Oztop and Dagtekin [11]. Deng et al. [12] have investigated the effects of heat and mass transport of double-diffusive mixed convection in a ventilated enclosure. Erturk et al. [13] have proposed numerical solutions of a 2-D steady incompressible flow in a driven cavity. Numerical solutions of a 2-D steady incompressible flow in a driven skewed cavity was investigated by Erturk and Dursun [14]. Natural convection in a rectangular cavity with partially active side walls was studied by Nithyadevi et al. [15]. Kandaswamy et al. [17] have investigated natural convection in enclosures with partially thermally active side walls containing internal heat sources. Double-diffusive mixed convective flow in a rectangular enclosure was numerically simulated by Teamah and Maghlany [18]. Tian and Yu [19] have proposed an efficient compact finite difference scheme for steady incompressible Navier-Stokes equations. Laminar mixed convection flow in the presence of a magnetic field in a top sided lid-driven cavity heated by a corner heater was investigated by Oztop et al. [20].

Alam et al. [21] have numerically investigated natural convection in a rectangular enclosure with partial heating and cooling at vertical walls. Buoyancy driven natural convection in an enclosure with two discrete heating from below was studied by

Zaman et al. [22]. Natural convection in a rectangular enclosure by a discrete heat source was presented computationally by Qarnia et al. [23]. Ambethkar and Manoj Kumar [24] have presented numerical solutions of a 2-D incompressible flow in a driven square cavity. Alleborn et al. [25] have investigated a lid-driven cavity with heat and mass transport. Ambethkar et al. [26] have investigated numerical solutions of a 2-D incompressible flow in a driven square cavity using streamfunction-vorticity formulation. Nithyadevi et al. [27] have investigated the effect of a Prandtl number on natural convection in a rectangular enclosure with discrete heaters. The solutions of a 2-D steady incompressible viscous flow with heat transfer in a driven square cavity using streamfunction-vorticity formulation was studied by Ambethkar and Manoj Kumar [28] numerically. Lax and Richtmyer [29] have presented a survey on the stability of linear finite difference equations.

The present review of literature cited above revealed the research progress made on natural convection and mixed convective flow in rectangular enclosures. In addition, it has also thrown some light on highly accurate numerical solutions using efficient compact numerical schemes for natural and mixed convective flows. However, in these investigations, the objective was to study the heat transfer rate and therefore, only the availability of heat sources have been considered on the walls of the rectangular enclosures. But no attempt has been made to consider concentration sources on the walls that allow for the study of mass transfer rate. Moreover, existing relevant literature doesn't reveal availability of a method that solves the continuity, x and y-momentum, energy and mass transfer equations which are the governing equations of a 2-D steady viscous incompressible flow in a driven square cavity with heat and concentration sources on its side wall. Since heat and concentration sources usually co-exist indoors as evidenced by numerous applications mentioned above, the present work is to propose a method that allows for the investigation of the problem of a 2-D steady viscous incompressible flow in a driven square cavity with heat and concentration sources on its side wall. To the best of our knowledge, this problem has never been attempted to investigate. So, we have been motivated to attempt to solve this problem in the present work.

The objective of this paper is to propose a method to study a numerically, viscous incompressible two-dimensional steady flow in a driven square cavity with heat and concentration sources placed on its side wall. The method proposed here is based on streamfunction-vorticity $(\psi - \xi)$ formulation. We have modified this formulation in such a way that it suits to solve the continuity, x and y-momentum, energy and mass transfer equations which are the governing equations of the problem under investigation in this study. In order to compute numerically, the streamfunction ψ, vorticityfunction ξ, temperature θ, concentration C and pressure P at different low, moderate and high Reynolds numbers, a general algorithm was proposed. The sequence of steps involved in this general algorithm is executed in a computer code, developed and run in a C compiler. The stability criterion of the numerical method used for solving the Poisson, vorticity transportation, energy and mass transfer has been given. Based on this criterion, we ought to choose appropriate time and space steps in numerical computations and thereby, we may obtain the desired accurate numerical solutions.

2. Mathematical formulation

2.1. Geometry description

The geometry of steady, 2-D incompressible viscous flow in a driven square cavity ABCD with heat and concentration sources placed on its side wall is shown in Figure 1. No-slip boundary conditions for velocity component v is considered on all four sides of the square cavity. No-slip boundary conditions for temperature θ and concentration C are considered on walls AD, DC and BC. Slip boundary condition for velocity $u = 10$, $u = -10$ is considered on walls AD and BC. The wall temperature and concentration of $\theta = 100$ and $C = 100$ are considered on wall AB. At all the four corner points of the square cavity, the velocity components (u, v), temperature θ, concentration C and pressure P assumed to be vanished.

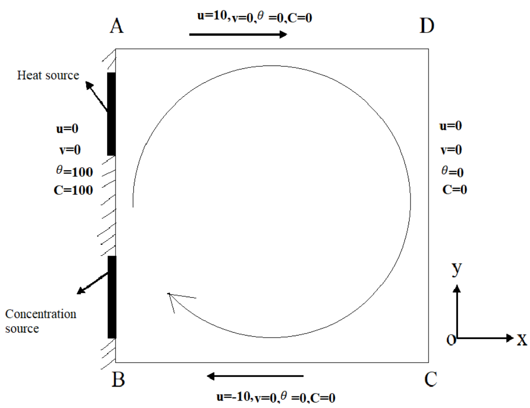

Fig. 1. Schematic diagram of the physical problem

2.2. Governing equations

The problem under consideration is governed by the equation of continuity, x and y components of momentum equations, equations of energy and mass transfer. In dimensional form, these equations can be written as follows:

Continuity equation:
$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

x-momentum equation:
$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial p}{\partial X} + \nu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

y-momentum equation: $$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial p}{\partial Y}+\nu\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right), \quad (3)$$

Energy equation: $$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\alpha\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right), \quad (4)$$

Concentration equation: $$U\frac{\partial c}{\partial X}+V\frac{\partial c}{\partial Y}=D\left(\frac{\partial^2 c}{\partial X^2}+\frac{\partial^2 c}{\partial Y^2}\right). \quad (5)$$

where U, V, p, T, α, ν, c, ρ and D are the velocity components along the x and y axis, pressure, temperature, thermal diffusivity, kinematic viscosity, concentration, density and mass diffusivity respectively. Using the following dimensionless variables:

$$(x,y)=\frac{(X,Y)u_0}{\nu},\quad (u,v)=\frac{(U,V)}{u_0},\quad P=\frac{p}{\rho u_0^2},\quad \theta=\frac{T-T_c}{T_h-T_c},$$

$$C=\frac{c-c_c}{c_h-c_c},\qquad Pr=\frac{\nu}{\alpha},\qquad Sc=\frac{\nu}{D},\qquad Re=\frac{u_0 L}{\nu}.$$

The boundary conditions in dimensional form are

$$\left.\begin{array}{lll}\text{left wall on AB:} & \text{at } X=0,\ U=V=0,\ T=100T_h,\ c=100c_h \\ \text{right wall on DC:} & \text{at } X=L,\ U=V=0,\ T=T_c,\ c=c_c \\ \text{top wall on AD:} & \text{at } Y=L,\ U=10u_0,\ V=0,\ T=c=0 \\ \text{bottom wall on BC:} & \text{at } Y=0,\ U=-10u_0,\ V=0,\ T=c=0\end{array}\right\} \quad (6)$$

The dimensionless governing equations become

Continuity equation: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (7)$$

x-momentum equation: $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \quad (8)$$

y-momentum equation: $$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \quad (9)$$

Energy equation: $$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right), \quad (10)$$

Concentration equation: $$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{1}{Sc}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right). \quad (11)$$

where the non-dimensional variables u, v, P, θ, C, L, u_0, Re, Pr and Sc are the velocity components along the x and y axis, pressure, temperature, concentration, characteristic length, characteristic velocity, Reynolds number, Prandtl number and

Schmidt number respectively. The dimensionless boundary conditions reduces to:

$$\left.\begin{array}{lll} \text{left wall on AB:} & \text{at } x=0, & u=v=0,\ \theta=100,\ C=100 \\ \text{right wall on DC:} & \text{at } x=1, & u=v=0,\ \theta=0,\ C=0 \\ \text{top wall on AD:} & \text{at } y=1, & u=10,\ v=0,\ \theta=C=0 \\ \text{bottom wall on BC:} & \text{at } y=0, & u=-10,\ v=0,\ \theta=C=0 \end{array}\right\} \quad (12)$$

2.3. Determination of pressure for viscous flow

In the streamfunction-vorticity method, to obtain pressure at each grid point for viscous flow, it is necessary to solve an additional equation for pressure. This equation is derived by differentiating the x-momentum equation with respect to x and y-momentum equation with respect to y and adding them together. By using the equation of continuity, the resulting equation reduces to

$$\nabla^2 P = 2\left[\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right) - \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)\right]$$

$$\text{or} \quad \nabla^2 P = S, \ \text{ where } S = 2\left[\left(\frac{\partial^2 \psi}{\partial x^2}\right)\left(\frac{\partial^2 \psi}{\partial y^2}\right) - \left(\frac{\partial^2 \psi}{\partial x \partial y}\right)^2\right] \quad (13)$$

Equation (13) is known as Poisson equation for pressure. A suitable second-order difference representation for the right hand side of equation (13) is given as

$$S_{i,j} = 2\left[\left(\frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{\Delta x^2}\right)\left(\frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta y^2}\right) - \left(\frac{\psi_{i+1,j+1} - \psi_{i+1,j-1} - \psi_{i-1,j+1} + \psi_{i-1,j-1}}{4\Delta x \Delta y}\right)^2\right]. \quad (14)$$

3. Numerical method

3.1. Discretization

Our objective in this paper is to propose a numerical method that solves the equation of continuity, x and y components of momentum equations, equations of energy and mass transfer subject to the given boundary conditions. Based on stream function-vorticity $(\psi - \xi)$ formulation [8, p. 121], the governing equations of a steady 2-D incompressible viscous flow in a driven cavity with heat and concentration sources on its side wall reduces to:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\xi, \tag{15}$$

$$\frac{\partial \xi}{\partial t} = -u\frac{\partial \xi}{\partial x} - v\frac{\partial \xi}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right), \tag{16}$$

$$\frac{\partial \theta}{\partial t} = -u\frac{\partial \theta}{\partial x} - v\frac{\partial \theta}{\partial y} + \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right), \tag{17}$$

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} - v\frac{\partial C}{\partial y} + \frac{1}{Sc}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right). \tag{18}$$

Essentially, the system is composed of the Poisson equation for streamfunction (15), the vorticity-transport equation (16), the energy equation (17) and the mass transfer equation (18). It is intended to obtain the steady state solution from the discretized equations of (15) to (18) in a time marching fashion.

To obtain the numerical solutions, the coupled equations (15) and (18) need to be solved in an iterative manner. Thus, we have used the method developed by Torrance [1] for solving natural convection (Torrance and Rockett [2]) and rotating flow (Kopecky and Torrance [3]) problems, to carry out the numerical computations of the unknown flow variables: ψ, ξ, u, v, P, θ and C for the present problem.

Consider a square numerical grid of size 1×1 having n horizontal interior grid lines and an equal number of vertical grid lines as shown in Figure 2. Applying the second order central difference approximation to both space derivatives in equation (14).

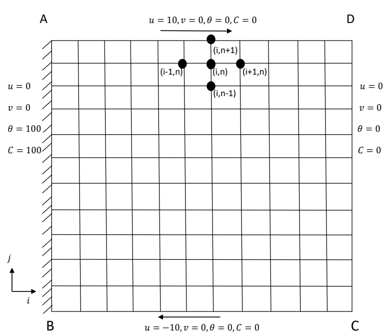

Fig. 2. Finite difference grid of a Square cavity

The discretized Poisson equation (15) for ψ by choosing $\Delta x = \Delta y = h$ reduces

$$\psi_{i+1,j}^{t+1} + \psi_{i-1,j}^{t+1} + \psi_{i,j+1}^{t+1} + \psi_{i,j-1}^{t+1} - 4\psi_{i,j}^{t+1} = -\xi_{i,j}^{t+1} h^2 \tag{19}$$

Now, to solve the vorticity-transport equation (16), the energy equation (17) and the mass transfer equation (18) computationally stable upwind-differencing scheme is used to approximate the first two terms on the right-hand side of this equation.

We first define u_f and u_b as the average x-directional velocities evaluated, respectively, at half a grid point forward and backward from the point (i,j) in x direction, given as

$$u_f = \frac{1}{2}(u_{i+1,j} + u_{i,j}), \quad u_b = \frac{1}{2}(u_{i,j} + u_{i-1,j}) \tag{20}$$

and, similarly, for v

$$v_f = \frac{1}{2}(v_{i,j+1} + v_{i,j}), \quad v_b = \frac{1}{2}(v_{i,j} + v_{i,j-1}) \tag{21}$$

Further defining,

$$\xi_1 = (u_f - |u_f|)\xi_{i+1,j} + (u_f + |u_f| - u_b + |u_b|)\xi_{i,j} - (u_b + |u_b|)\xi_{i-1,j} \tag{22}$$

$$\xi_2 = (v_f - |v_f|)\xi_{i,j+1} + (v_f + |v_f| - v_b + |v_b|)\xi_{i,j} - (v_b + |v_b|)\xi_{i,j-1} \tag{23}$$

the upwind differencing form is preserved. The terms multiplied by $\frac{1}{Re}$ are approximated by central-differencing schemes. For them we let

$$\xi_3 = \xi_{i+1,j} + \xi_{i-1,j} + \xi_{i,j+1} + \xi_{i,j-1} - 4\xi_{i,j} \tag{24}$$

Finally, a forward-differencing scheme is used to approximate the time derivative, so that

$$\left(\frac{\partial \xi}{\partial t}\right)_{i,j} = \frac{(\xi'_{i,j} - \xi_{i,j})}{\Delta t} \tag{25}$$

in which Δt is the size of the time increment and a prime is used to denote the value of a variable evaluated at time $t + \Delta t$. Thus after rearranging terms, (16) becomes

$$\xi'_{i,j} = \xi_{i,j} + \frac{\Delta t}{2h}\left(-\xi_1 - \xi_2 + 2\frac{\xi_3}{Re\ h}\right) \tag{26}$$

Similarly, for the energy equation (17) and mass transfer (18), we obtain

$$\theta'_{i,j} = \theta_{i,j} + \frac{\Delta t}{2h}\left(-\theta_1 - \theta_2 + 2\frac{\theta_3}{Pr\ h}\right), \quad C'_{i,j} = C_{i,j} + \frac{\Delta t}{2h}\left(-C_1 - C_2 + 2\frac{C_3}{Sc\ h}\right). \tag{27}$$

3.2. Specification of boundary conditions

The specification of boundary conditions on the walls *AB*, *DC*, *AD* and *BC* are respectively as follows:

$$\left.\begin{aligned}
\xi_{0,j} &= \frac{2(\psi_{0,j} - \psi_{1,j} + \frac{\partial\psi}{\partial x}|_{0,j}\Delta x)}{\Delta x^2}, \quad \xi_{n+1,j} = \frac{2(\psi_{n+1,j} - \psi_{n,j} - \frac{\partial\psi}{\partial x}|_{n+1,j}\Delta x)}{\Delta x^2}\\
\xi_{i,n+1} &= \frac{2(\psi_{i,n+1} - \psi_{i,n} - \frac{\partial\psi}{\partial y}|_{i,n+1}\Delta y)}{\Delta y^2}, \quad \xi_{i,0} = \frac{2(\psi_{i,0} - \psi_{i,1} + \frac{\partial\psi}{\partial y}|_{i,0}\Delta y)}{\Delta y^2}
\end{aligned}\right\}$$

Elliptic equations with Neumann boundary conditions for pressure $\frac{\partial P}{\partial n} = 0$ on all boundaries, such as the pressure equation (13), present an indeterminate problem, as the coefficient matrix of the finite-difference representation of the equation has one zero eigenvalue [30, p. 269]. Consequently, the resulting system of equations are linearly dependent and can not be solved uniquely. This can be alleviated by assigning a constant value to pressure at one reference point in the solution domain. We have assigned a constant value 5 at one reference point in the solution domain. The resulting equation will have a unique solution for an arbitrary value of the constant.

3.3. Stability

The stability of the numerical scheme (24) used in the numerical discretization has been proved in this section based on the criteria suggested by [30, p. 239]. The equation (24) can be rewritten as

$$\xi'_{i,j} = a_1\ \xi_{i+1,j} + a_2\ \xi_{i-1,j} + a_3\ \xi_{i,j} + a_4\ \xi_{i,j+1} + a_5\ \xi_{i,j-1} \tag{28}$$

where:

$$a_1 = \left[-\frac{\Delta t}{2h}\left(u_f - |u_f|\right) + \frac{\Delta t}{Re\ h^2}\right], \quad a_2 = \left[\frac{\Delta t}{2h}\left(u_b + |u_b|\right) + \frac{\Delta t}{Re\ h^2}\right],$$

$$a_3 = \left[1 - \Delta t\left\{\frac{1}{2h}\left(u_f + |u_f| - u_b + |u_b| + v_f + |v_f| - v_b + |v_b|\right) + \frac{4}{Re\ h^2}\right\}\right],$$

$$a_4 = \left[-\frac{\Delta t}{2h}\left(v_f - |v_f|\right) + \frac{\Delta t}{Re\ h^2}\right], \quad a_5 = \left[\frac{\Delta t}{2h}\left(v_b + |v_b|\right) + \frac{\Delta t}{Re\ h^2}\right].$$

Clearly all the coefficients except for a_3 are positive, no matter what the flow direction is. According to the quasilinear analysis of Lax and Richtmyer [29], the scheme is stable if every coefficient in (25) is positive or, equivalently, if $a_3 \geq 0$, which shows that this requirement gives the stability criterion that

$$\Delta t \leq \left[\frac{1}{2h}\left(u_f + |u_f| - u_b + |u_b| + v_f + |v_f| - v_b + |v_b|\right) + \frac{4}{Re\, h^2}\right]^{-1}. \quad (29)$$

Similarly, the stability criteria to the energy and mass transfer equations reduce to

$$\Delta t \leq \left[\frac{1}{2h}\left(u_f + |u_f| - u_b + |u_b| + v_f + |v_f| - v_b + |v_b|\right) + \frac{4}{Pr\, h^2}\right]^{-1}, \quad (30)$$

$$\Delta t \leq \left[\frac{1}{2h}\left(u_f + |u_f| - u_b + |u_b| + v_f + |v_f| - v_b + |v_b|\right) + \frac{4}{Sc\, h^2}\right]^{-1}. \quad (31)$$

4. Numerical computations

The numerical computation of the unknown flow variables ψ, ξ, u, v, P, θ and C for the present problem could be obtained with the aid of a computer programme developed and run on C compiler. The input data for the relevant parameters in the governing equations like the Reynolds number Re, Prandtl number Pr and Schmidt number Sc should be properly chosen to be incompatible with physical significance of the present problem. The value of the Prandtl number $Pr = 6.75$ and $Sc = 0.60$ have been chosen for water.

4.1. General algorithm

The general algorithm for computing the numerical solutions using the proposed method consists of the following sequence of steps:

Step 1. Specify the initial values for ξ, ψ, u, θ, C and v at time $t = 0$.

Step 2. Solve equation (16) for ξ at each interior grid point at time $t + \Delta t$.

Step 3. Solve equation (17) for θ and (18) for C at each interior grid point at time $t + \Delta t$.

Step 4. Iterate for new ψ values by solving (15) using new ξ values at interior points.

Step 5. Find the velocity components $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

Step 6. Determine the values of ξ on the boundaries using ψ and ξ values at interior points.

Step 7. Return to step 2 if the solution is not converged.

Step 8. Solve the Poisson equation (13) for P using the calculated ψ values at each grid point.

5. Results and discussion

A numerical method to study viscous, two-dimensional, incompressible, steady flow in a driven square cavity with heat and concentration sources placed on its side wall has been proposed in this study. This method, proposed here, has been implemented and verified to be well suited to solve the equation of continuity, x and y components of momentum equations, equations of energy and mass transfer subject to the given boundary conditions in this work. The qualitative study of results from this study is presented in terms of the importance and need of a method that solves the complex system of semi-linear p.d.e's along with the given boundary conditions which defines the physical problem mentioned above at different instances.

In Section 2.2, we have defined the governing equations (1) to (5) in dimensional form. By defining and using appropriate dimensionless variables and parameters in the dimensional form of the governing equations, we have obtained and presented the governing equations (7) to (11) in the dimensionless form. By considering the concentration source in addition to a heat source on the wall, we obtained an additional equation (11) with an appropriate boundary condition for concentration variable which needs to be solved in addition to the other three coupled equations subject to the boundary conditions. The purpose here is to discuss and analyze the flow patterns, isotherms, iso-concentrations for different heating and concentration sections of the square cavity. The rate of heat and mass transfer in the cavity is measured in terms of the average Nusselt and Sherwood number. In order to investigate the qualitative study of these results from this study, the governing equations (7) to (11) reduces to (15) to (18) [As mentioned in Section 3.1] after using the stream function-vorticity $(\psi - \xi)$ formulation [8, p. 121].

Since our target here is to determine the numerical solutions, the coupled equations need to be solved in an iterative manner. We propose here that one can use the method of finite differences developed by Torrance [1], Torrance and Rockett [2] for solving natural convection in an enclosure and Kopecky and Torrance [3] for rotating flow problems, to carry out the numerical computations of the unknown flow variables: ψ, ξ, u, v, P in the present problem. To ensure the accuracy of numerical solutions of these flow variables, we need to establish the stability of the numerical scheme used in the numerical discretization. We have proved the stability of the scheme based on the criteria suggested in [29, p. 239]. We have obtained the stability criteria for the x- and y-momentum, energy and mass transfer equations based on the quasilinear analysis of Lax and Richmayer [29]. By choosing appropriate time and space steps in numerical computations, we obtain the desired accurate numerical solutions. These results are presented in section 3.3. In section 4.1, we have presented a general algorithm and the sequence of steps involved that enable us to execute a computer code developed and run in C compiler. We propose that, with the help of this code, one can easily compute the numerical solutions of the flow variables such as velocity, pressure, temperature, concentration, streamfunction, vorticityfunction and thereby depict and analyze streamlines, vortex lines, isotherms, isobars, in the driven square cavity for low, moderate and high Reynolds numbers. We have chosen suitable Prandtl and Schmidt numbers which enable us to define the average Nusselt and Sherwood numbers as

$$\overline{Nu} = -\int_0^1 \left(\frac{\partial \theta}{\partial X}\right)_{X=0} dY, \quad \overline{Sh} = -\int_0^1 \left(\frac{\partial C}{\partial X}\right)_{X=0} dY$$

respectively. From this, one can discuss heat and mass transfer rates from the left wall of the cavity.

6. Conclusions

In this study, we proposed a method to study a numerically, viscous incompressible two-dimensional steady flow in a driven square cavity with heat and concentration sources placed on its side wall. We have modified the streamfunction-vorticity $(\psi - \xi)$ formulation in such a way that it suits to solve the continuity, x and y-momentum, energy and mass transfer equations which are the governing equations of the problem under investigation in this study. By defining appropriate dimensionless variables and parameters and using them in the dimensional form of the governing equations, we have obtained and presented the governing equations of the present problem in the dimensionless form. The purpose here is to discuss and analyze the flow pattern, isotherms, iso-concentrations for different heating and concentration sections of the square cavity. The rate of heat and mass transfer in the cavity is measured with the help of the average Nusselt and Sherwood number. We have obtained the stability criteria for the x and y- momentum, energy and mass transfer equations based on the quasilinear analysis. We have presented a general algorithm that enable to execute a computer code developed and run in C compiler. We propose that, with the help of this code, one can easily compute the numerical solutions of the flow variables such as velocity, pressure, temperature, concentration, stream function, vorticity function and thereby depict and analyze streamlines, vortex lines, isotherms, isobars, in the driven square cavity for low, moderate and high Reynolds numbers. We propose that the heat and mass transfer rates from the left wall of the cavity are studied by choosing suitable Prandtl, Schmidt, the average Nusselt and Sherwood numbers.

Nomenclature

Δx, Δy	grid spacing along x and y-axis
u_f, u_b	average x-directional velocities evaluated at half a grid point forward and backward from the point (i, j)

v_f, v_b	average y-directional velocities evaluated at half a grid point forward and backward from the point (i, j)
$\lvert u_f\rvert$, $\lvert u_b\rvert$, $\lvert v_f\rvert$, $\lvert v_b\rvert$	absolute value of u_f, u_b, v_f, v_b
$\psi_{i,j}^{t+1}$, $\xi_{i,j}^{t+1}$	streamfunction and vorticityfunction at (i, j) node at time $t+1$
c_h, c_c	concentration at left and right wall of the cavity
T_h, T_c	hot and cold wall temperature
t, ∇^2	time level and Laplacian operator

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