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A RESEARCH STUDY ON UNSTEADY STATE CONVECTION DIFFUSION FLOW WITH ADOPTION OF THE FINITE VOLUME TECHNIQUE

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Abstract. This research paper is an attempt to solve the unsteady state convection diffusion one dimension equation. It focuses on the fully implicit hybrid differencing numerical finite volume technique as well as the fully implicit central differencing numerical finite volume technique. The simulation of the unsteady state convection diffusion problem with a known actual solution is also used to validate both the techniques, respectively, the fully implicit hybrid differencing numerical finite volume technique as well as the fully implicit central differencing numerical finite volume technique by giving a particular example and solving it using the appropriate, particular technique. It is observed that the numerical scheme is an outstanding deal with the exact solution. Numerical results and graphs are presented for different Peclet numbers.

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1. Introduction

Many physical problems involve the combination of convective and diffusive processes. They occur in fields where mathematical modelling is important. The application of convective as well as diffusive processes is observed across industries and across countries, such as the engineering sector, the farming sector, biological research, and the heat transportation sector, in the special case of fluid dynamics [1, 2]. In the field of applied mathematics, problems related to boundary values like unsteady state convection-diffusion equations, second order partial differential equations have great significance [3, 4]. In order to solve this equation, a domain must contain specific conditions for the unresolved functions satisfying at its boundaries. The Dirichlet boundary condition is specified at a particular section of the boundary [5]. When the identification of the function of normal derivative is subject to specific cases, Neumann boundary conditions are those located on parts of the boundary. In test related problems, the entire boundary condition can be the Dirichlet boundary condition. Consider the one-dimensional unsteady state convection-diffusion flow equation [4] as

$$\frac{\partial(\rho T)}{\partial t}+\frac{\partial(\rho u T)}{\partial x} = \frac{\partial}{\partial x}\left(\Gamma\frac{\partial T}{\partial x}\right)+S, \quad t\geq 0,\ x\in[0,L] \tag{1}$$

$$\text{subject to initial condition, } T(x,0) = f(x),\ x\in[0,L], \tag{2}$$

$$\begin{aligned} \text{and boundary condition, } T(0,t) &= g_0(t),\ t\geq 0, \\ T(L,t) &= g_1(t),\ t\geq 0 \end{aligned} \tag{3}$$

Equation (1) is a convective diffusive fluid flow linear partial differential equation. The variables u and Γ are the velocity and diffusion coefficient, respectively.

The problem of unsteady state convection-diffusion has more applications in heat transfer and fluid dynamics [6]. In the field of applied mathematics, apart from others, fluid dynamics is a scientific discipline of great importance, in which relevant equations are usually Partial Differential equations. Such an equation has an analytic solution are only possible for a restricted and limited number of cases [7]. Due to this, several numerical techniques have been developed for the numerical approximations of convection-diffusion problems [8,9]. The finite difference approximation method, the finite element method, as well as finite volume techniques are most widely used for computational fluid dynamics (CFD) [10, 11]. The previous extensive research work was carried out by many researchers where numerical approximation of conviction diffusion questions was compared with finite difference as well as finite volume techniques [12,13]. Research work carried out by Kaya [3] provided insight into developing a polynomial based differential orthogonal method and some applications compared results with explicit and implicit finite difference approximation methods. It also found that differential quadrature gave better results [10]. Computational related research work for three numerical schemes based on the finite difference method has been used for third order transfer schemes, fourth order transfer schemes, as well as non-standard finite difference schemes. Yaghoubi [13] also analyzed high-order finite difference schemes. Aswin et al. [12] attempted to provide a description of the relative study of three numerical schemes and concluded that PDQM provides more accurate time and space results between the three schemes.

The finite volume scheme is one of the most important numerical methods among others with wide adaptation in numerical methods, which is widely used in numerical fluid dynamics to solve the convection-diffusion problem [14, 15]. The finite volume technique specialises in types of heat and fluid flow problems. Considering the wide adaptation, finite volume can be an appropriate method to determine numerical solution diffusion caused by convection [16]. For computational fluid dynamics (CFD) related research studies, finite volume has universal acceptance, and computational fluid dynamics (CFD) is a pre-requisite for solving partial differential equations. The Finite volume scheme provides freedom in terms of its key advantage for performing flow estimation on the element boundary [4, 17, 18].

This research article consists of the one dimensional convective diffusion flow equation under unsteady state conditions with a finite volume technique applied. In Section 2, a detailed discussion of the formulation and discretized form of the unsteady state one-dimensional problem for one dimension of the unsteady convective diffusive flow finite volume method is introduced in this article. The numerical scheme is discussed in Section 3, and the corresponding initial and boundary conditions have accurate solutions, and it is sufficient to analyze the physical requirements. For boundary conditions, Section 4 discusses the results of diffusive flow caused by convection under unsteady conditions, including the diffusive flow diagram caused by convection. Lastly, in Section 5, this paper concludes.

2. Finite volume method

The finite volume numerical method is representing and evaluating partial differential equations in terms of algebraic equations [2]. The finite volume method is easy to understand and also suitable for appropriate physical interpretation. This is the biggest advantage of this method. Control volume is an arithmetic area divided into discrete (non-overlapping) elements. Therefore, each control body is contained in a node of the computing grid. Finite volume techniques pool the small finite volume around each node. After applying the Gaussian divergence theorem, we obtain the surface integral. The flow entering the finite volume is exactly the same as the flow leaving the adjacent volume. That is why those elements are calculated as the flow velocity of every final finite volume surface. There is a lot of evidence that these plans are conservative. Numerical finite volume methods are becoming more and more popular in the estimated solution of partial differential equations (PDEs). Readers can find extensive information in [1, 4]. The finite volume analysis consists of three main steps.

Step 1) Grid Generation:

Let us divide the domain into every grid point in this space, which is managemental by grid precipitation or volume, therefore the control volume limit and physical limit overlap as presented in Figure 1

Step 2) Discretization:

The blending of governing equations over a control volume is a necessary component of the finite volume method. The technique of integrating governed equations over the control volume yields discretization forms of the simultaneous linear equa-

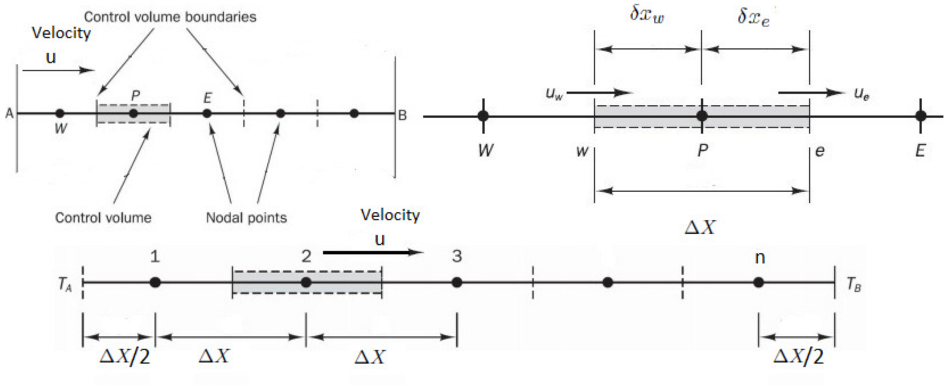

Fig. 1. Grid generation

tions at the grid points and then we will have an inclination to discretization, Eq. (1), after implementing integration over the entire control volume that's assumed to be non-deforming with a further integration over a finite time step ("Δt")

$$\int_{cv}\int_{t}^{(t+\Delta t)}\frac{\partial(\rho T)}{\partial t}dtdv+\int_{t}^{(t+\Delta t)}\int_{cv}\frac{\partial(\rho uT)}{\partial x}dvdt$$
$$=\int_{t}^{(t+\Delta t)}\int_{cv}\frac{\partial}{\partial x}\left(\Gamma\frac{\partial T}{\partial x}\right)dvdt+\int_{t}^{(t+\Delta t)}\int_{cv}Sdvdt$$

$$\int_{cv}\rho(T_P-T_P^0)dv+\int_{t}^{(t+\Delta t)}(\rho uAT)_e-(\rho uAT)_w dt$$
$$=\int_{t}^{(t+\Delta t)}\left(\Gamma A\frac{\partial T}{\partial x}\right)_e-\left(\Gamma A\frac{\partial T}{\partial x}\right)_w dt+\int_{t}^{(t+\Delta t)}\overline{S}\Delta v dt \qquad (4)$$

In equation (4), Face area of control volume is denoted by A, the volume Δv is equal to $A\Delta x$, $\overline{S}$ is average source strength, T_P^0 denoted as temperature distribution at time t and T_P denoted as temperature distribution at time $(t+\Delta t)$.

To obtain a discretized equation for the one dimensional unsteady state convection diffusion equation, we have to estimate the term in equation (4) [1,4]. The flux per unit per area and conductive diffusion at the cell face are necessary to define variables F as well as D. It is necessary to define variables F as well as D given by $F=\rho u$ and $D=\frac{\Gamma}{\delta x}$. The variables F and D (cell face values) are often written as $F_{cellface}=(\rho\ u)_{cellface}$, $D_{cellface}=\left(\frac{\Gamma_{cellface}}{\delta x_{cellface}}\right)$, and for one dimensional unsteady convection-diffusion flow, cell face values as west (w) and east (e). For finite volume numerical techniques, $A_e = A_w = A$, thus we get

$$\rho \cdot \left(T_P - T_P^0\right)\Delta x + \int_t^{(t+\Delta t)} (F_e T_e - F_w T_w) dt$$
$$= \int_t^{(t+\Delta t)} D_e(T_E - T_P) - D_w(T_P - T_W) dt + \int_t^{(t+\Delta t)} \overline{S}\Delta v dt \qquad (5)$$

To evaluate the convection and diffusion terms in equation (5) over time level t to $(t+\Delta t)$, we have to make associated suppositions regarding the variability of T_E, T_P and T_W with time. The weighting parameter $\theta \in [0,1]$ and I_T denote the integral of temperature with time t defined in the following way:

$$I_T = \int_t^{(t+\Delta t)} T_P \, dt = (\ \theta T_P + (1-\theta) T_P^0\ )\ \Delta t \qquad (6)$$

by using the equation (6) approach we get

$$\rho(T_P - T_P^0)\Delta x + [\theta(F_e T_e - F_w T_w) + (1-\theta)(F_e T_e - F_w T_w)]\Delta t = [\theta(D_e(T_E - T_P)$$
$$-D_w(T_P - T_W)) + (1-\theta)(D_e(T_E - T_P) - D_w(T_P - T_W))]\Delta t + \overline{S}\Delta x \Delta t$$

For fully implicit finite volume method, consider weighting parameter $\theta = 1$, then we get

$$\rho \cdot (T_P - T_P^0)\Delta x + (F_e T_e - F_w T_w)\Delta t = [D_e(T_E - T_P) - D_w(T_P - T_W)]\Delta t + \overline{S}\Delta x \Delta t$$
$$\frac{\rho \Delta x}{\Delta t}(T_P - T_P^0) = [D_e(T_E - T_P) - D_w(T_P - T_W)] - (F_e T_e - F_w T_w) + \overline{S}\Delta x \qquad (7)$$

2.1. Methodology for fully implicit central difference scheme

The numerical approximation central differencing scheme is used to describe the diffusion terms values which occur on the right hand side of an equation (7) [4], and it is interpolated linearly to evaluate the left hand side cell face values for the convection terms in an equation. We can write the cell face values for a uniform grid of T as

$$T_e = \frac{T_P + T_E}{2}, \qquad T_w = \frac{T_W + T_P}{2}$$

In the convective term, substitute the expressions value of T_e and T_w into equation (7). We get

$$\frac{\rho\Delta x}{\Delta t}(T_P - T_P^0) = [D_e(T_E - T_P) - D_w(T_P - T_W)]$$
$$-\left(F_e\frac{T_P + T_E}{2} - F_w\frac{T_W + T_P}{2}\right) + \overline{S}\Delta x$$

This can be rearranged to give the discretized form of the fully implicit central differencing numerical scheme is illustrated as follows, [1,4,19]

$$a_P T_P = a_W T_W + a_E T_E + a_P^0 T_P^0 + S_u \tag{8}$$

From the linear source term $\overline{S}\Delta x = S_u + S_P T_P$, we obtained the value of the term S_u and S_P with use of boundary b. The boundary temperature T_A and T_B are denoted as T_b.

$$a_E = D_e + \frac{F_e}{2}, \quad a_W = D_w + \frac{F_w}{2}, \quad S_P = -(2D \pm F), \quad S_u = -(2D \pm F)T_b,$$
$$a_P^0 = \frac{\rho\Delta x}{\Delta t}, \quad a_P = a_W + a_E + a_P^0 + F_e - F_w - S_P$$

2.2. Methodology for fully implicit hybrid scheme

The combination of upwind and central schemes is the hybrid difference numerical scheme. The second order accurate central difference scheme is used for small Peclet numbers $\left(Pe = \frac{F}{D} < 2\right)$, and the first order convergence upwind scheme is used for large Peclet number $\left(Pe = \frac{F}{D} \geq 2\right)$. Fully implicit hybrid discretization of differential equation of unsteady convection diffusion one dimensional equation [4], which is expressed as the equation (9) [1,4,19]

$$a_P T_P = a_W T_W + a_E T_E + a_P^0 T_P^0 + S_u \tag{9}$$

where

$$a_p = a_W + a_E + a_P^0 + F_e - F_w - S_P$$

From the linear source term $\overline{S}\Delta x = S_u + S_P T_P$, we obtained the value of the term S_u and S_P with use of boundary b. The boundary temperature T_A and T_B are denoted as T_b.

$$a_W = \max\left[F_w, \left(\frac{F_w}{2} + D_w\right), 0\right], \; a_E = \max\left[-F_e, \left(-\frac{F_e + D_e}{2}\right), 0\right],$$
$$a_P^0 = \frac{\rho\Delta x}{\Delta t}, \quad S_p = -(2D \pm F), \quad S_u = (2D \pm F)T_b \tag{10}$$

Step 3) Execution and Implementation:

We have a simultaneous linear algebraic equation after discretization over every control volume. Generally, simultaneous linear algebraic equations are in tri-diagonal form, which can be solved using numerical methods like the tri-diagonal matrix algorithm (TDMA), Gauss-Siedel technique, and successive over-relaxation (SOR) method [20].

3. Numerical results

Problem: 1 Unsteady state convection diffusion equation Eq. (1) is subject to initial condition

$$T(x,0) = f(x) = \sin(2\pi x), \; x \in [0,1] \tag{11}$$

and to boundary condition

$$\begin{aligned} T(0,t) = g_0(t) &= e^{(-4\Gamma\pi^2 t) sin(-2\pi\, ut)}, \\ T(1,t) = g_1(t) &= e^{(-4\Gamma\pi^2 t) sin(2\pi\,(1-ut))},\; t \in [0,T] \end{aligned} \tag{12}$$

In this example, consider $\Gamma = 0.005$, $\rho = 1$ and $u = 2.5$. Analytic solution [12] of Eq. (1) is subject to initial condition (11) & boundary condition (12) is

$$T(x,t) = e^{(-4\Gamma\pi^2 t) sin(2\pi(x-ut))} \tag{13}$$

3.1. Fully implicit central scheme numerical result

For the numerical result of the fully-implicit central differencing scheme, let us consider the equation (1) is subject to boundary and initial conditions given in (11) and (12) respectively. Now, coefficients and source terms of the fully implicit central differencing scheme for equation (1) are in Table 1, where $\Delta t = 0.001$ and $\Delta x = 0.05$. After substitution of coefficient numerical values of Table 1 in equation

Table 1. Source term and coefficients for all nodes of central method

Nodes	a_W	a_E	S_P	S_u
1	0	$\left(D_e - \frac{F_e}{2}\right) = -1.15$	$-(2D+F) = -2.7$	$(2D+F)T_b$
2-19	$\left(D_w + \frac{F_w}{2}\right) = 1.35$	$\left(D_e - \frac{F_e}{2}\right) = -1.15$	0	0
20	$\left(D_w + \frac{F_w}{2}\right) = 1.35$	0	$-(2D-F) = 2.3$	$(2D-F)T_b$

(8) for different values of time t, then for the various nodes we have discretized equations. These simultaneous linear equations are evaluated in MATLAB [20] by using the successive-over relaxation (SOR) method, and then temperature profile values for all nodes (x) are represented in Table 3.

3.2. Fully implicit hybrid scheme numerical result

For the numerical result of the fully implicit hybrid scheme, let us consider the equation (1) is subject to boundary and initial conditions given in (11) and (12) respectively. Now, coefficients and source terms of the fully implicit hybrid differencing scheme for equation (1) are represented in Table 2, where $\Delta t = 0.001$ and $\Delta x = 0.05$, After substitution of coefficient numerical values of Table 2 in equation (9) for different values of time t, then for the various nodes we have discretized equations. These simultaneous linear equations are evaluated in MATLAB [20] using the successive-over relaxation (SOR) method, and then temperature profile values for all nodes (x) are represented in Table 3.

Table 2. The source term and coefficients for all nodes of hybrid

Nodes	a_W	a_E	S_P	S_u
1	0	$\max\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right] = 0$	$-(2D+F) = -2.7$	$(2D+F)T_b$
2-19	$\max\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right] = 2.5$	$\max\left[-F_e, \left(D_e - \frac{F_e}{2}\right), 0\right] = 0$	0	0
20	$\max\left[F_w, \left(D_w + \frac{F_w}{2}\right), 0\right] = 2.5$	0	$-(2D-F) = -0.2$	$(2D-F)T_b$

3.3. Norm based error for L_p

Using the fully-implicit central and the fully-implicit hybrid numerical schemes, the numerical solution of the convective-diffusive heat flow equation in the unsteady state is obtained. According to the defined rules of norm L_2 and L_∞, the accuracy of the proposed scheme for error measurement of numerical schemes is high.

$$\begin{aligned} L_2 &= \left[\frac{1}{\Delta x}\sum_{i=1}^{N}\left[T_i^{(numerical)} - T_i^{(exact)}\right]^2\right]^{\frac{1}{2}} \\ L_\infty &= max\left\{\left|T_i^{(numerical)} - T_i^{(exact)}\right| / 1 \le i \le N\right\} \end{aligned} \tag{14}$$

The MATLAB programming simulated results of the given problem: 1 can be found in Tables 3 and 4 and Figures 2-9 under the initial and boundary conditions.

Table 3. Numerical results of the convection diffusion unsteady state equation

Nodes (x)	time (t)	Central	Hybrid	Analytical	Error(Hybrid)
0.125	0.004	0.655719046	0.661261289	0.66078992	0.000713341
	0.006	0.62975134	0.637423385	0.636669502	0.001184104
	0.008	0.603701626	0.613005535	0.611939953	0.001741317
	0.01	0.577618809	0.588030761	0.586626155	0.002394379
0.325	0.004	0.907927903	0.916183577	0.917030282	0.000923312
	0.006	0.914942698	0.927443186	0.928675955	0.001327447
	0.008	0.920997355	0.937803734	0.939396162	0.001695161
	0.01	0.92609594	0.947256467	0.949181058	0.002027633
0.725	0.004	–0.966217972	–0.974914905	–0.975146513	0.000237511
	0.006	–0.954236006	–0.967142016	–0.967436696	0.000304599
	0.008	–0.941449385	–0.958456158	–0.958778447	0.000336146
	0.01	–0.927878735	–0.948866769	–0.949181058	0.000331115
0.925	0.004	–0.502840222	–0.507511154	–0.508639651	0.002218658
	0.006	–0.526172296	–0.533520488	–0.535192563	0.003124248
	0.008	–0.548755316	–0.559002413	–0.561196472	0.00390961
	0.01	–0.570574987	–0.583937656	–0.586626155	0.004582985

Table 4. At different time level norm based errors for problem:1 with $\Delta t = 0.001$

	Hybrid Scheme		Central Scheme	
Time	L_∞ Norm	L_2 Norm	L_∞ Norm	L_2 Norm
0.001	0.000298862	0.000202731	0.006741794	0.002267787
0.002	0.000621497	0.000405627	0.013159978	0.004481622
0.003	0.000968218	0.000608858	0.019269456	0.006646206
0.004	0.001339323	0.00081259	0.025084361	0.008765737
0.005	0.001735094	0.001016991	0.0306181	0.010843963
0.006	0.002155793	0.001222225	0.035883389	0.012884228
0.007	0.002601667	0.001428457	0.04089229	0.014889517
0.008	0.003072945	0.00163585	0.045656243	0.016862491
0.009	0.003569835	0.001844564	0.050186101	0.018805523
0.01	0.00409253	0.002054758	0.054492159	0.020720729

It was found that the fully implicit hybrid numerical scheme gives better results than the fully implicit central numerical scheme shown in Table 3.

4. Results and discussion

To analyse the approximate value of the finite-volume of the convective diffusion flow in an unsteady state, observe the temperature profile at each node point in Figures 2-9. In Figures 2 and 3 show the stable plot of the fully implicit central difference scheme and fully implicit hybrid difference scheme plot for Peclet number, $Pe = 25$ of one dimensional convection and diffusion equation (1) subject to initial condition (11) and boundary condition (12) for two different time slots of $t = 0.004, 0.008$.

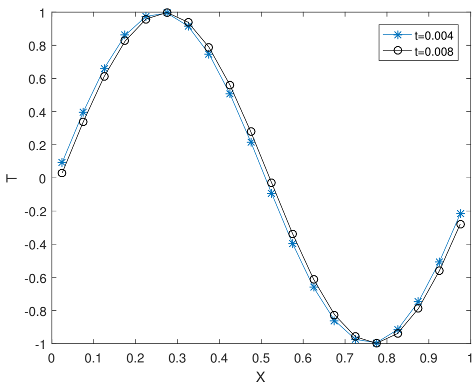

Fig. 2. Hybrid temperature profile for Pe = 25

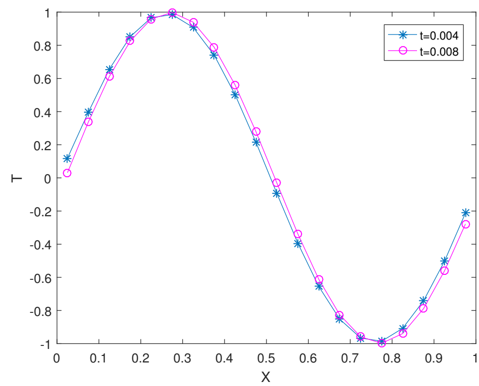

Fig. 3. Central temperature profile for Pe = 25

Figures 4 and 5 are two kinds of norm L_2 and L_∞ errors that behave in completely

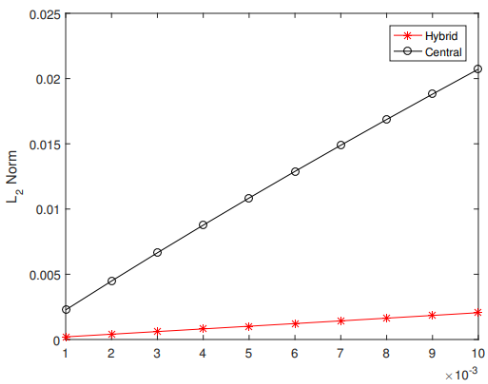

Fig. 4. L_2 norm error

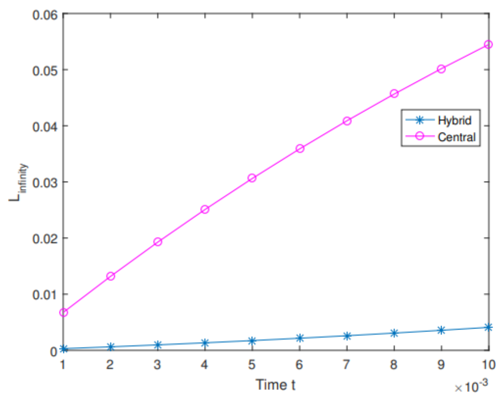

Fig. 5. L_∞ norm error

different ways. This estimation of errors tells us the convergence order of the numerical scheme. Here, in this research paper, the tendency to determine that the fully implicit hybrid numerical scheme provides higher convergence as compared to the fully implicit central numerical scheme, and Figures 6 and 7 represented the temperature profile for Peclet number, $Pe = 500$ of both numerical scheme for t = 0.01. This Peclet number was achieved by taking domain length $L = 5$, node $N = 100$, velocity $u = 50$ and density $\rho = 1$ subject to initial condition (11) & boundary condition (12).

Figures 8 and 9 represented the temperature profile for Peclet number, $Pe = 2000$ of both numerical schemes for t = 0.01. This Peclet number is achieved by taking domain length $L = 10$, node $N = 200$, velocity $u = 50$ and density $\rho = 1$ subject to initial condition (11) and boundary condition (12).

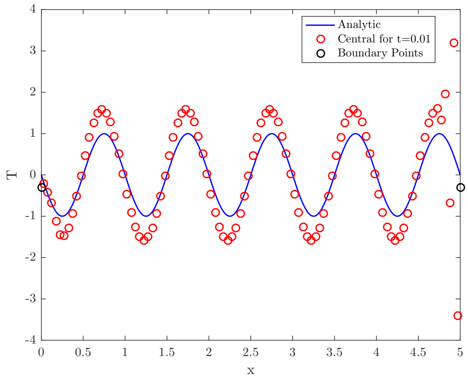

Fig. 6. Central temperature profile for Pe = 500

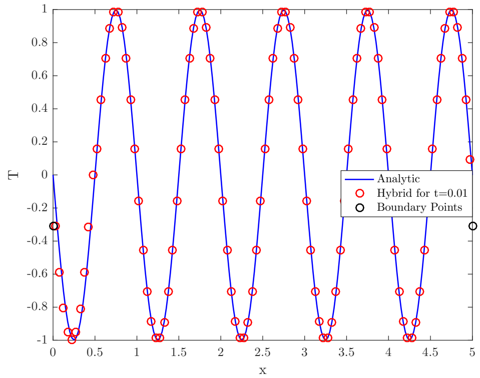

Fig. 7. Hybrid temperature profile for Pe = 500

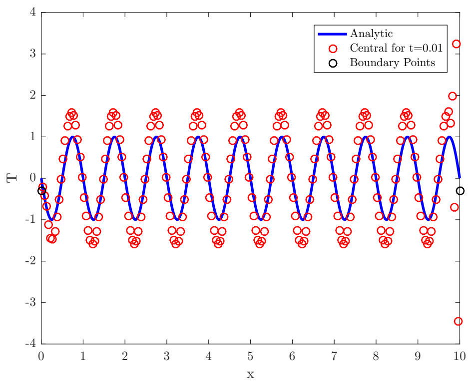

Fig. 8. Central temperature profile for Pe = 2000

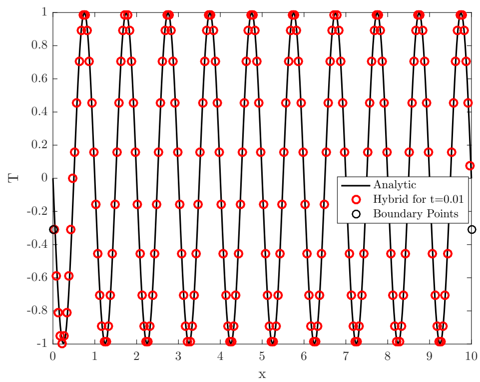

Fig. 9. Hybrid temperature profile for Pe = 2000

5. Conclusion

The analysis of obtained results confirms the good compatibility of the two techniques, which proves their correctness. Changes in temperature distribution due to time are clearly visible. A fully implicit central difference scheme and a fully implicit hybrid difference scheme for the unsteady state convection-diffusion one dimensional equation have been presented. For the test examples studied, it has been found that the fully implicit hybrid difference scheme gives better point-wise solutions than the fully implicit central difference scheme. The hybrid difference scheme derives benefits from the favorable properties of the central and upwind schemes. It is switched to upwind differencing when central differencing produces inaccurate results at high Peclet number (Pe). The coefficients are always positive because the scheme is fully conservative and it is unconditionally bounded. Using the upwind formulation for large values of Peclet number (Pe), it satisfies the transportive requirements.

The scheme produces physically realistic solutions and is unconditionally stable. Built techniques may be modified in a simple way to solve two-dimensional and three-dimensional unsteady state convection-diffusion problems. In this case, the analytical method will be limited to simple geometry, but in the case of a numerical model, such constraints do not exist.

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