SENSITIVITY ANALYSIS OF BURN INTEGRALS WITH RESPECT TO THICKNESS OF EPIDERMIS

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Abstract. In the paper the numerical analysis of thermal process proceeding in the domain of one-dimensional skin tissue subjected to an external heat source is presented. The degree of the skin burn can be predicted on the basis of Henriques integrals. Main subject of paper is the sensitivity analysis of these integrals with respect to the thicknesses of epidermis and dermis. On the stage of numerical realization the boundary element method has been used.

1. Governing equation

The skin is treated as a multilayer domain, in which one can distinguish the following sub-domains: epidermis Ω_1 of thickness $L_1 - L_0$ [m], dermis Ω_2 of thickness $L_2 - L_1$ and sub-cutaneous region Ω_3 of thickness $L_3 - L_2$ - Figure 1.

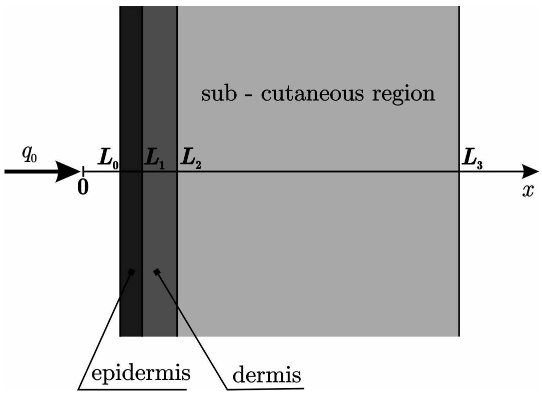

Fig. 1. Skin tissue domain

The transient bioheat transfer in the domain of skin is described by the following system of Pennes equations [1]:

$$x \in \Omega_e : \quad c_e \frac{\partial T_e}{\partial t} = \lambda_e \frac{\partial^2 T_e}{\partial x^2} + k_e (T_B - T_e) + Q_{me} \tag{1}$$

where e identifies the epidermis, dermis and sub-cutaneous region, λ_e [W/mK] is the thermal conductivity and c_e [J/m^{3}K] is specific heat per unit of volume, $k_e = G_e c_B$ is the product of blood perfusion rate and volumetric specific heat of blood, T_B is the blood temperature and Q_{me} is the metabolic heat source. It should be pointed out that for the epidermis sub-domain ($e = 1$) $G_1 = 0$ and $Q_{m1} = 0$.
On the contact surfaces between sub-domains considered the continuity conditions are given, namely

$$x \in \Gamma_{e,e+1}: \quad \begin{cases} -\lambda_e \dfrac{\partial T_e}{\partial x} = \lambda_{e+1} \dfrac{\partial T_{e+1}}{\partial x}, \quad e = 1,2 \\ T_e = T_{e+1} \end{cases} \tag{2}$$

Additionally

$$x \in \Gamma_0: \quad q = \begin{cases} q_0, & t \le t_0 \\ \alpha(T - T^{\infty}), & t > t_0 \end{cases} \tag{3}$$

where $q = \lambda_1 \partial T_1/\partial x$, q_0 is the given boundary heat flux, t_0 is the exposure time, α is the heat transfer coefficient, T^{∞} is the ambient temperature. For conventionally assumed boundary limiting the system the no-flux condition

$$x \in \Gamma_3: \quad q_3 = 0 \tag{4}$$

can be accepted. For $t = 0$ the initial temperature distribution is known, namely

$$t = 0: \quad T_1 = T_{1p}(x), \quad T_2 = T_{2p}(x), \quad T_3 = T_{3p}(x) \tag{5}$$

A quadratic initial temperature distribution between 32.5°C at the surface and 37°C at the base of the sub-cutaneous region was introduced [1].
Thermal damage of skin begins when the temperature at the basal layer (the interface between epidermis and dermis) rises above 44°C (317 K). Henriques [2] found that the degree of skin damage could be predicted on the basis of the integrals

$$I_b = \int_0^{\tau} P_b(T_b) \exp\left(-\frac{\Delta E}{R T_b(t)} \right) \mathrm{d}t \tag{6}$$

and

$$I_d = \int_0^{\tau} P_d(T_d) \exp\left(-\frac{\Delta E}{R T_d(t)} \right) \mathrm{d}t \tag{7}$$

where $\Delta E/R$ [K] is the ratio of activation energy to universal gas constant, P_b, P_d [1/s] are the pre-exponential factors, while T_b, T_d [K] are temperatures of basal

layer (the surface between epidermis and dermis) and dermal base (the surface between dermis and sub-cutaneous region).
First degree burn are said to occur when the value of the burn integral (6) is from the interval $0.53 < I_b \le 1$, while the second degree burn when $I_b > 1$ [1, 2]. Third degree burn are said to occur when $I_d > 1$. So, in order to determine the values of integrals I_b, I_d the heating and next the cooling curves for the basal layer and dermal base must be known.

2. Shape sensitivity analysis

In the paper [3] the sensitivity analysis of temperature field in domain of skin tissue with respect to thermophysical parameters of skin has been presented. Here, the shape sensitivity analysis of temperature distribution and burn integrals is discussed. Similar problem has been presented in [4], but only the sensitivities of burn integrals with respect to shape design parameter L_0 have been calculated. Here, the modification of parameter L_1 is also discussed.
Using the concept of material derivative we can write [5, 6]

$$\frac{\mathrm{D}T_e}{\mathrm{D}b_s} = \frac{\partial T_e}{\partial b_s} + \frac{\partial T_e}{\partial x} v_s \tag{8}$$

where $v_s = v_s(x, b_s)$ is the velocity associated with design parameter $b_1 = L_0$ or $b_2 = L_1$.
For the material derivative following formulas can be derived (c.f. equation (8)) [4]:

$$\frac{\mathrm{D}}{\mathrm{D}b_s}\left(\frac{\partial T_e}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b_s}\right) - \frac{\partial T_e}{\partial x}\frac{\partial v_s}{\partial x} \tag{9}$$

$$\frac{\mathrm{D}}{\mathrm{D}b_s}\left(\frac{\partial T_e}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b_s}\right) \tag{10}$$

$$\frac{\mathrm{D}}{\mathrm{D}b_s}\left(\frac{\partial^2 T_e}{\partial x^2}\right) = \frac{\partial^2}{\partial x^2}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b_s}\right) - 2\frac{\partial^2 T_e}{\partial x^2}\frac{\partial v_{es}}{\partial x} - \frac{\partial T_e}{\partial x}\frac{\partial^2 v_{es}}{\partial x^2} \tag{11}$$

If the direct approach of sensitivity method is applied [4-6] then the equations (1) are differentiated with respect to parameters b_s, $s = 1, 2$.
Introducing the functions $U_{es} = \mathrm{D}T_e/\mathrm{D}b_s$ and using the formulas (9), (10), (11) one has

$$x \in \Omega_e: \quad c_e \frac{\partial U_{es}}{\partial t} = \lambda_e \left(\frac{\partial^2 U_{es}}{\partial x^2} - 2\frac{\partial^2 T_e}{\partial x^2}\frac{\partial v_s}{\partial x} - \frac{\partial T_e}{\partial x}\frac{\partial^2 v_s}{\partial x^2}\right) - k_e U_{es} \tag{12}$$

or (c.f. equation (1))

$$c_e \frac{\partial U_{es}}{\partial t} = \lambda_e \frac{\partial^2 U_{es}}{\partial x^2} - k_e U_{es} - 2\left[c_e \frac{\partial T_e}{\partial t} - k_e (T_B - T_e) - Q_{me} \right] \frac{\partial v_s}{\partial x} - \lambda_e \frac{\partial T_e}{\partial x} \frac{\partial^2 v_s}{\partial x^2} \quad (13)$$

In similar way the boundary - initial conditions are differentiated with respect to shape parameters b_s. So, for surface of the skin one has

$$x \in \Gamma_0 : \quad \frac{\mathrm{D} q}{\mathrm{D} b_s} = \lambda_1 \frac{\mathrm{D}}{\mathrm{D} b_s} \left(\frac{\partial T_e}{\partial x} \right) = \begin{cases} \dfrac{\mathrm{D} q_0}{\mathrm{D} b_s} = 0, & t \le t_0 \\ \alpha \dfrac{\mathrm{D} T_1}{\mathrm{D} b_s}, & t > t_0 \end{cases} \quad (14)$$

this mean

$$x \in \Gamma_0 : \quad W_{1s} = \lambda_1 \frac{\partial U_{1s}}{\partial x} = \begin{cases} \lambda_1 \dfrac{\partial T_1}{\partial x} \dfrac{\partial v_s}{\partial x}, & t \le t_0 \\ \alpha U_{1s} - \lambda_1 \dfrac{\partial T_1}{\partial x} \dfrac{\partial v_s}{\partial x}, & t > t_0 \end{cases} \quad (15)$$

or

$$x \in \Gamma_0 : \quad W_{1s} = \begin{cases} q_0 \dfrac{\partial v_s}{\partial x}, & t \le t_0 \\ \alpha U_{1s} - q_1 \dfrac{\partial v_s}{\partial x}, & t > t_0 \end{cases} \quad (16)$$

For boundary limiting the system:

$$x \in \Gamma_3 : \quad W_3 = -\lambda_3 \left(\frac{\partial U_{3s}}{\partial x} \right) = 0 \quad (17)$$

Differentiating the continuity conditions (2) one obtains (c.f. formula (9))

$$x = L_e : \quad \begin{cases} W_{es} - q_e \dfrac{\partial v_s}{\partial x} = W_{e+1,s} - q_{e+1} \dfrac{\partial v_s}{\partial x} & e = 1, 2 \\ U_{es} = U_{e+1,s} \end{cases} \quad (18)$$

where: $q_e = -\lambda_e \partial T_e / \partial x$, $W_{es} = -\lambda_e \partial U_{es} / \partial x$ for $x = L_e$, $e = 1, 2$ and $q_e = \lambda_e \partial T_e / \partial x$, $W_{es} = \lambda_e \partial U_{es} / \partial x$ for $x = L_{e-1}$, $e = 2, 3$.

Consistent with the formula (8) the initial condition takes a form

$$U_{es} = \frac{\partial T_{ep}}{\partial x} v_s \quad (19)$$

In the case of sensitivity analysis with respect to shape parameter $b_1 = L_0$ we assume the following form of velocity

$$v_1(x,b_1)=\begin{cases}\dfrac{L_1-x}{L_1-b_1}, & L_0\le x\le L_1\\ 0, & L_1\le x\le L_3\end{cases} \tag{20}$$

while for the problem concerning the sensitivity analysis with respect to shape parameter $b_2 = L_1$

$$v_2(x,b_2)=\begin{cases}\dfrac{x-L_0}{b_2-L_0}, & L_0\le x\le L_1\\ \dfrac{L_2-x}{L_2-b_2}, & L_1\le x\le L_2\\ 0, & L_2\le x\le L_3\end{cases} \tag{21}$$

Taking into account the forms (6), (7) of functionals I_b, I_d, the sensitivity of these integrals with respect to the parameters b_s is calculated using the formulas

$$\frac{\mathrm{D}I_r}{\mathrm{D}b_s}=\int_0^{\tau}P_r\frac{\Delta E}{RT_r^2}\exp\left(-\frac{\Delta E}{RT_r}\right)U_{rs}\,\mathrm{d}t \tag{22}$$

where $r = p$ or $r = s$ and $T_b = T_1(L_1, t) = T_2(L_1, t)$, $T_d = T_2(L_2, t) = T_3(L_2, t)$, $U_{bs} =$ $= U_{1s}(L_1, t) = U_{2s}(L_1, t)$, $U_{ds} = U_{2s}(L_2, t) = U_{3s}(L_2, t)$ (c.f. equations (2)).
The change of burn integrals connected with the change of parameters b_s results from the Taylor formula limited to the first-order sensitivity, this means

$$I_r(b_s\pm\Delta b_s)=I_r(b_s)\pm\frac{\mathrm{D}I_r}{\mathrm{D}b_s}\Delta b_s \tag{23}$$

3. Boundary element method

The primary and also the additional problems resulting from the sensitivity analysis have been solved using the 1st scheme of the BEM for 1D transient heat diffusion [7]. The boundary integral equations (for successive layers of skin - $e = 1, 2, 3$) corresponding to the primary problem and the transition $t^{f-1}\to t^f$ are of the form [3, 4, 7]

$$T_e(\xi,t^f)+\left[\frac{1}{c_e}\int_{t^{f-1}}^{t^f}T_e^*(\xi,x,t^f,t)q_e(x,t)\,\mathrm{d}\,t\right]_{x=L_{e-1}}^{x=L_e}=$$

$$=\left[\frac{1}{c_e}\int_{t^{f-1}}^{t^f}q_e^*(\xi,x,t^f,t)T_e(x,t)\,\mathrm{d}\,t\right]_{x=L_{e-1}}^{x=L_e}+\int_{L_{e-1}}^{L_e}T_e^*(\xi,x,t^f,t^{f-1})T_e(x,t^{f-1})\,\mathrm{d}\,x+ \tag{24}$$

$$+\frac{1}{c_e}\int_{L_{e-1}}^{L_e}\left[k_eT_B-k_eT_e(x,t^{f-1})+Q_{me}\right]\int_{t^{f-1}}^{t^f}T_e^*(\xi,x,t^f,t)\,\mathrm{d}\,t\,\mathrm{d}\,x$$

where T_e^* are the fundamental solutions given by formulas

$$T_e^*(\xi,x,t^f,t)=\frac{1}{2\sqrt{\pi a_e(t^f-t)}}\exp\left[-\frac{(x-\xi)^2}{4a_e(t^f-t)}\right] \tag{25}$$

where ξ is the point in which the concentrated heat source is applied and $a_e=\lambda_e/c_e$. The heat fluxes resulting from the fundamental solutions are equal to

$$q_e^*(\xi,x,t^f,t)=-\lambda_e\frac{\partial T_e^*(\xi,x,t^f,t)}{\partial x}=\frac{\lambda_e(x-\xi)}{4\sqrt{\pi}\left[a_e(t^f-t)\right]^{3/2}}\exp\left[-\frac{(x-\xi)^2}{4a_e(t^f-t)}\right] \tag{26}$$

For $\xi\to L_{e-1}^{+}$ and $\xi\to L_e^{-}$ for each domain considered one obtains the system of equations

$$\begin{bmatrix}g_{11}^e & g_{12}^e\\ g_{21}^e & g_{22}^e\end{bmatrix}\begin{bmatrix}q_e(L_{e-1},t^f)\\ q_e(L_e,t^f)\end{bmatrix}=\begin{bmatrix}h_{11}^e & h_{12}^e\\ h_{21}^e & h_{22}^e\end{bmatrix}\begin{bmatrix}T_e(L_{e-1},t^f)\\ T_e(L_e,t^f)\end{bmatrix}+\begin{bmatrix}p_e(L_{e-1})\\ p_e(L_e)\end{bmatrix}+\begin{bmatrix}z_e(L_{e-1})\\ z_e(L_e)\end{bmatrix} \tag{27}$$

and then the final form of resolving system results from the continuity conditions (2) ad conditions given for $x=L_0$, and $x=L_3$ (equations (3), (4)):

$$\begin{bmatrix}-h_{11}^1 & -h_{12}^1 & g_{12}^1 & 0 & 0 & 0\\ -h_{21}^1 & -h_{22}^1 & g_{22}^1 & 0 & 0 & 0\\ 0 & -h_{11}^2 & g_{11}^2 & -h_{12}^2 & g_{12}^2 & 0\\ 0 & -h_{21}^2 & g_{21}^2 & -h_{22}^2 & g_{22}^2 & 0\\ 0 & 0 & 0 & -h_{11}^3 & g_{11}^3 & -h_{12}^3\\ 0 & 0 & 0 & -h_{21}^3 & g_{21}^3 & -h_{22}^3\end{bmatrix}\begin{bmatrix}T_1(L_0,t^f)\\ T_b(L_1,t^f)\\ q_b(L_1,t^f)\\ T_d(L_2,t^f)\\ q_d(L_2,t^f)\\ T_3(L_3,t^f)\end{bmatrix}=\begin{bmatrix}-g_{11}^1q_0+p_1(L_0)+z_1(L_0)\\ -g_{21}^1q_0+p_1(L_1)+z_1(L_1)\\ p_2(L_1)+z_2(L_1)\\ p_2(L_2)+z_2(L_2)\\ p_3(L_2)+z_3(L_2)\\ p_3(L_3)+z_3(L_3)\end{bmatrix} \tag{28}$$

In similar way one can solve the additional sensitivity problems.

4. Examples of computations

In numerical computations the following values of parameters have been assumed [1]: $\lambda_1 = 0.235$ W/mK, $\lambda_2 = 0.445$ W/mK, $\lambda_3 = 0.185$ W/mK, $c_1 = 4.3068\times$ $\times10^6$ J/m^{3}K, $c_2 = 3.96\cdot10^6$ J/m^{3}K, $c_3 = 2.674\cdot10^6$ J/m^{3}K, $c_B = 3.9962\cdot10^6$ J/m^{3}K, $T_B = 37$°C, $G_1 = 0$, $G_e = 0.00125$ (m^{3}blood/ s)/m^{3}tissue for $e = 2, 3$, $Q_{m1} = 0$, $Q_{me} = 245$ W/m^3 for $e = 2, 3$ [1]. Pre-exponential factors: $P_b = 1.43\cdot10^{72}$ 1/s for $T_b \geq 317$ K and $P_b = 0$ for $T_b < 317$ K, while $P_d = 2.86\cdot10^{69}$ 1/s for $T_d \geq 317$ K and $P_d = 0$ for $T_d < 317$ K. The ratio of activation energy to universal gas constant: $\Delta E/R = 55000$ K. The thicknesses of sub-domains: epidermis $L_1 - L_0 = 0.0001$ m, dermis $L_2 - L_1 = 0.002$ m and sub-cutaneous region $L_3 - L_2 = 0.01$ m. Heat transfer coefficient $\alpha = 8$ W/m^{2}K, and the ambient temperature $T^\infty = 20$°C. Time step: $\Delta t = 0.05$ s. The sensitivity of burns integrals has been calculated under the assumption that $\Delta L_0 = 10^{-5}$ m and $\Delta L_1 = 10^{-5}$ m (c.f. equation (23)).

In the first example of computations the heat flux $q_0 = 6500$ W/m^2 on the skin surface has been assumed, the exposure time: $t_0 = 18$ s. The successive skin layers have been divided into 10, 40 and 120 internal cells.

In Figure 2 the temperature distribution in the skin domain is shown. In Figures 3 and 4 the course of burn integral I_b and its courses found on the basis of sensitivity analysis with respect to parameters L_0 and L_1 are shown (c.f. equations (6), (23)). The times to the first and second degree burns predicted for the basic value of epidermis thickness are equal 16.25 s and 17.8 s, respectively. It is visible, that the thickness of epidermis has essential influence on the times of burns appearance.

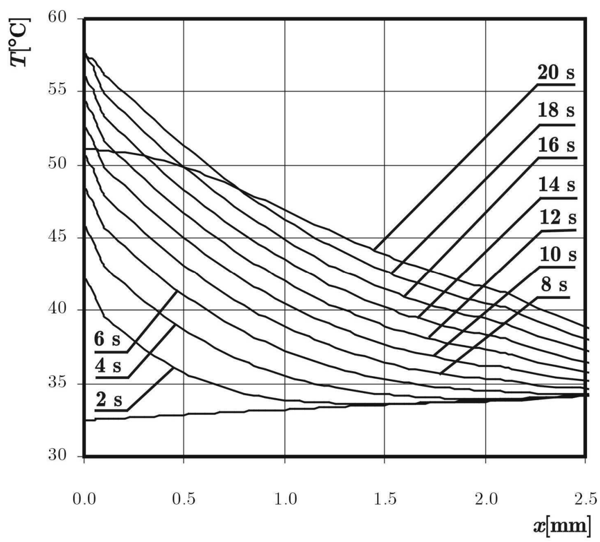

Fig. 2. Temperature distribution ($q_0 = 6500$ W/m^2, $t_0 = 18$ s)

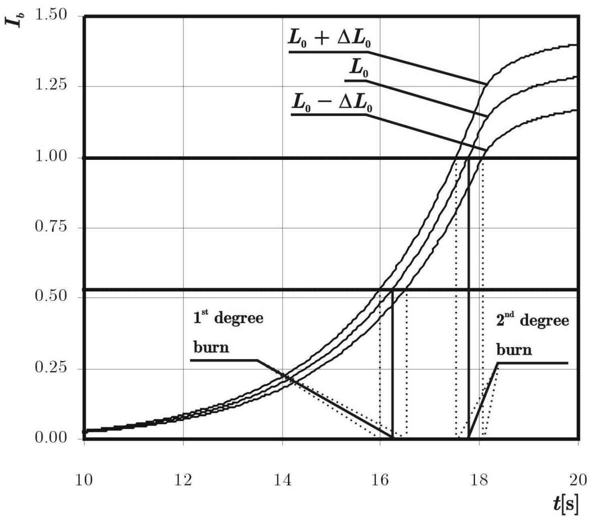

Fig. 3. Course of burn integral I_b - change of L_0

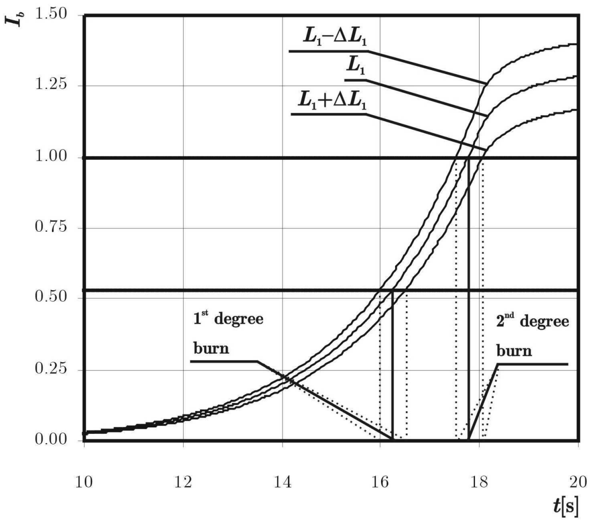

Fig. 4. Course of burn integral I_b - change of L_1

In the second example of computations the heat flux q_0 = 80000 W/m^2 on the skin surface has been assumed, the exposure time: t_0 = 5 s. The successive skin layers have been divided into 5, 20 and 60 internal cells.

In Figure 5 the distribution of temperature in the skin domain is shown. In Figures 6 and 7 the course of burn integral I_d and its courses found on the basis of sensitivity analysis with respect to the parameters L_0 and L_1 are shown (c.f. equations (7), (23)). The time to the third degree burn predicted for the basic values of epidermis and dermis thickness is equal 14.75 s. As previously, it is visible, that the thickness of epidermis has essential influence on the time of burn appearance.

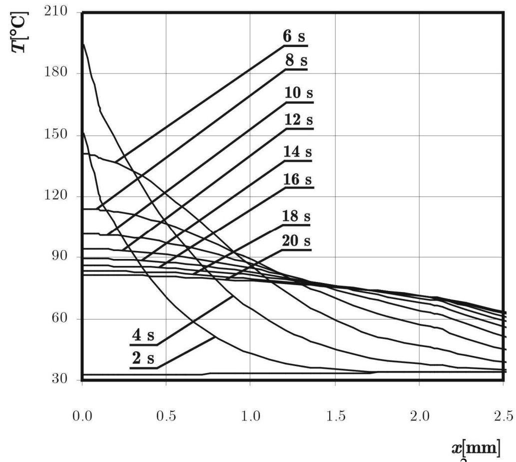

Fig. 5. Temperature distribution (q_0 = 80000 W/m^2, t_0 = 5 s)

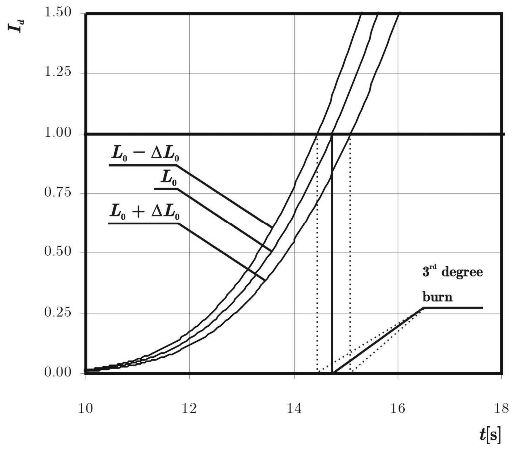

Fig. 6. Course of burn integral I_d - change of L_0

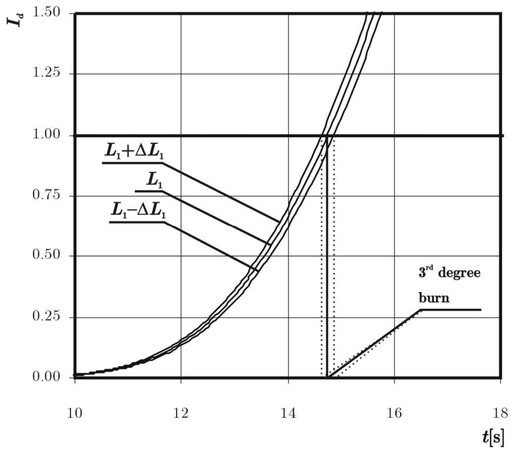

Fig. 7. Course of burn integral I_b - change of L_1

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