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SENSITIVITY ANALYSIS OF TEMPERATURE FIELD IN THE TISSUE WITH A TUMOR

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Abstract. The numerical algorithm based on the boundary element method is used for the temperature field computations in the non-homogeneous domain of healthy tissue and the tumor region. Thermophysical parameters of tumor region, in particular the perfusion coefficient and the metabolic heat source are essentially bigger than for healthy tissue. The values of these parameters are assumed to be constant. From the mathematical point of view the problem is described by the system of two Poisson's equations supplemented by the adequate boundary conditions. The main subject of the paper is the sensitivity analysis of temperature distribution with respect to the constant source functions in the sub-domains considered. In the final part the examples of computations are shown.

1. Formulation of the problem

The heat transfer process in non-homogeneous domain healthy tissue - tumor region is described by following system of equations [1, 2]

$$x \in \Omega_e: \quad \lambda_e \nabla^2 T_e(x) + Q_e = 0 \tag{1}$$

where e = 1, 2 identifies the subdomains of tissue and tumor - c.f. Figure 1, λ_e is the thermal conductivity, T_e denotes the temperature, $Q_e = Q_{perf\ e} + Q_{met\ e}$ are the heat sources ($Q_{perf\ e}$ is the perfusion heat source, $Q_{met\ e}$ is the metabolic heat source).
On the surface between tissue and tumor the ideal thermal contact is assumed:

$$x \in \Gamma_c: \quad \begin{cases} T_1(x) = T_2(x) \\ q_1(x) = -q_2(x) \end{cases} \tag{2}$$

where $q_e(x) = -\lambda_e \partial T_e(x) / \partial n_e$ is the heat flux, $\partial T_e(x)/\partial n_e$ denotes the directional derivative at the boundary point considered, while $n_e = \left[\cos\alpha_1^e, \cos\alpha_2^e, \cos\alpha_3^e\right]$ is the external unit normal vector.

On the remaining parts of the boundary the following conditions can be accepted

$$\begin{aligned} x \in \Gamma_1 : &\quad T_1(x) = T_b \\ x \in \Gamma_2 : &\quad q_1(x) = 0 \end{aligned} \tag{3}$$

where T_b is the boundary temperature.

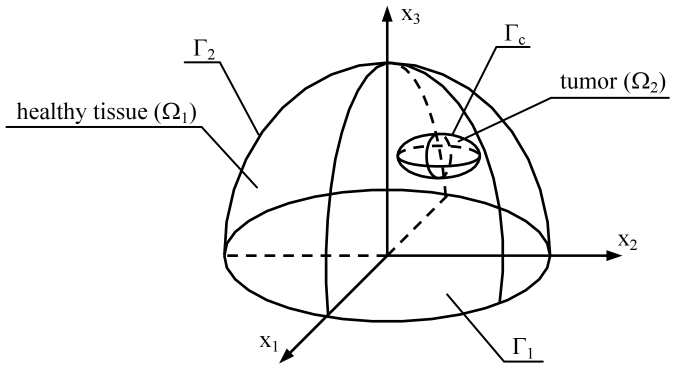

Fig. 1. Domain considered

2. Parameter sensitivity analysis

In this paper the sensitivity analysis of heat transfer process with respect to the source functions Q_e, $e = 1, 2$ is done. If the direct method of sensitivity analysis is used [3], then we should consider two additional boundary problems ($s = 1, 2$):

$$\begin{aligned} x \in \Omega_1 : &\quad \lambda_1 \nabla^2 Z_{1s}(x) + \delta_{1s} = 0 \\ x \in \Omega_2 : &\quad \lambda_2 \nabla^2 Z_{2s}(x) + \delta_{2s} = 0 \\ x \in \Gamma_c : &\quad \begin{cases} Z_{1s}(x) = Z_{2s}(x) \\ V_{1s}(x) = -V_{2s}(x) \end{cases} \\ x \in \Gamma_1 : &\quad Z_{1s}(x) = 0 \\ x \in \Gamma_2 : &\quad V_{1s}(x) = 0 \end{aligned} \tag{4}$$

where:

$$
\begin{aligned}
Z_{es} &= \frac{\partial T_e}{\partial Q_s} \\
V_{es} &= -\lambda_e \frac{\partial Z_{es}}{\partial n_e}
\end{aligned} \tag{5}
$$

and δ_{es} is the Kronecker delta, $e = 1, 2$.

3. Method of solution

The basic problem described by equations (1)-(3) and additional problems (4) have been solved using the boundary element method [6]. In order to avoid the discretization of the interior of sub-domains, the following substitutions for the basic boundary problem is introduced

$$
U_e(x) = \lambda_e T_e(x) + \frac{Q_e}{6}\sum_{k=1}^{3} x_k^2 \tag{6}
$$

In this way the equations (1) are replaced by the Laplace ones. The boundary conditions should be also transformed. So, the continuity condition (2) takes a form

$$
x \in \Gamma_c: \quad \begin{cases} W_2(x) = -W_1(x) - \dfrac{1}{3} Q_2 \displaystyle\sum_{k=1}^{3} x_k \cos\alpha_k^2 - \dfrac{1}{3} Q_1 \sum_{k=1}^{3} x_k \cos\alpha_k^1 \\ U_2(x) = \dfrac{\lambda_2}{\lambda_1} U_1(x) + \dfrac{1}{6}\left(Q_2 - \dfrac{\lambda_2}{\lambda_1} Q_1\right) \displaystyle\sum_{k=1}^{3} x_k^2 \end{cases} \tag{7}
$$

where $W_e(x) = -\partial U_e(x)/\partial n_e$. The remaining boundary conditions (3) are expressed as follows

$$
\begin{cases} x \in \Gamma_1: \quad U_1(x) = U_b = \lambda_1 T_b + \dfrac{1}{6} Q_1 \displaystyle\sum_{k=1}^{3} x_k^2 \\ x \in \Gamma_2: \quad W_1(x) = W_b = -\dfrac{1}{3} Q_1 \displaystyle\sum_{k=1}^{3} x_k \cos\alpha_k^1 \end{cases} \tag{8}
$$

For the additional problems resulting from the sensitivity analysis with respect to the source functions Q_1, Q_2 the substitution is following

$$
U_e(x) = \lambda_e Z_{e1}(x) + \frac{\delta_{e1}}{6}\sum_{k=1}^{3} x_k^2 \tag{9}
$$

and

$$U_e(x) = \lambda_e Z_{e2}(x) + \frac{\delta_{e2}}{6}\sum_{k=1}^{3} x_k^2 \tag{10}$$

Of course, the boundary conditions (4) should be also transformed.

In this way, in order to solve the basic problem and the additional ones the standard algorithm of the boundary method for the system of Laplace's equations can be used [4].

4. Results of computations

In numerical computations has been made studies of temperature distribution of healthy tissue and the tissue with tumor, and the sensitivity analysis of temperature distribution with respect to the constant source functions.

The real shape of biological tissue (breast) has been approximated by half of an ellipsoid with the semi-axises of lengths 0.05, 0.04, 0.06 m - Figure 1. The discretization of healthy tissue is presented in Figure 2. The tumor has been placed in the first quadrant of the domain of tissue. Generally, the tumor takes a form of ellipsoid - Figure 3.

In numerical computations the following lengths of semi-axises of tumor have been assumed: $a_t = 0.008$ m, $b_t = 0.006$ m, $c_t = 0.005$ m.

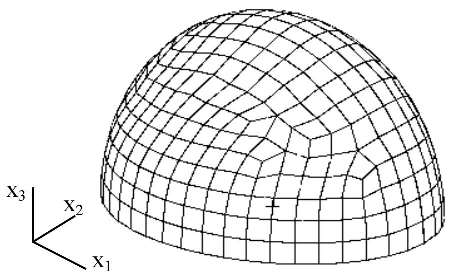

Fig. 2. Discretization of healthy tissue

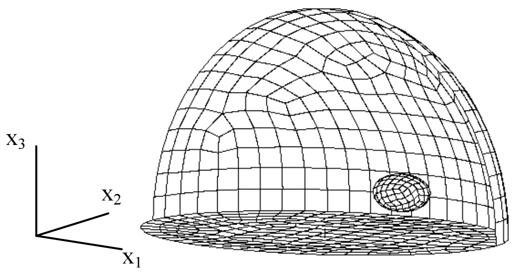

Fig. 3. Discretization of tumor and healthy tissue

The following input data have been taken into account [2, 5]: for healthy tissue thermal conductivity $\lambda_1 = 0.5$ W/mK, heat source $Q_1 = 220$ W/m^3, for tumor region thermal conductivity $\lambda_2 = 0.6$ W/mK, heat source $Q_2 = 3400$ W/m^3, boundary temperature $T_b = 37$°C.

In Figures 4 and 5 the temperatures fields on the skin surface in the form of contours projected on the plane (x_1, x_2) for healthy tissue and tissue with tumor are shown.

In Figure 6 the sensitivity function $(\partial T/\partial Q) * \Delta Q$ for healthy tissue is presented ($\Delta Q = 25$ W/m^3).

Figures 7 and 8 illustrate the changes of temperature in the domain healthy tissue - tumor region due to the changes of Q_1, Q_2 under the assumption that $\Delta Q_1 = 25$ W/m^3, $\Delta Q_2 = 350$ W/m^3.

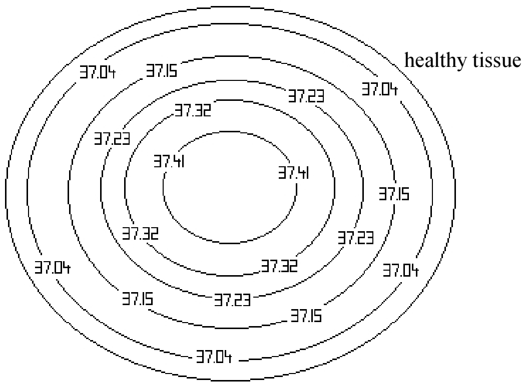

Fig. 4. Temperature distribution on the skin surface - healthy tissue

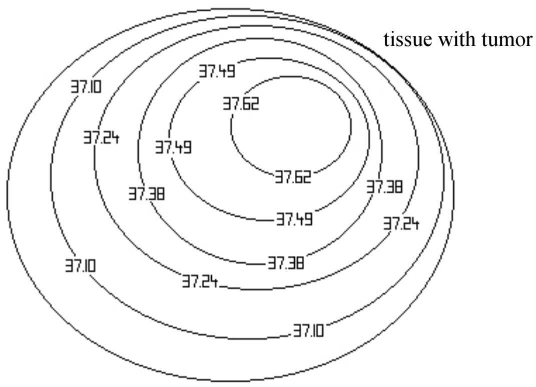

Fig. 5. Temperature distribution on the skin surface - tissue with tumor

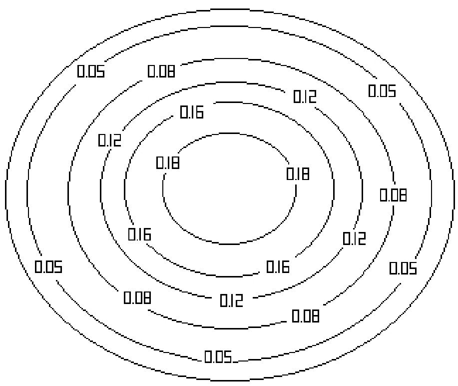

Fig. 6. Difference temperature distribution on the skin surface due to the change of source function Q for healthy tissue ($\Delta Q = 25$ W/m^3)

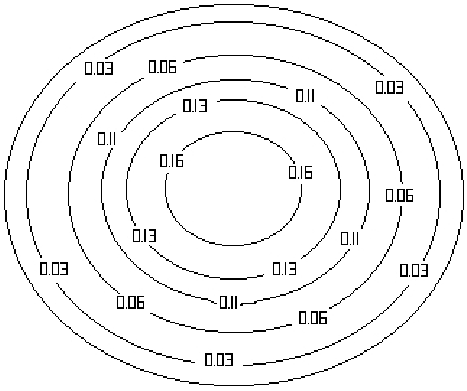

Fig. 7. Difference temperature distribution on the skin surface due to the change of source function Q_1 for tissue with tumor (ΔQ_1 = 25 W/m^3)

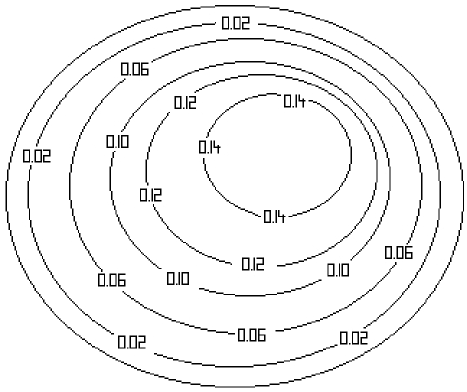

Fig. 8. Difference temperature distribution on the skin surface due to the change of source function Q_2 for tissue with tumor (ΔQ_2 = 350 W/m^3)

It is visible that the changes of temperature due to the changes of source functions Q_1 and Q_2 are essential.

References

[1] Huang H.W., Chan C.L., Roemer R.B., Analytical solutions of Pennes bio-heat transfer equation with a blood vessel, Journal of Biomechanical Engineering 1994, 116, 208-121.

[2] Majchrzak E., Mochnacki B., Analysis of thermal processes occurring in tissue with a tumor region using the BEM, Journal of Theoretical and Applied Mechanics 2002, 1, 40, 101-102.

[3] Dems K., Sensitivity analysis in thermal problems - II: structure shape variation, Journal of Thermal Stresses 1987, 10, 1-16.

[4] Majchrzak E., Boundary element method in heat transfer, Publ. of the Technological University of Czestochowa, Czestochowa 2001 (in Polish).

[5] Liu J., Xu L.X., Boundary information based diagnostics on the thermal states of biological bodies, International Journal of Heat and Mass Transfer 2000, 43, 2827-2839.