THE STABILITY OF MOTION OF SLAB IN CONTINUOUS CASTING OF STEEL FORCED BY OSCILLATING CASTER

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Abstract. In this paper the motion of the slab has been analysed. The equation of motion for the slab was solved numerically. The stability of the obtained solutions was determined with regard to the dependence of the casting speed and the intensity of heat exchange in the primary and secondary cooling zone.

1. Introduction

The equation of motion for the analysed slab - caster system is a partial differential equation of second order with appropriate unique conditions (geometrical, boundary and initial). The motion of the slab is caused by the vibrations of the caster. The caster has a given harmonic motion $\delta = \delta_0 \sin(\omega t)$, which has an effect on the slab with a specified axial load $s(x,t)$, per unit length. This load is the sum of the normal interactions between the solidifying slab layer and the caster, and the friction conditions between the slab and the caster [1, 2]. The withdrawal force of the slab is realised by a pulling roller. The rolls are pressed down on to the slab with normal force and they produce the determined withdrawal force via the coefficient of rolling friction. The angular velocity of the rolls is chosen purposely to obtain the required velocity of slab withdrawal w_0 (casting speed). The constant casting velocity is realised when the angular velocity of the rolls is constant in respect to slide absence. It is assumed that the displacements of the liquid and solid parts of the slab are the same, and that the total load of the slab is carried only by the solidified part of the slab. It is also assumed that solidified part of the slab is viscoelastic with linear elastic (E^s) and viscous (μ^s) characteristics. Taking into account the above assumptions, one can obtain the equation of motion for forced vibrations in the following form:

$$(\rho^s A^s + \rho^l A^l)\frac{\partial^2 u}{\partial t^2} = \left(\int_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} \frac{\partial \mu^s}{\partial x} y \, dy - 2(\rho^s A^s + \rho^l A^l)w_0 \right)\frac{\partial^2 u}{\partial t \partial x} +$$

$$+\left(\int\limits_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} E^s y\,dy + w_0 \int\limits_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} \frac{\partial \mu^s}{\partial x} y\,dy - (\rho^s A^s + \rho^l A^l)w_0^2\right)\frac{\partial^2 u}{\partial x^2} +$$

$$+\frac{\partial u}{\partial x}\int\limits_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} \frac{\partial E^s}{\partial x} y\,dy + \frac{\partial^3 u}{\partial x^2 \partial t}\int\limits_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} \mu^s y\,dy + \frac{\partial^3 u}{\partial x^3}w_0 \int\limits_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} \mu^s y\,dy + s(x,t)$$

$$(1)$$

where:

$u = u(x,t)$- displacement of the slab,

   ρ^l, ρ^s - density of the liquid and solid phase, respectively,

   A^l, A^s - cross-section area of the liquid and solid part of the slab, respectively.

The boundary conditions of the equation are specified by the free and of the slab and the end restrained by the pulling roller action ($x = L$), which gives:

$$\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} = 0; \quad \left.u(x,t)\right|_{x=L} = 0 \tag{2}$$

Initial conditions are equal to

$$\left.u(x,t)\right|_{t=0} = 0, \quad \left.\frac{du_0(x,t)}{dt}\right|_{t=0} = 0 \tag{3}$$

An effective solution of the above equation, due to its of complication can only be obtained with the use of approximation methods [1, 5]. The commonly used Galerkin's method was applied in this paper. According to this method, the solution of equation (1) takes the form of the following series

$$u(x,t) = \sum_{i=1}^{\infty} S_i(t)U_i(x) \tag{4}$$

where $U_i(x) = \cos\left(\left(i - \frac{1}{2}\right)\frac{\pi}{L}x\right)$ are basic functions chosen to fulfil given bound-

ary conditions. The predicted solution is substituted into the motion equation (1), then consecutively multiplied by all the basic functions and finally integrated in the interval $(0,L)$. Thus the equation of motion is transformed into a system of ordinary differential equations in relation to unknown time functions $S_i(t)$. The time functions are solved in relation to the second time derivative. This system can be written in a matrix form

$$\ddot{\mathbf{S}} = \mathbf{G}\dot{\mathbf{S}} + \mathbf{H}\mathbf{S} + \mathbf{K} \tag{5}$$

where the presented matrixes are: $\mathbf{G} = \mathbf{A}^{-1}\mathbf{B}$; $\mathbf{H} = \mathbf{A}^{-1}\mathbf{C}$; $\mathbf{K} = \mathbf{A}^{-1}\mathbf{D}$, $\mathbf{A}^{-1}$ is an inverse matrix of matrix $\mathbf{A}$ and:

$$\mathbf{A} = [a_{ij}]; \;\; \mathbf{B} = [b_{ij}]; \;\; \mathbf{C} = [c_{ij}]; \;\; \mathbf{D} = [d_j]$$

and stated coefficients are:

$$a_{ij} = \int_0^L (\rho^l A^l(x) + \rho^s A^s(x))U_i(x)U_j(x)dx$$

$$
\begin{aligned}
b_{ij} = {} & \int_0^L \frac{d\mu^s}{dx}(\rho^l A^l(x) + \rho^s A^s(x))U_i'(x)U_j(x)dx + \\
& -2w_0 \int_0^L (\rho^l A^l(x) + \rho^s A^s(x))U_i'(x)\mathrm{U}_j(x)dx + \\
& +\int_0^L \mu^s(x)(\rho^l A^l(x) + \rho^s A^s(x))U_i''(x)U_j(x)dx
\end{aligned}
$$

$$
\begin{aligned}
c_{ij} = {} & \int_0^L \left(\int_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} E^s y\,dy \right)U_i''(x)U_j(x)dx + w_0 \int_0^L \frac{d\mu^s}{dx}(\rho^l A^l + \rho^s A^s)U_i''(x)U_j(x)dx + \\
& -2w_0^2 \int_0^L (\rho^l A^l + \rho^s A^s)U_i''(x)U_j(x)dx + \int_0^L \left(\int_{\frac{a}{2}-\eta(x)}^{\frac{a}{2}} \frac{\partial E^s}{\partial x} y\,dy \right)U_i'(x)U_j(x)dx + \; \backslash \\
& + w_0 \int_0^L \mu_s(x)(\rho^l A^l + \rho^s A^s)U_i'''(x)U_j(x)dx
\end{aligned}
$$

$$d_j = \int_0^L s(x,t)U_j(x)dx \tag{6}$$

2. The numerical solution of the system of ordinary differential equations. Analysis of the stability of the solutions

It follows from the conducted tests that only the two first terms of the series should be taken into account. The summing the other terms of the series has practi-

cally no significant meaning, because those quantities are very small. For the two first terms of series the system (5) takes the form:

$$\ddot{S}_1 = G_{11}\dot{S}_1 + G_{12}\dot{S}_2 + H_{11}S_1 + H_{12}S_2 + K_1\pi$$
$$\ddot{S}_2 = G_{21}\dot{S}_1 + G_{22}\dot{S}_2 + H_{21}S_1 + H_{22}S_2 + K_2$$

(7)

The coefficients, present in the above equation for the two first terms of the series, can be written as:

$$G_{11} = \frac{b_{11}a_{22} - b_{12}a_{21}}{a_{11}a_{22} - a_{12}a_{21}} \qquad G_{12} = \frac{b_{21}a_{22} - b_{22}a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$G_{21} = \frac{b_{12}a_{11} - b_{11}a_{12}}{a_{11}a_{22} - a_{12}a_{21}} \qquad G_{22} = \frac{b_{22}a_{11} - b_{21}a_{12}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$H_{11} = \frac{c_{11}a_{22} - c_{12}a_{21}}{a_{11}a_{22} - a_{12}a_{21}} \qquad H_{12} = \frac{c_{21}a_{22} - c_{22}a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$H_{21} = \frac{c_{12}a_{11} - c_{11}a_{12}}{a_{11}a_{22} - a_{12}a_{21}} \qquad H_{22} = \frac{c_{22}a_{11} - c_{21}a_{12}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$K_1 = \frac{d_1a_{22} - d_2a_{21}}{a_{11}a_{22} - a_{12}a_{21}} \qquad K_2 = \frac{d_2a_{11} - d_1a_{12}}{a_{11}a_{22} - a_{12}a_{21}}$$

(8)

Substituting the dependence: $S_1 = y_1$; $S_2 = y_2$; $\dot{S}_1 = \dot{y}_1 = y_3$; $\dot{S}_2 = \dot{y}_2 = y_4$, into the system (7) transforms the system of second order differential equations into a system of first order differential equations

$$\begin{cases} \dot{y}_1 = y_3 \\ \dot{y}_2 = y_4 \\ \dot{y}_3 = G_{11}y_3 + G_{12}y_4 + H_{11}y_1 + H_{12}y_2 + K_1 \\ \dot{y}_4 = G_{21}y_3 + G_{22}y_4 + H_{21}y_1 + H_{22}y_2 + K_2 \end{cases}$$

(9)

such a system is solved numerically with the use of the Runghe-Kutha-Merson method. Unknown time functions and their derivatives are determined, and the displacement of the slab in an arbitrary cross-section is calculated according to equation (4).

The solutions are stable or unstable [1, 3, 4] with regard to the dependency on the value of given real parameters of motion and initial conditions of the solution. In general, stability is defined as follows: if defined motion exists with given initial conditions and the motion differs insignificantly from the motion without

disturbances after small changes in the initial conditions or after small disturbances then the motion is said to be stable. If small changes in the initial conditions or small disturbances cause a radical change in motion, the motion is unstable. It is essential to estimate the stability of motion, because unexpected disturbance always occur when we give determined motions to mechanical systems and we want to know if they cause an undesirable radical change in motion. Let us take into account the forced motion of a system described by heterogeneous linear equations:

$$\frac{dx_i}{dt} = \sum_{j=1}^{n} a_{ij} x_j + f_i(t), \quad i = 1, 2, ..., n \tag{10}$$

Let us consider two solutions $x_{i1}(t)$ and $x_{i2}(t)$ close to each other. Their difference is expressed by equation:

$$\xi_i(t) = x_{i1}(t) - x_{i2}(t), \quad i = 1, 2, ..., n \tag{11}$$

After differentiating and substituting the right side of equation (10), we obtain:

$$\frac{d\xi_i}{dt} = \frac{dx_{i1}}{dt} - \frac{dx_{i2}}{dt} = \sum_{j=1}^{n} a_{ij}(x_{j1} - x_{j2}) = \sum_{j=1}^{n} a_{ij}\xi_j, \quad i = 1, 2, ..., n \tag{12}$$

This means that the test of stability of the heterogeneous system leads to a test of stability of an appropriate homogeneous system. Particular solutions to this system are searched for in the form of function:

$$x_i(t) = A_i e^{\lambda t}, \quad i = 1, 2, ..., n \tag{13}$$

After substituting into the tested system of equations and reducing by $A_i e^{\lambda t}$, we obtain the system of algebraic equations:

$$A_i(-\lambda + a_{ii}) + \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} A_j = 0, \quad i = 1, 2, ..., n \tag{14}$$

A characteristic determinant of this system is that it should equal zero:

$$\Delta = \begin{vmatrix} a_{11} - \lambda & a_{12} & . & a_{1n} \\ a_{12} & a_{22} - \lambda & . & a_{2n} \\ . & . & . & . \\ a_{n1} & a_{n2} & . & a_{nn} - \lambda \end{vmatrix} = 0 \tag{15}$$

The radicals of the characteristic equation are determined by the solution of determinant (15). The sign of the real part of these radicals decides the stability of system. If all the radicals of the characteristic equation have negative real parts, all particular solutions drop to zero when time increases unlimitedly. The same happens in case of the general solution. This solution is asymptotically stable if the difference between two arbitrary solutions drops to zero while time increases to infinity. If only one radical has a real part greater than zero, then the appropriate solution increases unlimitedly while $t \to \infty$, so the general solution is unstable.

The above analysis allows us to estimate the stability of the system solution (9) on the basis of knowing the character of the radicals in the characteristic equation. That yields:

$$\varDelta = \begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ H_{11} & H_{12} & G_{11} - \lambda & G_{12} \\ H_{21} & H_{22} & G_{21} & G_{22} - \lambda \end{vmatrix} = 0 \qquad (16)$$

Radicals λ_i are determined from the above equation and the stability of the solution is educed from the sign of the real part of these radicals.

3. Examples of computations

The system slab - caster is considered. The caster has a given oscillating motion $\delta_0 \sin \omega t$. Tangent force $s(x,t)$, directed towards the slab, is determined by equation:

$$s(x,t) = (f - f_{sr})g\rho_l \frac{a}{2}x \qquad (17)$$

where: $f_{sr} = \dfrac{\omega}{\pi} \displaystyle\int_0^{\pi/\omega} \left(f_k + (f_s - f_k)e^{-\alpha|v|} \right) \mathrm{sgn}(v)dt$

and ρ_l is the density of the slab liquid phase, g - gravitational acceleration, f_s, f_k - coefficients of static and kinetic friction, respectively, v - relative velocity of the slab in relation to the caster. The tangent force depends on the relative velocity of the slab in relation to the caster and on the load force developed by the solidifying slab layer. These quantities were specified in paper [7]. Young's modulus and coefficient of viscosity, occurred in the above equation, depend on temperature and are determined by equations [2, 6]:

$$E(T(x,y)) = \frac{59.4 \cdot 10^6}{T_K - 1134}(T_K - T)^2 \qquad (18)$$

$$\mu(T(x,y)) = \mu^k \frac{(T - T_E)^2}{(T_K - T_E)^2} \qquad (19)$$

where: $E(T)$ - modulus of elasticity, Pa, T_K - temperature of slab solidification, K, T_E - ambient temperature and $T = T(x,y)$ - slab temperature expressed by the following relationship:

$$T(x,y) = T_K - (T_K - T_p(x))\left(1 - \left(\frac{a}{2} - y\right)\frac{1}{\eta(x)}\right) \qquad (20)$$

Temperature of the slab surface $T_p(x)$ and thickness of the solidifying slab layer $\eta(x)$ is determined in the way shown in the paper [7].

Numerical computations were made for given values of system parameters: $\rho^s = 7850$ kg/m^3, $\delta_0 = 0.01$ m, $\omega = 1.2$ rad/s, $\mu^k = 5 \cdot 10^7$ Ns/m^2, $L = 8,0$ m, $L_{kr} = 0.7$ m, $f_s = 0.1$; $f_k = 0.3$; temperature distributions along the length of the slab were determined for the following values of parameters: $L_s = 8$ m, $\lambda_M = 349$ W/mK, $T_E = 323$ W/mK, $T_E = 323$ K, $T_K = 1808$ K, $T_w = 293$ K, $a = 0.14$ m, $h_M = 0.04$ m, $L = 268 \cdot 10^3$ J/kg, $\lambda_M = 349$ W/mK, $\lambda = 29$ W/mK; $\beta = 800$ W/m^{2}K; $c = 800$ J/kgK; $\alpha_w = 1500$ W/m^{2}K; $\alpha_E = 2750$ W/m^{2}K.

The stability of the system was tested in terms of its dependence on the intensity of heat exchange in the primary cooling zone (coefficient α_W - cooling in caster) and in the secondary cooling zone (coefficient α_E - cooling with direct water spray) for different withdrawal velocities of the slab. The zones of stable and unstable solutions were determined (Fig. 1). They change insignificantly with the change in the value of α_W coefficient.
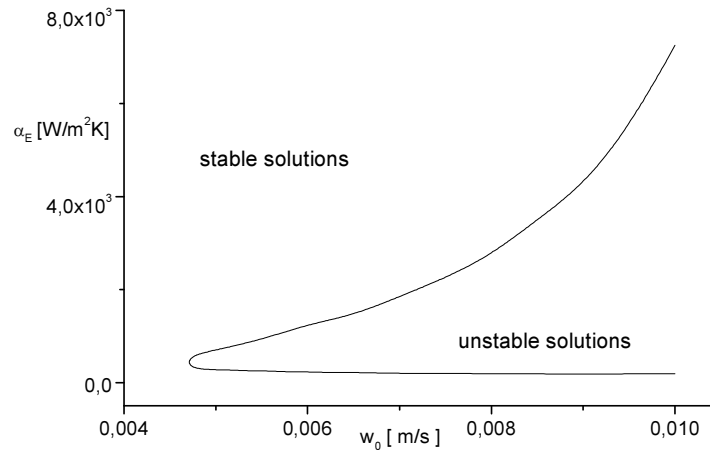

Fig. 1. Zone of stable and unstable solutions of the slab - caster system

References

[1] Ciekot A., Parkitny R., Analiza stateczności rozwiązań równania ruchu wlewka COS, Scientific Research of the Institute of Mathematics and Computer Science 2002, 1.

[2] Gudaszari E.G., Panowko G.Ja., Tieorija wibracionnych tiechnołogiczeskich processow pri nie-kulonowom trienii, Nauka, Moskwa 1988.

[3] Gutowski R., Równania różniczkowe zwyczajne, WNT, Warsaw 1971.

[4] Brimacombe J.K., Empowerment with Knowlege - Toward the Intelligent Mold for the Continuous Casting of Steel Billets, Iron and Steelmaker 1993, 11, 35-47.

[5] Savage J., Pearson J., Rolling characteristics of multiple section continuously cast steel billets, Continuous Casting, ed. D.L. McBride, T.E. Dancy, Interscience Publisher, New York 1962, 179-195.

[6] Schwerdtfeger K., Sha H., Depth of Oscillation Marks Forming in Continuous Casting of Steel, Metallurgical and Materials Transactions B 2000, 31B, 8, 813-826.

[7] Ciekot A., Analiza oddziaływań mechanicznych krystalizatora wykonującego ruchy oscylacyjne na prostoliniowy wlewek ciągłego odlewania, Praca doktorska, Czestochowa University of Technology, Częstochowa 2003.