MODELLING OF FREEZING PROCESS OF THE TISSUE SUBJECTED TO THE ACTION OF SPHERICAL CRYOPROBES

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Abstract. In the paper the freezing process of biological tissue subjected to the action of two internal spherical cryoprobes is discussed. The problem is strongly non-linear because the parameters appearing in the mathematical model of the process are temperature--dependent. In order to solve the task considered, the finite element method for 3D domain oriented in the Cartesian co-ordinate systems has been used. In the final part of the paper the example of computations is shown.

1. Governing equations

From the mathematical point of view the biological tissue freezing process can be described by the following equation [1-4]

$$x \in \Omega: \quad c(T)\frac{\partial T(x,t)}{\partial t} = \operatorname{div}\left[\lambda(T)\operatorname{grad} T(x,t)\right] + L_V \frac{\partial f_S(x,t)}{\partial t} \tag{1}$$

where c is the specific heat per unit of volume, λ is the thermal conductivity, L_V is the volumetric latent heat, f_S is the frozen state fraction at the point considered, T, $x = \{x_1, x_2, x_3\}$, t denote temperature, spatial co-ordinates and time.

If we assume that the dependence between and the $f_s(x,t)$ temperature for the interval $[T_2, T_1]$ (the beginning and the end of freezing) is known then

$$L_V \frac{\partial f_S(x,t)}{\partial t} = L_V \frac{\mathrm{d} f_S(T)}{\mathrm{d} t}\frac{\partial T(x,t)}{\partial t} \tag{2}$$

and the equation (1) can be written in the form

$$x \in \Omega: \quad C(T)\frac{\partial T(x,t)}{\partial t} = \operatorname{div}\left[\lambda(T)\operatorname{grad} T(x,t)\right] \tag{3}$$

where

$$C(T) = c(T) - L_V \frac{\partial f_S(x,t)}{\partial t} \tag{4}$$

is called the substitute thermal capacity of intermediate region. The energy equation in the form (3) can be extended on the whole domain considered, because for $T > T_1 : f_S(T) = 0$, while for $T < T_2 : f_S(T) = 1$ and $C(T) \rightarrow c(T)$. This property of equation (3) constitutes a base of the so-called fixed domain approach [5, 6]. Summing up, the equation discussed describes the heat transfer processes in the whole conventionally homogenous domain. The problem is strongly non-linear - both the parameters $C(T)$ and $\lambda(T)$ are temperature dependent [1, 7].

On the cryoprobes surfaces Γ_1 and Γ_2 (c.f. Figure 1) the Dirichlet boundary condition can be accepted

$$x \in \Gamma_1 \cup \Gamma_2 : \quad T(x, t) = T_C \tag{5}$$

where T_C is the cryoprobes temperature. On the arbitrary assumed external surface Γ_0, limiting the domain considered the no-flux condition is assumed

$$x \in \Gamma_0 : \quad q(x, t) = -\lambda \frac{\partial T(x, t)}{\partial n} = 0 \tag{6}$$

where $\partial T(x,t)/\partial n$ is the normal derivative at the boundary point x.
For $t = 0$ the initial temperature field is known, namely

$$t = 0 : \quad T(x, 0) = T_0(x) \tag{7}$$

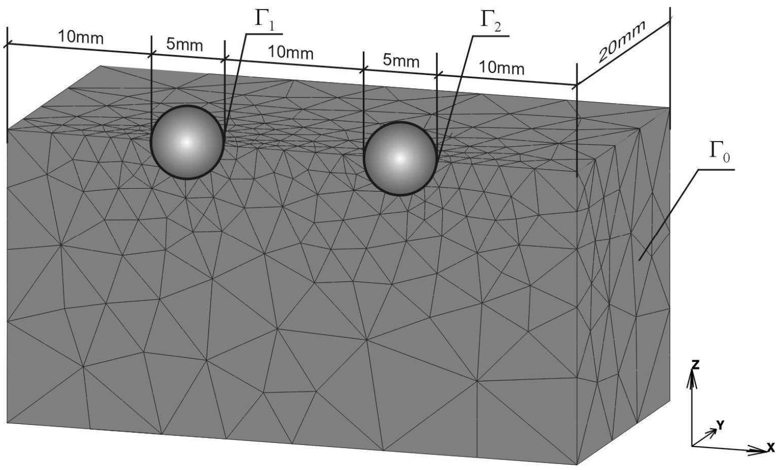

Fig. 1. Domain considered

2. Finite element method

The problem discussed has been solved using the finite element method. At first the time grid is introduced

$$0 = t^0 < t^1 < ... < t^{f-1} < t^f < ... < t^F < \infty \tag{8}$$

with a constant step $\Delta t = t^f - t^{f-1}$.

The weighted residual criterion for equation (3) and domain Ω oriented in Cartesian co-ordinate system has the following form [6]

$$\int_\Omega \left\{ \sum_{e=1}^{3} \frac{\partial}{\partial x_e} \left[\lambda(T) \frac{\partial T(x, t^S)}{\partial x_e} \right] - C(T) \left. \frac{\partial T(x, t)}{\partial t} \right|_{t=t^S} \right\} w(x) \, \mathrm{d}\Omega = 0 \tag{9}$$

where $t^S \in \left[t^{f-1}, t^f \right]$, $x = (x_1, x_2, x_3)$ and $w(x)$ is the weighting function.

Using the Gauss-Green-Ostrogradski theorem, after a certain mathematical manipulations one has

$$\begin{gathered} \int_\Omega \sum_{e=1}^{3} \lambda(T) \frac{\partial T(x, t^S)}{\partial x_e} \frac{\partial w(x)}{\partial x_e} \mathrm{d}\Omega = \\ = \int_\Gamma \lambda(T) \frac{\partial T(x, t^S)}{\partial n} w(x) \mathrm{d}\Gamma - \int_\Omega C(T) \left. \frac{\partial T(x, t)}{\partial t} \right|_{t=t^S} w(x) \mathrm{d}\Omega \end{gathered} \tag{10}$$

where $\Gamma = \Gamma_0 \cup \Gamma_1 \cup \Gamma_2$.

In order to solve the equation (10), the domain Ω of biological tissue has been divided into N finite elements and the integrals in equation (10) have been substituted by the sum of integrals over the finite elements

$$\begin{gathered} \sum_{i=1}^{N} \int_{\Omega_i} \sum_{e=1}^{3} \lambda(T) \frac{\partial T(x, t^S)}{\partial x_e} \frac{\partial w(x)}{\partial x_e} \mathrm{d}\Omega_i = \\ = \sum_{i=1}^{N} \int_{\Gamma_i} \lambda(T) \frac{\partial T(x, t^S)}{\partial n} w(x) \mathrm{d}\Gamma_i - \sum_{i=1}^{N} \int_{\Omega_i} C(T) \left. \frac{\partial T(x, t)}{\partial t} \right|_{t=t^S} w(x) \mathrm{d}\Omega_i \end{gathered} \tag{11}$$

In this paper the 10-nodal tetrahedral finite elements have been used - Figure 2. In order to transform the finite element Ω_i into the standardized tetrahedron the following substitution can be introduced

$$x_e = \eta_1 x_e^1 + \eta_2 x_e^2 + \eta_3 x_e^3 + (1 - \eta_1 - \eta_2 - \eta_3) x_e^4, \quad e = 1,2,3 \tag{12}$$

where (x_1^1, x_2^1, x_3^1), (x_1^2, x_2^2, x_3^2), (x_1^3, x_2^3, x_3^3), (x_1^4, x_2^4, x_3^4) are the co-ordinates of the finite element nodes 1, 2, 3, 4 and $0 \le \eta_1 \le 1$, $0 \le \eta_2 \le 1 - \eta_1$, $0 \le \eta_3 \le 1 - \eta_1 - \eta_2$.

The unknown function T is approximated in the following way

$$T = \sum_{k=1}^{10} N_k T_k^s \tag{13}$$

where T_k^s are the nodal values of temperature in the finite element considered, while:

$$\begin{gathered} N_1 = \eta_1(2\eta_1 - 1), \quad N_2 = \eta_2(2\eta_2 - 1), \quad N_3 = \eta_3(2\eta_3 - 1) \\ N_4 = (1-\eta_1-\eta_2)(1-2\eta_1-2\eta_2), \quad N_5 = 4\eta_1\eta_2 \\ N_6 = 4\eta_2\eta_3, \; N_7 = 4\eta_1\eta_3, \quad N_8 = 4\eta_1(1-\eta_1-\eta_2-\eta_3) \\ N_9 = 4\eta_2(1-\eta_1-\eta_2-\eta_3), \quad N_{10} = 4\eta_3(1-\eta_1-\eta_2-\eta_3) \end{gathered} \tag{14}$$

are the shape functions. The weighting function w is defined as follows

$$w = \sum_{k=1}^{10} \beta_k N_k \tag{15}$$

where β_k are the unknown coefficients.

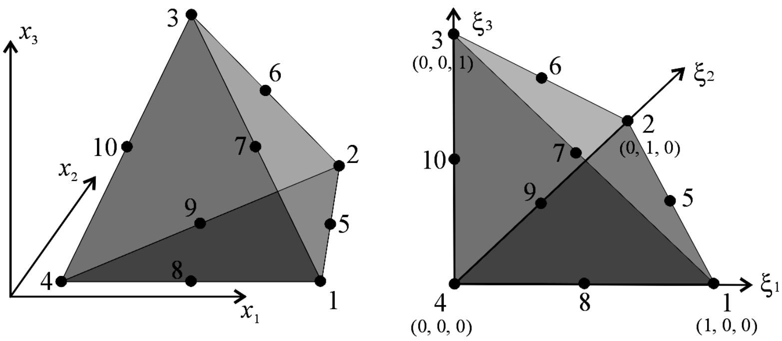

Fig. 2. 10-nodal tethrahedral element

Finally, one obtains the following system of equations [6]

$$\left(\mathbf{K} + \frac{1}{\Delta t}\mathbf{P}\right) \cdot \mathbf{T}^f = \frac{1}{\Delta t}\mathbf{P} \cdot \mathbf{T}^{f-1} + \mathbf{W} \tag{16}$$

where **K** is the conductivity matrix, **P** is the thermal capacity matrix, **W** is the matrix connected with boundary conditions, Δt is the time step.

3. Results of computations

The dimensions of domain considered are presented in Figure 1. The computations have been done using the MSC PATRAN/MARC software. Initial temperature of biological tissue equals 37°C, the beginning of the freezing process corresponds to the temperature $T_1 = -1°C$, the end of the freezing process corresponds to the temperature $T_2 = -8°C$, time step $\Delta t = 1$ s. In Figure 3 the temperature field in domain considered for times 5 and 60 s is shown.

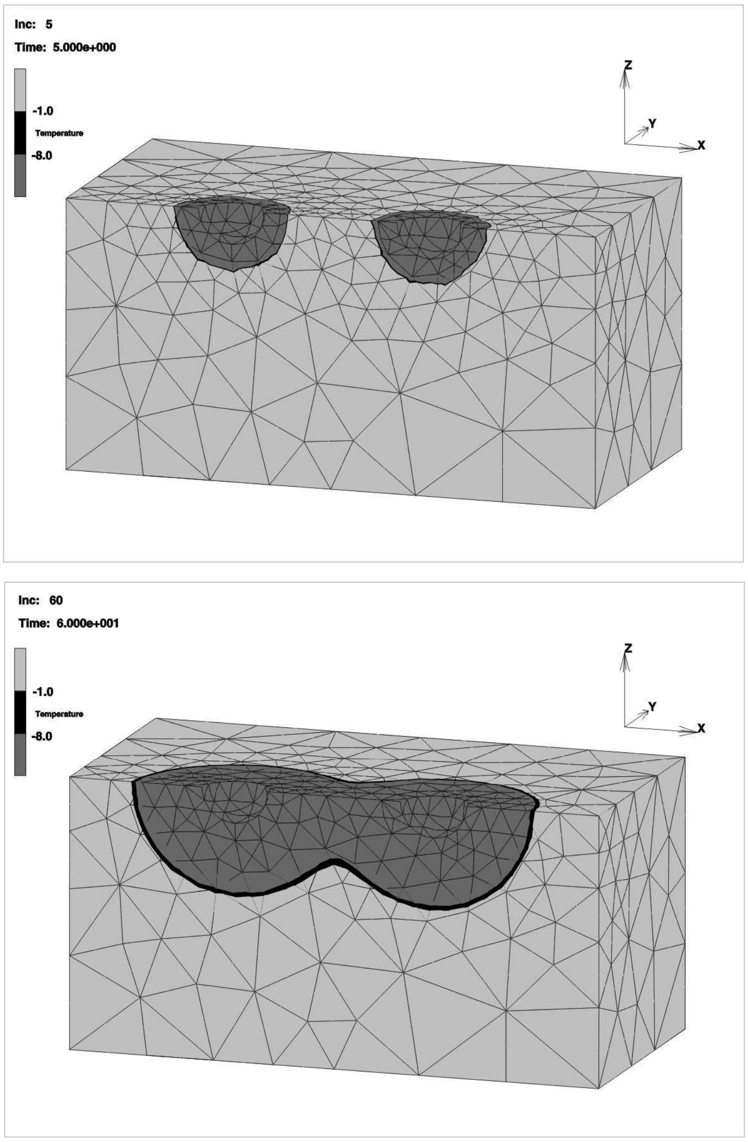

Fig. 3. Temperature field in domain considered

Figures 4 illustrates the temperature distribution in the distinguished cross section for time 60 s.

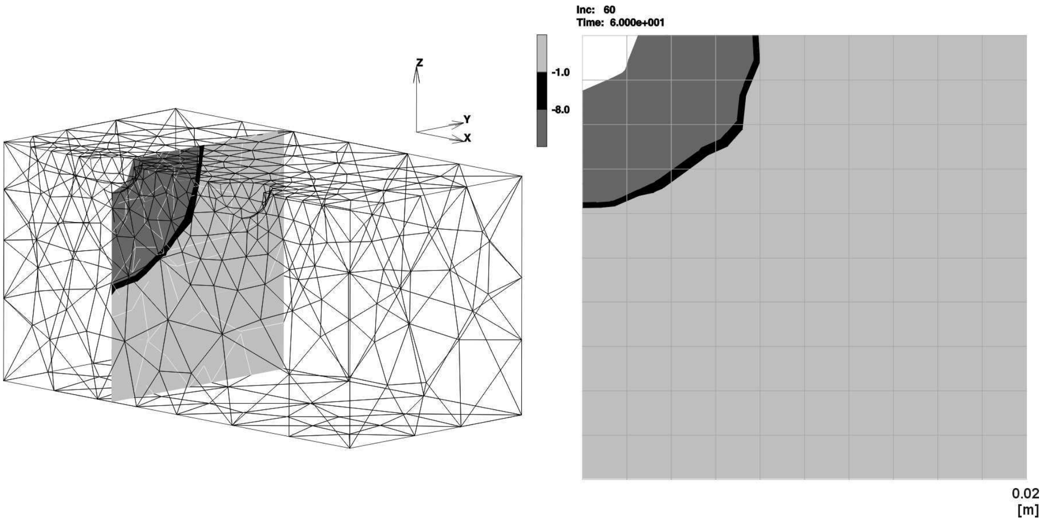

Fig. 4. Temperature distribution for time 60 s

Summing up, the MSC PATRAN/MARC software allows to solve the strongly non-linear problem connected with the modelling of biological tissue freezing process.

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References

[1] Comini G., Del Giudice L., Thermal aspects of cryosurgery, Journal of Heat Transfer 1976, 98, 543-549.

[2] Dziewoński M., Mochnacki B., Szopa R., Numerical model of biological tissue freezing, Acta of Bioengineering and Biomechanics 1999, 1, Suppl. 1, 139-142.

[3] Majchrzak E., Dziewoński M., Numerical simulation of freezing process using the BEM, Computer Assisted Mechanics and Engineering Sciences 2000, 7, 667-679.

[4] Majchrzak E., Mochnacki B., The BEM application for numerical solution of non-steady and non-linear thermal diffusion problems, Computer Assisted Mechanics and Engineering Sciences 1996, 3, 4, 327-346.

[5] Bonacina C., Comini C., Fasano A., Primacero M., On the estimation of thermophysical properties in nonlinear heat conduction problems, International Journal of Heat and Mass Transfer 1974, 17, 861-867.

[6] Majchrzak E., Mochnacki B., Metody numeryczne. Podstawy teoretyczne, aspekty praktyczne i algorytmy, Wyd. Politechniki Śląskiej, Gliwice 2004.