PARAMETER SENSITIVITY ANALYSIS OF BURN INTEGRALS USING THE ADJOINT APPROACH

***Ewa Majchrzak*[1, 2]*, Marek Jasiński*[1]**

[1]*Silesian University of Technology, Gliwice*
[2]*Czestochowa University of Technology, Czestochowa*

Abstract. In the paper the numerical analysis of thermal processes proceeding in the domain of biological tissue subjected to an external heat source is presented. Heat transfer in the skin tissue was assumed to be transient and one-dimensional. The degree of the skin burn can be predicted on the basis of so called Henriques integrals and the main subject of the paper is the sensitivity analysis of these integrals with respect to the thermophysical parameters. Here the adjoint approach of parameter sensitivity analysis has been applied. On the stage of numerical computations the boundary element method has been used. In the final part of the paper the results obtained are shown.

1. Introduction

The skin is treated as a multilayer 1D domain, in which one can distinguish the following sub-domains: epidermis of thickness L_1, m, dermis of thickness $L_2 - L_1$ and sub-cutaneous region Ω_3 of thickness $L_3 - L_2 - L_1$. The thermophysical parameters of successive layers equal λ_e, W/(mK) (thermal conductivity), and c_e, J/(m^3 K) (specific heat per unit of volume).

Thermal damage of skin begins when the temperature at the basal layer (the interface between epidermis and dermis) rises above 44°C (317 K). Henriques [1] found that the degree of skin damage could be predicted on the basis of the integrals

$$I_b = \int_0^{t^F} P_b(T_b)\exp\left(-\frac{\Delta E}{RT_b(t)}\right)\mathrm{d}t \tag{1}$$

and

$$I_d = \int_0^{t^F} P_d(T_d)\exp\left(-\frac{\Delta E}{RT_d(t)}\right)\mathrm{d}t \tag{2}$$

where $\Delta E/R$, K, is the ratio of activation energy to universal gas constant, P_b, P_d, 1/s, are the pre-exponential factors, while T_b, T_d, K, are temperatures of basal layer (the surface between epidermis and dermis) and dermal base (the surface between dermis and sub-cutaneous region).

First degree burn are said to occur when the value of the burn integral [1] is from the interval $0.53 < I_b \le 1$, while the second degree burn when $I_b > 1$ [2]. Third degree burn are said to occur when $I_d > 1$. So, in order to determine the values of integrals I_b, I_d the heating and next the cooling curves for the basal layer and dermal base must be known.

2. Governing equations

We consider the following mathematical model of bioheat transfer in the skin tissue [2-5]

$$\begin{cases} x \in \Omega_e: & c_e \dfrac{\partial T_e(x,t)}{\partial t} = \lambda_e \dfrac{\partial^2 T_e(x,t)}{\partial x^2} - k_e T_e(x,t) + Q_e \\ x = L_0 = 0: & \begin{cases} T_1(x,t) = T_0, & t \le t_0 \\ q_1(x,t) = -\lambda_1 \dfrac{\partial T_1(x,t)}{\partial x} = 0, & t > t_0 \end{cases} \\ x = L_1: & \begin{cases} T_1(x,t) = T_2(x,t) = T_b(t) \\ -\lambda_1 \dfrac{\partial T_1(x,t)}{\partial x} = -\lambda_2 \dfrac{\partial T_2(x,t)}{\partial x} = q_b(t) \end{cases} \\ x = L_2: & \begin{cases} T_2(x,t) = T_3(x,t) = T_d(t) \\ -\lambda_2 \dfrac{\partial T_2(x,t)}{\partial x} = -\lambda_3 \dfrac{\partial T_3(x,t)}{\partial x} = q_d(t) \end{cases} \\ x = L_3 = L: & q_3(x,t) = -\lambda_3 \dfrac{\partial T_3(x,t)}{\partial x} = 0 \\ t = 0: & T_e(x,t) = T_{ep}(x) \end{cases} \tag{3}$$

where $e = 1, 2, 3$ correspond to the epidermis, dermis and sub-cutaneous region, $Q_e = k_e T_B + Q_{me}$, $k_e = G_e c_B$ is the product of blood perfusion rate and volumetric specific heat of blood, T_B is the blood temperature and Q_{me} is the metabolic heat source, T_0 is the given boundary temperature, t_0 is the exposure time and $T_{ep}(x)$ is the initial temperature distribution.

3. Adjoint approach of sensitivity analysis

In the paper, using adjoint approach [6, 7], the sensitivity of the integrals I_b and I_d with respect to the thermophysical parameters is analyzed. These parameters we denote by p_s, $s = 1, 2, 3,\ldots, 9$, this means $p_1 = \lambda_1$, $p_2 = \lambda_2,\ldots$, $p_4 = c_1$, $\ldots$, $p_9 = k_3$. For the problem considered the following adjoint equations are defined [8]

$$L_{e-1} < x < L_e: \quad c_e \frac{\partial W_e(x,\tau)}{\partial \tau} = \lambda_e \frac{\partial^2 W_e(x,\tau)}{\partial x^2} - k_e W_e(x,\tau) \tag{4}$$

where $0 \leq \tau \leq \tau = t^F - t$.

The equations (4) are supplemented by the initial condition:

$$L_{e-1} < x < L_e, \quad \tau = 0: \quad W_e(x,0) = 0 \tag{5}$$

boundary condition for $x = L_0$:

$$x = 0: \quad W_1(x,\tau) = 0 \tag{6}$$

and boundary condition for $x = L$:

$$x = L: \quad V_3(x,\tau) = -\lambda_3 \frac{\partial W_3(x,\tau)}{\partial x} = 0 \tag{7}$$

For the functional (1) the continuity conditions for $x = L_1$ and $x = L_2$ have the following form [8]

$$x = L_1: \quad \begin{cases} W_1(x,\tau) = W_2(x,\tau) = W_b(\tau) \\ V_1(x,\tau) = V_2(x,\tau) - P_b(T_b) \dfrac{\Delta E}{R T_b^2(\tau)} \exp\left[-\dfrac{\Delta E}{R T_b(\tau)}\right] \end{cases} \tag{8}$$

and

$$x = L_2: \quad \begin{cases} W_2(x,\tau) = W_3(x,\tau) = W_d(\tau) \\ V_2(x,\tau) = V_3(x,\tau) = V_d(\tau) \end{cases} \tag{9}$$

while for the functional (2) one obtains

$$x = L_1: \quad \begin{cases} W_1(x,\tau) = W_2(x,\tau) = W_b(\tau) \\ V_1(x,\tau) = V_2(x,\tau) = V_b(\tau) \end{cases} \tag{10}$$

and

$$x = L_2: \quad \begin{cases} W_2(x,\tau) = W_3(x,\tau) = W_d(\tau) \\ V_2(x,\tau) = V_3(x,\tau) - P_d(T_d) \dfrac{\Delta E}{R T_d^2(\tau)} \exp\left[-\dfrac{\Delta E}{R T_d(\tau)}\right] \end{cases} \tag{11}$$

In above equations $V_e(x, \tau) = -\lambda \partial W_e(x, \tau)/\partial x$.

The sensitivity of integral (1) with respect to the parameter p_s is calculated using the formula [8]:

$$\frac{\partial I_b}{\partial p_s}=\int\limits_0^{t^F}\left\{\sum_{e=1}^{3}\int\limits_{L_{e-1}}^{L_e}\left[-\frac{\partial W_e}{\partial x}\frac{\partial \lambda_e}{\partial p_s}\frac{\partial T_e}{\partial x}-W_e\frac{\partial c_e}{\partial p_s}\frac{\partial T_e}{\partial t}-W_e\frac{\partial k_e}{\partial p_s}T_e+W_e\frac{\partial Q_e}{\partial p_s}\right]\mathrm{d}x\right\}\mathrm{d}t \quad (12)$$

similarly:

$$\frac{\partial I_d}{\partial p_s}=\int\limits_0^{t^F}\left\{\sum_{e=1}^{3}\int\limits_{L_{e-1}}^{L_e}\left[-\frac{\partial W_e}{\partial x}\frac{\partial \lambda_e}{\partial p_s}\frac{\partial T_e}{\partial x}-W_e\frac{\partial c_e}{\partial p_s}\frac{\partial T_e}{\partial t}-W_e\frac{\partial k_e}{\partial p_s}T_e+W_e\frac{\partial Q_e}{\partial p_s}\right]\mathrm{d}x\right\}\mathrm{d}t \quad (13)$$

Of course, in the case (12) the function $W_e(x, \tau)$ is the solution of problem described by equations (4)-(9), while in formula (13) the function $W_e(x, \tau)$ is the solution of problem described by equations (4)-(7), (10), (11).

4. Results of computations

The basic problem and additional ones have been solved using the 1st scheme of the BEM for multilayer 1D domain [8, 9].

In numerical computations the following mean values of parameters have been assumed [2, 8]: $\lambda_1 = 0.235$ W/(mK), $\lambda_2 = 0.445$ W/(mK), $\lambda_3 = 0.185$ W/(mK), $c_1 = 4.3068 \cdot 10^6$ J/(m^{3}K), $c_2 = 3.96 \cdot 10^6$ J/(m^{3}K), $c_3 = 2.674 \cdot 10^6$ J/(m^{3}K), $c_B = 3.9962 \cdot 10^6$ J/(m^{3}K), $T_B = 37°$C, $G_1 = 0$, $G_e = 0.00125$ (m^{3}blood/ s)/m^3 tissue for $e = 2, 3$, $Q_{m1} = 0$, $Q_{me} = 245$ W/m^3 for $e = 2, 3$ [2]. Pre-exponential factors: $P_b = 1.43 \cdot 10^{72}$ 1/s for $T_b \geq 317$ K and $P_b = 0$ for $T_b < 317$ K, while $P_d = 2.86 \cdot 10^{69}$ 1/s for $T_d \geq 317$ K and $P_d = 0$ for $T_d < 317$ K. The ratio of activation energy to universal gas constant: $\Delta E/R = 55000$ K. The thicknesses of successive skin layers: 0.1, 2 and 10 mm. These layers have been divided into 10, 40 and 120 internal cells. Time step: $\Delta t = 0.05$ s.

We assume that on the skin surface acts the external heat source of duration $t_0 = 5$ s which generates on this surface the temperature $T_0 = 60°$C. The times to first and second degree burn predicted for these values are equal 3.8 s and 5 s, respectively. In Figure 1 the temperature distribution for $x \in [0, 12$ mm] and time 1, 2, 3,…, 10 s is shown. The sensitivity analysis of burn integral I_b has been done with respect to the all thermophysical parameters. It turned out that especially essential in the case considered are the changes of thermal conductivity and volumetric specific heats of the epidermis and dermis sub-domain. In Figures 3, 4 and 5 the courses of burn integral I_b found on the basis of sensitivity analysis with respect to λ_1, λ_2 and c_2 are shown. Here it was assumed that $\Delta\lambda_1 = 0.025$, $\Delta\lambda_2 = 0.075$, $\Delta c_2 = 120000$ and the following formula has been taken into account

$$I_b(t, p_s \pm \Delta p_s) = I_b(t, p_s) \pm \frac{\partial I_b}{\partial p_s}\Delta p_s \quad (14)$$

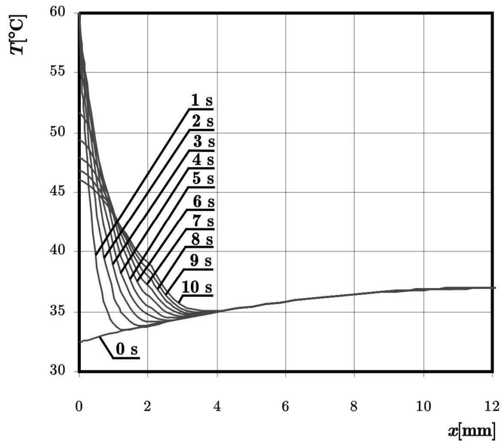

Fig. 1. Temperature distribution

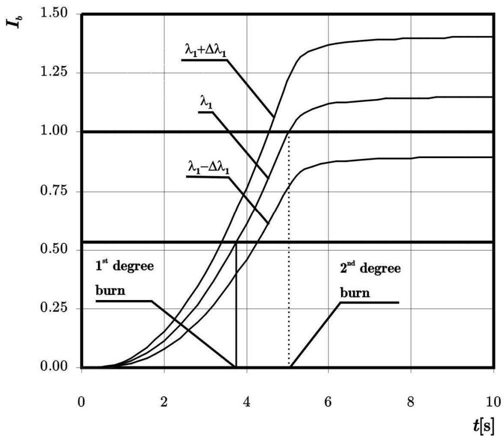

Fig. 2. Integral I_b - change of λ_1

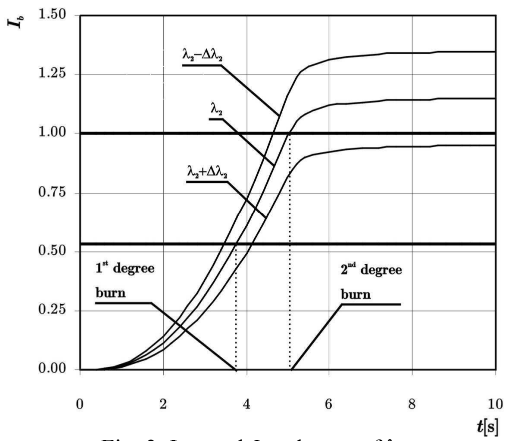

Fig. 3. Integral I_b - change of λ_2

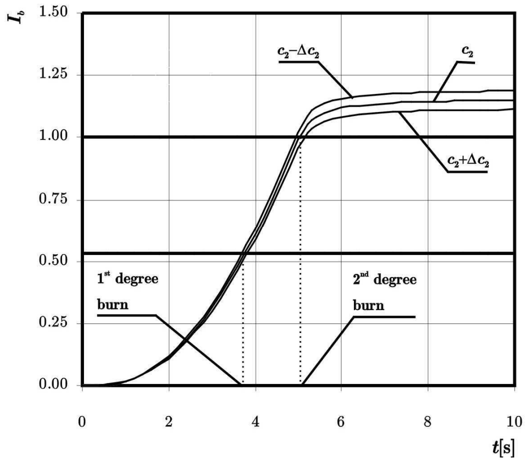

Fig. 4. Integral I_b - change of c_2

The paper was sponsored by KBN (Grant No 3 T11F 018 26).

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