AN INVESTIGATION INTO THE INFLUENCE OF ACTIVITY DURATION TIME FUZZINESS ON CRITICAL PATH PARAMETERS

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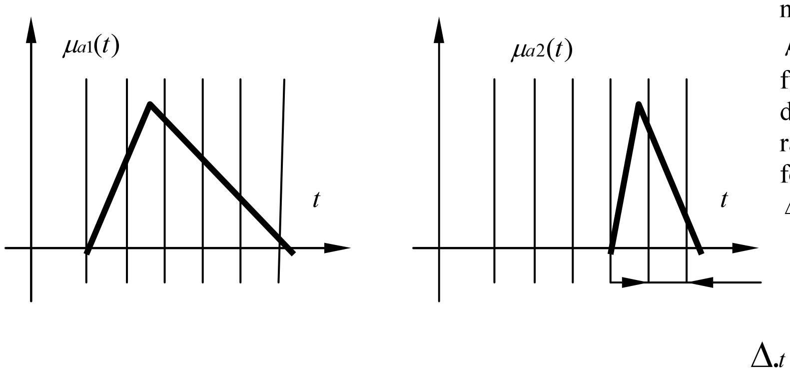
Abstract. Resources and fuzziness influence in a crucial way critical path parameters such as length and configuration. An algorithm establishing the order and range of the analysis by changing the activity duration time values, resulting from the fuzziness range in each node, will be formulated to investigate this influence. The research strategy selection has an essential influence upon the results, their range and quality and their reliability. The obtained results, applying the canons of engineering backwards, can be used to regulate the fuzziness range thereby controlling the course and length of the critical path.

1. Principles used in investigations of activity diagrams as an element of the analytic procedures algorithmization

A full file of sets of the individual activities duration time has to be used to create the full picture of all possible configurations of the critical path. Different data for the concrete set, created with the use of time increments (time slices), result from the time fuzziness.



$a1...a5$ - activity nodes,
$\mu_{ai}(t)$ - belonging function of activity duration time to range determined for ai node
Δ_t - time slice

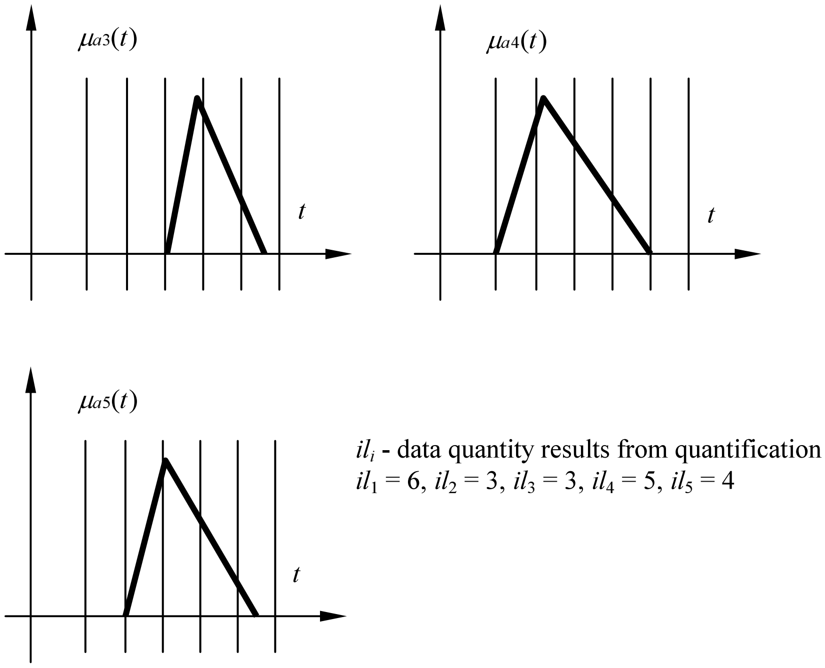


Fig. 1. An example of the set of activity duration fuzzy times

An analysis of the activity network has a discrete character and does not allow the precise determination of the duration time set parameters for which the change of critical path configuration occurs. The analysis in the example from Figure 1 includes the variation of sets, i.e. $V = il_1 \cdot il_2 \cdot il_3 \cdot il_4 \cdot il_5 = 6 \cdot 3 \cdot 3 \cdot 5 \cdot 4 = 1080$. On one hand, the lack of continuity makes it impossible to catch the sore points exactly (points of the critical path configuration change), on the other hand, it is not necessary to analyse all cases characterized by the variation algorithm. The spots of the critical path „trajectory" change are the nodes to which more than one directed edge is reaching (Fig. 2).

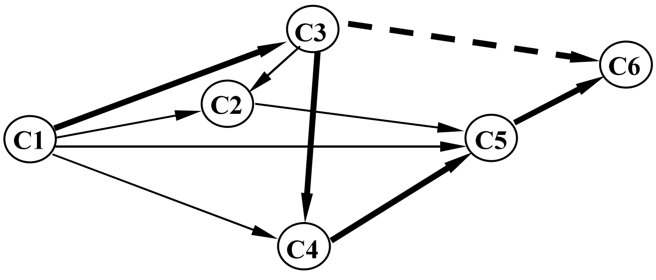


Fig. 2. Activity network with four sore nodes: C2, C4, C5, C6

Node C2 is reached from C1 or C3, C4 from C1 or C3, C5 from C1, C2 or C4, C6 from C3 or C5. The shape of critical path will mark out C1-C3-C4-C5-C6 points if the following deterministic data are assumed: C1-C2 = 9 (time unit - t. u.), C1-C3 = 6, C1-C4 = 4, C1-C5 = 2, C2-C5 = 2, C3-C4 = 5, C3-C6 = 11, C4 = C5 = = 1, C5-C6 = 7. The length of critical path amounts to 19 t. u. (the heavy line in Figure 2). However, if the activity described by C3-C6 nodes will grow longer for example up to 22 t. u., as a result of the fuzziness, then the critical path changes its shape and will be determined by C1-C3-C6 nodes and its length amounts to 28 t. u. The longest way to the individual forks (sore points) in the classical algorithm is determined, from the last one which is a target node. It is as assumed that carrying out the analysis to the last point to which more then one edge reaches is only sensible. This phase of the algorithm allows the determination of the critical path length as well as the so-called earliest start and end activity. Certainly, the change of activity duration time changes the critical path configuration, which results from a non-deterministic (uncertain) knowledge to this point, i.e. from the time data fuzziness. The classical algorithm of the critical path search is considered below (it is written in the source code - Algorithm 1). Both the data and the results are shown in the form of the table *a*[*i*,*j*], where *i* represents the activity number, and *j* represents successively: - start node, - end node, - activity duration time, - the earliest moment of activity start, - the earliest moment of activity end, - the latest moment of activity start, - the latest moment of activity end, - time reserve.

```
readln(f,n);
for i:=1 to n do
for j:=1 to 8 do a[i,j]:=0;
for i:=1 to n do
begin
for j:=1 to 3 do
read(f,a[i,j]); readln(f)
end;
i:=1;l:=1;b[i]:=a[1,1];
qq:for k:=1 to n do
begin
if a[k,1]=b[i] then a[k,6]:=a[k,3]+a[k,4] else goto q;
for j:=1 to n do
if a[j,1]=a[k,2] then
if a[k,6]>a[j,4] then begin
a[j,4]:=a[k,6];a[j,6]:=a[j,3]+a[j,4] end;
for m:=1 to l do
if a[k,2]=b[m] then goto q;l:=l+1;b[l]:=a[k,2];
q:end;
if i<l then begin i:=i+1; goto qq end;
max:=a[n,6];m:=n;
```

```
for i:=n-1 downto 1 do
if max<a[i,6] then begin max:=a[i,6];m:=i end;
for i:=1 to n do
if a[i,2]=a[m,2] then begin
a[i,7]:=a[m,6];a[i,5]:=a[i,7]-a[i,3] end;
l:=1;i:=1;b[i]:=a[m,2];
t:for k:=n downto 1 do
begin
if a[k,2]=b[i] then
begin
a[k,5]:=a[k,7]-a[k,3];
for j:=n downto 1 do
if a[j,2]=a[k,1] then
if (((a[j,7]>a[k,5]) then
begin a[j,7]:=a[k,5];a[j,5]:=a[j,7]-a[j,3];end;
for m :=1 to l do
if a[k,1]=b[m] then goto tt;l:=l+1;b[l]:=a[k,1]
end;
tt:end;
if i<l then begin i:=i+1;goto t end;
for i:=1 to n do a[i,8]:=a[i,7]-a[i,6];
for k:=1 to n do
for j:=1 to n do
if ((a[j,1]=a[k,2]) and (a[k,7]>a[j,5])) then
begin a[k,7]:=a[j,5]; a[k,5]:=a[k,7]-a[k,3];end;
for i:=1 to n do a[i,8]:=a[i,7]-a[i,6];
for i:=1 to n do
begin
for j:=1 to 8 do write(g,a[i,j]:3,' ');writeln(g);
end;close(g);
```

Algorithm 1. Analyse algorithm of activity diagram and critical path creation (ACP)

The correction of the earliest time of the successive activity start is realized in this algorithm (column 4 of the results matrix, i.e. $a[j,4]$) - after each search for a longer path to the sore node. This is realized by the program statements set in which the time to reach the given node by the addition of a successive edge (activity) is compared to the hitherto existing maximal value of this time. These statements are marked off by bold print. Algorithm 1 can be used to carry out the analysis and search out the parameters for which the change of critical path shape in uncertain knowledge circumstances occurs concerning the activities duration time which are related by means of conjunctional and disjunctional connections on the activity diagram [9]. A research organization diagram, shown in Algorithm 2 (Fig. 3), can be used to carry out the analysis in the full range of activity duration time fuzziness. Algorithm 1 will be used here as a procedure of the critical path shape and length registration for each activity duration time set. The results of the procedure operation will be placed in tables (Table 1 shows an example of the obtained results).

Table 1

A1	A2	A3	A4	A5	A6	A7	A8
1	2	9	0	1	9	10	1
1	3	6	0	0	6	6	0
1	4	4	0	7	4	11	7
1	5	2	0	10	2	12	10
2	5	2	9	10	11	12	1
3	4	5	6	6	11	11	0
3	6	11	6	8	17	19	2
4	5	1	11	11	12	12	0
5	6	7	12	12	19	19	0

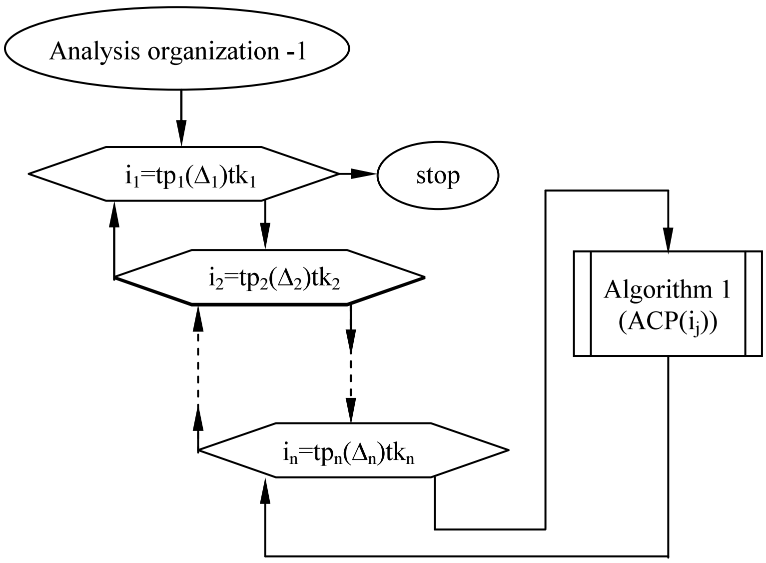


Fig. 3. Analysis organization diagram and registration of critical path parameters, where: t_p, t_k - start and end of fuzziness (Algorithm 2)

Algorithm 2 (Fig. 3) is a representation of the research organization method, which consists in the systematic search. Thus, it is not effective, because the number of possible paths is maximal equal to the forks product of all nodes and the sets of activity network parameters is equal to the product of the number of digitised

values of the all activities duration time ($il_1 \cdot il_2 \cdot ... \cdot il_n$) (Fig. 1). The computational (or temporal) complexity is proportional to this product.
Another assumption of the research organization may be an initial selection of all possible paths and then an examination of each of them by means of a digitisation of activity duration times which are part of a given path taking into consideration reserves resulting from conjunction connections [9]. The computational complexity of this algorithm reduces itself insignificantly in relation to Algorithm 2 (Fig. 3) (statistically: maximum twice).
The next variant of the activity diagram research organization is the most effective. It consists in an investigation of the influence of the duration time fuzziness of individual activities on the critical path configuration by minimal values of remaining activities (Fig. 4). Thanks to this, it is easier to appoint turning point moments in which a change of the critical path „track" occurs.

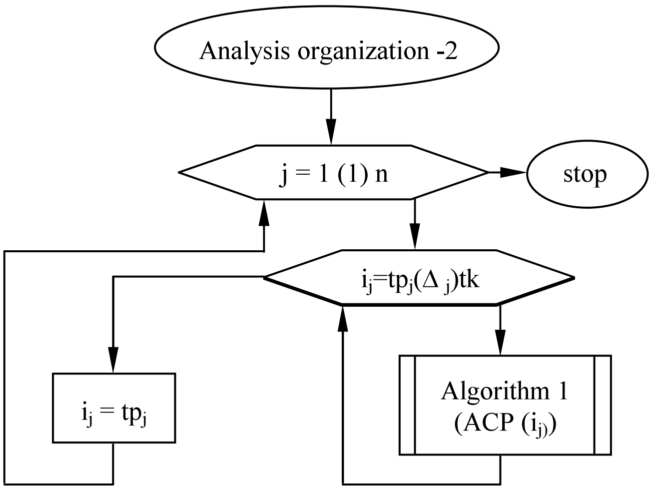


Fig. 4. Analysis organization diagram based on an investigation of the influence of the duration time fuzziness of individual activities on the critical path shape (Algorithm 3)

The analysis organization diagram is useful in the situation when the variation of the activity duration times does not exceed a dozen or so percent and this presents an essential limitation and disadvantage of this model. Its crucial advantage is the small computational complexity, which is proportional, not to the product but to the sum of the number of digitalized values of the individual activities duration times ($il_1 + il_2 + ... + il_n$).
As an example, the change of configuration and critical path length can be disclosed for the input data presented in Table 2. It is shown by way of an example in selected tables 3, 4, 5 and 6.

Table 2. Input data concerning duration time and fuzziness of the activities determined by edges (column 1-> node 1; column 2-> node 2)

9			
1	2	9	14
1	3	6	8
1	4	4	8
1	5	2	4
2	5	2	6
3	4	5	9
3	6	11	14
4	5	1	4
5	6	7	11

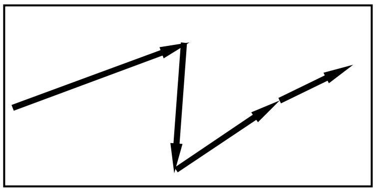
edges number

fuzziness upper limit

fuzziness lower limit

Tables 3, 4, 5 and 6. Exemplifying results describing the critical path configuration for different data from the range of activity duration time fuzziness (based on the diagram in Figure 2)

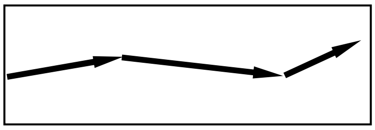
1	2	9	0	1	9	10	1
1	3	6	0	0	6	6	0
1	4	4	0	7	4	11	7
1	5	2	0	10	2	12	10
2	5	2	9	10	11	12	1
3	4	5	6	6	11	11	0
3	6	11	6	8	17	19	2
4	5	1	11	11	12	12	0
5	6	7	12	12	19	19	0



Critical path configuration
1 – 3 – 4 – 5 – 5 - 6

1	2	14	0	1	14	15	1
1	3	8	0	0	8	8	0
1	4	8	0	9	8	17	9
1	5	4	0	17	4	21	17
2	5	6	14	15	20	21	1
3	4	9	8	8	17	17	0
3	6	14	8	18	22	32	10
4	5	4	17	17	21	21	0
5	6	11	21	21	32	32	0

1	2	11	0	0	11	11	0
1	3	6	0	1	6	7	1
1	4	4	0	8	4	12	8
1	5	2	0	11	2	13	11
2	5	2	11	11	13	13	0
3	4	5	6	7	11	12	1
3	6	11	6	9	17	20	3
4	5	1	11	12	12	13	1
5	6	7	13	13	20	20	0



Critical path configuration
1 – 2 – 5 - 6

1	2	14	0	0	14	14	0
1	3	6	0	4	6	10	4
1	4	4	0	11	4	15	11
1	5	2	0	14	2	16	14
2	5	2	14	14	16	16	0
3	4	5	6	10	11	15	4
3	6	11	6	12	17	23	6
4	5	1	11	15	12	16	4
5	6	7	16	16	23	23	0

Other values of the duration time fuzziness lead to other configurations of the critical path; it can be path 1-3-6 (as shown in Figure 2). The conducted research can be used to optimisation level according to reserves and critical path length minimization criteria. An organization improvement and its costs decrease will be facilitated when both criteria are taken into consideration.

2. Scheduling optimisation in uncertain knowledge conditions concerning activity duration times

The minimization of the critical path length for a set of duration time fuzzy values consists in the selection of the activity set with the shortest periods of realization:

$$SET_{\min} = \{\min(tp_1),\min(tp_2),...,\min(tp_n)\}$$

The minimization of time reserves may be variously interpreted. Its implementation can be treated as intentional procedure or devoid of purpose. In the second case, these reserves can be effectively used for the realization of other tasks, for example, with a preparatory or protective character. The fuzziness of the activity duration time can be used for a time reserves decrease, if they can be treated as negative. Of course, the selection of which activities might be „lengthened" for the time reserves decrease depends on the undertaking organizer.

3. Conclusions

1. Fuzzy information to the point of an activity duration time make the activity connections system more flexible and allows the optimisation of the undertaking realization which is made up of conjunctional and disjunctional connections of these activities.
2. The problem of the algorithmization of the activity diagrams investigation with fuzzy time data remains open. The main task is to obtain all possible configurations of the critical path for the full data set.
3. One of the more effective algorithms is based on the investigation of the individual activities interaction (in the range of time fuzziness) with simultaneous preservation of duration time minimal values for remaining activities. It increases the correction probability of the critical path but does not secure the full review of time specifications.
4. The optimisation of undertaking realization organization treated as the activities set with the specific conjunctional and disjunctional connections is the combination of critical path length minimization and efficiency maximization of utilization of time reserves.

References

[1] Ajdukiewicz K., Language and cognition, Vol. I and II. PWN, Warsaw 1985 (in Polish).

[2] Buzan T., Memory in readiness, RAVI, Warsaw 2002 (in Polish).

[3] Cormen T.H., Introduction to algorithms, WNT, Warsaw 1997 (in Polish).

[4] Hanter I.M.L., Memory, facts and illusions, PWN, Warsaw 1963 (in Polish).

[5] Kacprzyk J., Fuzzy sets in system analysis, PWN, Warsaw 1986 (in Polish).

[6] Lachwa A., Fuzzy world of files, numbers, relations, facts, rules and decisions, Akademicka Oficyna Wydawnicza Exit, Warsaw 2001 (in Polish).

[7] Lorayane H., Secrets of supermemory, RAVI, Warsaw 2002 (in Polish).

[8] Piegat A., Fuzzy modelling and controlling, Akademicka Oficyna Wydawnicza Exit, Warsaw 2003 (in Polish).

[9] Syslo M., Deo N., Kowalik J., Discrete optimisation algorithms, WN PWN, Warsaw 1995 (in Polish).

[10] Rutkowska D., Piliński M., Rutkowski L., Neural networks, genetic algorithms and fuzzy systems. WN PWN, Warsaw 1997 (in Polish).

[11] Zadeh L.A., Fuzzy limitations calculus; design and systems; methodological problems, Ossolineum 1980 (in Polish).