ON THE MODELLING OF DYNAMIC BEHAVIOUR IN A LAMINATED MEDIUM WITH A SPACE-VARYING STRUCTURE*

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Abstract. In this contribution we propose an approach to the modelling of dynamic problems in a laminated solids with a space-varying structure. This approach is a certain generalization of the modelling procedure formulated in [3] for periodic structures, based on the simplicial subdivision of the unit cell and resulted in the finite difference form of the governing equations. Passage from the finite difference model to the discrete-continuum one is shown.

1. Introduction

In the existing literature the main attention in modelling of the dynamic behaviour of laminated media was given to the layered composites with a periodic structure [1, 2]. However in many situations met in engineering we have to deal with laminated solids with a space varying structure. This space-varying structure can be caused by:

– curvilinear shape of a structural element,
– variable volume percentage of constituents,
– desired arrangement of a reinforcement in a vicinity of holes and notches,
– designed distribution of material properties of a solid (gradient materials),
– complying with the cost constraints.

The aim of this contribution is to propose the mathematical model for solids with a two component laminated structure varying in space in one direction (z-axis direction) and homogeneous in every perpendicular direction (x-axis direction); an example of this type laminated solid is shown in Figure 1. The proposed approach constitutes a certain generalization of the approach to the modelling of periodic structures based on the periodic simplicial division of an elastic periodic medium and leading to the system of finite difference equations [3-6]. A new element of the proposed modelling method is a passage from the discrete to the discrete-continuum equations involving partial derivatives in argument x, t and finite differences in the z-axis direction. Symbol t stands for the time coordinate.

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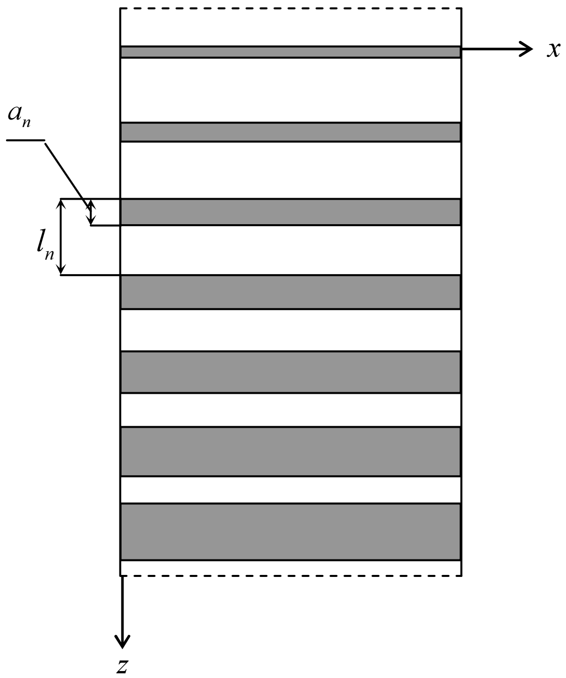

Fig. 1. Fragment of the gradient type laminated material

All consideration are carried out in the context of the linear elasticity theory under assumption of the perfect bonding between adjacent layers. For the sake of simplicity considerations will be restricted to the plane strain problems for an unbounded medium. We shall also assume that linear-elastic homogeneous layers with material planes z = const are elastic symmetry planes. Moreover the modelling procedure is restricted to problems in which the minimum length of the deformation pattern in the direction normal to the laminae is sufficiently large when compared to the maximum thickness of laminae.

Throughout the paper superscripts a, b take the value 1,2,...,m, subscript n run over 0,±1,±2,... . Index A takes the value 1, 3. The summation convention holds for index A.

2. Formulation of the finite difference simplicial model

In the first stage of modelling we formulate what will be called the finite difference simplicial model by applying a certain generalization of the approach proposed in [3] for medium with periodic structure. Let the laminated solid under consideration has a periodic structure determined by a vector $\mathbf{d}^2$ parallel to the laminae interfaces and varying in space structure determined by vectors $\mathbf{d}^1_n$, n = 0,±1,±2,..., as shown in Figure 2. The length of vector $\mathbf{d}^2$ is denoted by d and can be an arbitrary established because of a homogeneous structure of aforementioned laminate in x-axis direction. The length of vectors $\mathbf{d}_n^1$ are denoted by l_n and are determined by a space-varying structure of aforementioned laminate in z-axis direction. Here and hereafter it is also assumed that a certain simplicial division of every typical segment, which is shown in Figure 2, is known. In Figure 2 the simplest simplicial division is presented.

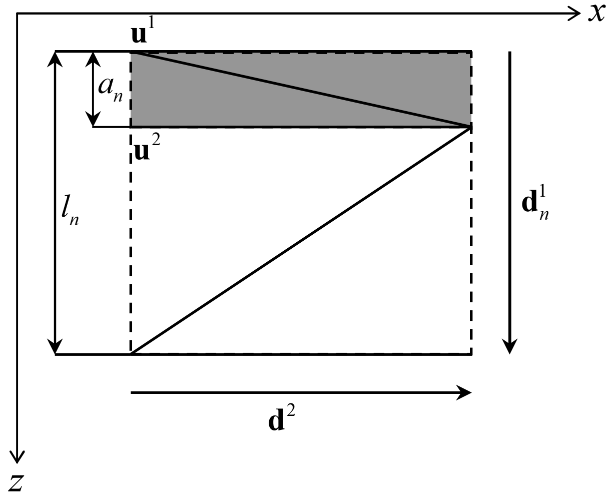

Fig. 2. The simplest simplicial division of a medium (periodic in the *x*-axis direction)

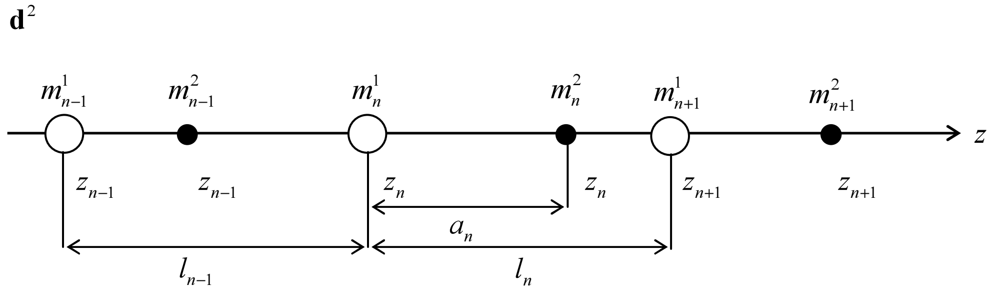

Fig. 3. System of masses for the simplest simplicial division

We shall also approximate the mass distribution in the laminated medium by a system of concentrated masses assigned to nodal points belonging to the typical segment for the related simplicial division. Hence there are systems of masses m_n^a, $a = 1,2,\ldots,m$, $n = 0,\pm1,\pm2$. For the simplest simplicial division there are two systems of masses m_n^1 and m_n^2 as shown in Figure 3. Let us denote by $\mathbf{u}^a(x,z_n,t)$, $x = nd$, $n = 0,\pm1,\pm2$ the displacement of mass points m_n^a. For the sake of application the results derived in [3] we introduce the system of vectors $\mathbf{d}_n^3 = \mathbf{d}_n^1 + \mathbf{d}^2$ and the following finite differences:

$$\begin{aligned}
&\Delta_{1n}\mathbf{u}^a\left(x, z_n, t\right) = \mathbf{u}^a\left(x, z_n + l_n, t\right) - \mathbf{u}^a\left(x, z_n, t\right) \\
&\bar{\Delta}_{1n}\mathbf{u}^a\left(x, z_n, t\right) = \mathbf{u}^a\left(x, z_n, t\right) - \mathbf{u}^a\left(x, z_n - l_{n-1}, t\right) \\
&\Delta_{3n}\mathbf{u}^a\left(x, z_n, t\right) = \mathbf{u}^a\left(x + d, z_n + l_n, t\right) - \mathbf{u}^a\left(x, z_n, t\right) \\
&\bar{\Delta}_{3n}\mathbf{u}^a\left(x, z_n, t\right) = \mathbf{u}^a\left(x, z_n, t\right) - \mathbf{u}^a\left(x - d, z_n - l_{n-1}, t\right) \\
&\Delta_2\mathbf{u}^a\left(x, z_n, t\right) = \mathbf{u}^a\left(x + d, z_n, t\right) - \mathbf{u}^a\left(x, z_n, t\right) \\
&\bar{\Delta}_2\mathbf{u}^a\left(x, z_n, t\right) = \mathbf{u}^a\left(x, z_n, t\right) - \mathbf{u}^a\left(x - d, z_n, t\right)
\end{aligned} \tag{1}$$

Hence under denotation $\mathbf{u}_n^a = \mathbf{u}^a\left(x, z_n, t\right)$ we conclude that the strain energy function for the laminated material under consideration is represented by

$$U_n = U_n\left(\Delta_{An}\mathbf{u}_n^a, \Delta_2\mathbf{u}_n^b, \mathbf{u}_n^a - \mathbf{u}_n^b\right) \tag{2}$$

where a, b = 1,2,...,m, n = 0,±1,±2, A = 1, 3. It can be shown [6], that for the unknowns $\mathbf{u}_n^a$ in the absence of body forces we obtain a system of equations

$$\bar{\Delta}_{An}\frac{\partial U_n}{\partial \Delta_{An}\mathbf{u}_n^a} + \bar{\Delta}_2\frac{\partial U_n}{\partial \Delta_2\mathbf{u}_n^a} - \frac{\partial U_n}{\partial \mathbf{u}_n^a} = m_n^a\ddot{\mathbf{u}}_n^a, \quad a = 1,2,...,m \tag{3}$$

which represent the finite difference simplicial model of the laminated medium under consideration.

3. Passage to a discrete-continuum model

In order to obtain a discrete-continuum model, due to the laminated structure under consideration is homogeneous in every direction parallel to laminate interfaces, we introduce assumption $\left|\mathbf{d}^2\right| \to 0$. Hence, under denotation $\partial(\cdot) = \frac{\partial}{\partial x}(\cdot)$, $x \in R$, we conclude that the strain energy function (2) can be approximated by

$$\tilde{U}_n = U_n\left(\Delta_{1n}\mathbf{u}_n^1, \partial\mathbf{u}_n^a, \mathbf{u}_n^a - \mathbf{u}_n^b\right) \tag{4}$$

Then it can be shown that the equations of motion (3) can be approximated by equations

$$\bar{\Delta}_{1n}\frac{\partial \tilde{U}_n}{\partial \Delta_{1n}\mathbf{u}_n^1} + \partial\frac{\partial \tilde{U}_n}{\partial\left(\partial\mathbf{u}_n^a\right)} - \frac{\partial \tilde{U}_n}{\partial \mathbf{u}_n^a} = \tilde{m}_n^a\ddot{\mathbf{u}}_n^a, \quad a = 1,2,...,m \tag{5}$$

where $\tilde{m}_n^a$ is the mass density. Equations (5) represent the discrete-continuum model of the laminated medium under consideration. The resulting equations are represented by system of partial differential equations combined with finite difference equations in the direction perpendicular to the laminae interfaces.

4. Conclusions and remarks

The main results of this contribution are:

1. A family of approximated mathematical models for analysis of dynamic behaviour of elastic laminates with a space-varying structure is formulated.
2. The obtained models are represented by system of PDEs combined with finite difference equations in the direction perpendicular to the lamina interfaces.
3. The proposed approach possible to investigate propagation of both short and long waves (for a periodic medium).
4. The derived models will be applied to the dynamic analysis of the laminated gradient materials.
5. The modelling approach proposed in this contribution constitutes a starting point for a formation of discrete-continuum models describing the dynamic behaviour of laminated media in which interfaces between adjacent laminae are not plane.

An example of application of the proposed models will be discussed in a separate paper.

References

[1] Herrmann G., Kaul R.K., Delph T.J., On Continuum Modelling of the Dynamic Behavior of Layered Composites, Arch. Mech. 1976, 28, 405-421.

[2] Fish J., Wen Chen, Higher-order Homogenization of Initial/boundary-value Problem, Journ. of Eng. Mech. 2001, 127, 1223-1230.

[3] Rychlewska J., Szymczyk J., Woźniak C., A Simplicial Model for Dynamic Problems in Periodic Media, J. Theor. Appl. Mech. 2000, 38, 3-13.

[4] Rychlewska J., Woźniak C., Simplicial modelling of dynamic problems in a micro-periodic composite material, PAAM Proc. Appl. Math. Mech. 2003, 3, 481-482.

[5] Rychlewska J., Szymczyk J., Woźniak C., On the modelling of dynamic behaviour of periodic lattice structures (in the course of publication).

[6] Woźniak C., Discrete Elasticity, Arch. Mech. 1971, 23, 801-816.