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TEMPERATURE DETERMINATION IN THE TISSUE WITH A TUMOR USING MRBEM AND FEM

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Abstract. The numerical algorithms based on the boundary element method and finite element method are used for the temperature field computations in the non homogenous domain being the composition of healthy tissue and the tumor region. The three dimensional problem is considered. Thermophysical parameters of sub-domains, in particular the perfusion coefficients, thermal conductivities and the metabolic heat sources are different. From the mathematical point of view the problem is described by the system of two Poisson's equations with temperature-dependent source functions. These equations are supplemented by the adequate boundary conditions. The algorithms discussed allow, among others, to determine the temperature distribution on the surface of the skin. In the final part of the paper the examples of computations are shown.

1. Governing equations

From the mathematical point of view the bioheat transfer processes in the domain of biological tissue are described by the Pennes equation [1, 2]. If we consider the steady-state problem then we obtain the following system of equations

$$x \in \Omega_e : \quad \lambda_e \nabla^2 T_e(x) + k_e \left[T_B - T_e(x) \right] + Q_{me} = 0 \tag{1}$$

where $e = 1, 2$ identifies the sub-domains of healthy tissue and tumor (Fig. 1), λ_e is the thermal conductivity, $k_e = G_e c_B$ is the perfusion coefficient (G_e is the blood perfusion rate, c_B is the volumetric specific heat of blood), T_B is the blood temperature, Q_{me} is the metabolic heat source.
On the surface between tissue and tumor the ideal thermal contact is assumed:

$$x \in \Gamma_c : \quad \begin{cases} T_1(x) = T_2(x) = T(x) \\ q_1(x) = -q_2(x) = q(x) \end{cases} \tag{2}$$

where $q_e(x) = -\lambda_e \partial T_e(x) / \partial n_e$ is the heat flux, $\partial T_e(x) / \partial n_e$ denotes the directional derivative at the boundary point considered, while n_e is the external unit normal vector.

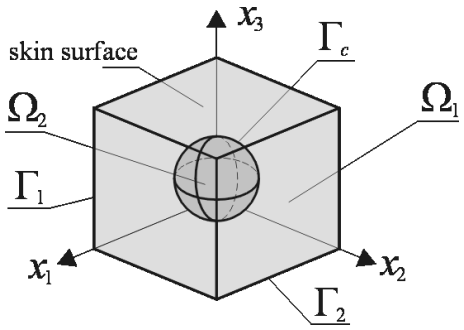

Fig. 1. Skin tissue with a tumor

On the remaining parts of the boundary the following boundary conditions (Fig. 1) can be accepted:

$$\begin{aligned} x \in \Gamma_1 &: \quad q_1(x) = 0 \\ x \in \Gamma_2 &: \quad T_1(x) = T_b \end{aligned} \tag{3}$$

where T_b is the known boundary temperature.

2. Multiple reciprocity boundary element method

From the mathematical point of view the Pennes equations (1) describing the steady temperature field in the system healthy tissue-tumor region are the Poisson equations in which the source functions are temperature-dependent. So, in the case of standard boundary element method application the boundary and also the interior of the domain considered should be discretized. If the multiple reciprocity BEM is used then only the boundary of the domain is discretized.
The algorithm is presented for the equations describing the temperature distribution in the tissue with tumor

$$x \in \Omega_e : \quad \lambda_e \nabla^2 T_e(x) - k_e T_e(x) + Q_e = 0 \tag{4}$$

where $Q_e = k_e T_B + Q_{me}$.

The standard boundary element method leads to the following integral equations [3-5]

$$\begin{aligned} &B(\xi) T_1(\xi) + \int_{\Gamma_e} V_{0e}^*(\xi, x) q_e(x) \mathrm{d}\Gamma_e = \\ &\int_{\Gamma_e} Z_{0e}^*(\xi, x) T_e(x) \mathrm{d}\Gamma_e + \int_{\Omega_e} \left[-k_e T_e(x) + Q_e \right] V_{0e}^*(\xi, x) \mathrm{d}\Omega_e \end{aligned} \tag{5}$$

where ξ is the observation point, $B(\xi) \in (0, 1]$, $V_{0e}^*(\xi, x)$ is the fundamental solution:

$$V_{0e}^{*}\left(\xi,x\right)=\frac{1}{4\pi\lambda_{e}r} \tag{6}$$

r is the distance between the points ξ and x

$$r=\sqrt{\left(x_1-\xi_1\right)^2+\left(x_2-\xi_2\right)^2+\left(x_3-\xi_3\right)^2} \tag{7}$$

while

$$Z_{0e}^{*}\left(\xi,x\right)=-\lambda_{e}\frac{\partial V_{0e}^{*}\left(\xi,x\right)}{\partial n} \tag{8}$$

and

$$q_{e}\left(x\right)=-\lambda_{e}\frac{\partial T_{e}\left(x\right)}{\partial n_{e}} \tag{9}$$

The heat flux resulting from fundamental solution can be calculated analytically and then

$$Z_{0e}^{*}\left(\xi,x\right)=\frac{d}{4\pi r^3} \tag{10}$$

where

$$d=\left(x_1-\xi_1\right)\cos\alpha_1+\left(x_2-\xi_2\right)\cos\alpha_2+\left(x_3-\xi_3\right)\cos\alpha_3 \tag{11}$$

while $\cos\alpha_1$, $\cos\alpha_2$, $\cos\alpha_3$ are the directional cosines of the normal outward vector n. The last component in equations (5) we denote by I. In multiple reciprocity method the domain integral I is transformed into the equivalent boundary integrals [2, 6-8]

$$\begin{aligned} I=\sum_{l=1}^{\infty}\left(\frac{k_e}{\lambda_{\mathrm{e}}}\right)^{l-1}\int_{\Gamma_e}\left[-\frac{Q_e}{\lambda_e}+\frac{k_e}{\lambda_e}T_e\left(x\right)\right]Z_{le}^{*}\left(\xi,x\right)\mathrm{d}\Gamma_e- \\ \sum_{l=1}^{\infty}\left(\frac{k_e}{\lambda_e}\right)^{l}\int_{\Gamma_e}V_{le}^{*}\left(\xi,x\right)q_e\left(x\right)\mathrm{d}\Gamma_e \end{aligned} \tag{12}$$

where

$$V_{le}^{*}\left(\xi,x\right)=\frac{1}{4\pi\lambda_e}r^{2l-1}C_l,\quad l=1,2,3,... \tag{13}$$

while:

$$C_0 = 1, \quad C_1 = \frac{1}{2}, \quad C_2 = \frac{1}{24}$$
$$C_l = \frac{1}{(2l-1)(2l-3)} C_{l-1}, \quad l = 3,4,5,... \tag{14}$$

and

$$Z_{le}^*(\xi, x) = -\frac{d}{4\pi}(2l-1) r^{2l-3} C_l \tag{15}$$

In numerical realization, the boundary Γ is divided into N boundary elements Γ_j, $j = 1, 2,..., N$. If the constant elements are used, then one obtains the following system of algebraic equations (c.f. equations (5), (6), (13), (14))

$$\sum_{j=1}^{N} G_{ij} q_j = \sum_{j=1}^{N} H_{ij} T_j + R_i, \qquad i = 1,2,...,N \tag{16}$$

where

$$G_{ij} = \sum_{l=0}^{\infty} \left(\frac{k_e}{\lambda_e} \right)^l \int_{\Gamma_j} V_l^*(\xi^i, x) \, d\Gamma_j \tag{17}$$

while

$$H_{ij} = \sum_{l=0}^{\infty} \left(\frac{k_e}{\lambda_e} \right)^l \left[\int_{\Gamma_j} Z_l^*(\xi^i, x) \, d\Gamma_j - \frac{1}{2} \delta_{ij} \right] \tag{18}$$

and

$$R_i = -\frac{Q_e}{\lambda_e} \sum_{j=1}^{N} \left[\sum_{l=1}^{\infty} \left(\frac{k_e}{\lambda_e} \right)^{l-1} \int_{\Gamma_j} Z_l^*(\xi^i, x) \, d\Gamma_j \right] \tag{19}$$

For the needs of further considerations the following denotations are introduced (c.f. Figure 1):

- $\mathbf{T}_1$, $\mathbf{q}_1$ are the vectors of function T and q on the boundary $\Gamma_1 \cup \Gamma_2$ of domain Ω_1,
- $\mathbf{T}_{c1}$, $\mathbf{T}_{c2}$, $\mathbf{q}_{c1}$, $\mathbf{q}_{c2}$ are the vectors of functions T and q on the contact surface Γ_c between domains Ω_1 and Ω_2.

Using above notations, one obtains the following systems of equations:

- for the healthy tissue domain

$$\begin{bmatrix}\mathbf{G}_1 & \mathbf{G}_{c1}\end{bmatrix}\begin{bmatrix}\mathbf{q}_1 \\ \mathbf{q}_{c1}\end{bmatrix} = \begin{bmatrix}\mathbf{H}_1 & \mathbf{H}_{c1}\end{bmatrix}\begin{bmatrix}\mathbf{T}_1 \\ \mathbf{T}_{c1}\end{bmatrix} + \mathbf{R}_1 \tag{20}$$

- for the tumor region

$$\mathbf{G}_{c2}\mathbf{q}_{c2} = \mathbf{H}_{c2}\mathbf{T}_{c2} + \mathbf{R}_2 \tag{21}$$

The condition (2) written in the form

$$\begin{cases}\mathbf{q}_{c1} = -\mathbf{q}_{c2} = \mathbf{q} \\ \mathbf{T}_{c1} = \mathbf{T}_{c2} = \mathbf{T}\end{cases} \tag{22}$$

should be introduced to the equations (22) and (23) and then

$$\begin{bmatrix}\mathbf{G}_1 & -\mathbf{H}_{c1} & \mathbf{G}_{c1} \\ \mathbf{0} & -\mathbf{H}_{c2} & -\mathbf{G}_{c2}\end{bmatrix}\begin{bmatrix}\mathbf{q}_1 \\ \mathbf{T} \\ \mathbf{q}\end{bmatrix} = \begin{bmatrix}\mathbf{H}_1\mathbf{T}_1 + \mathbf{R}_1 \\ \mathbf{R}_2\end{bmatrix} \tag{23}$$

In order to solve the system of equations (25), the remaining boundary conditions (3) should be taken into account.
The internal values of T_1 and T_2 can be determined on the basis of formulas, for Ω_1 and Ω_2, separately

$$T_i = \sum_{j=1}^{N} H_{ij}T_j - \sum_{j=1}^{N} G_{ij}q_j + R_i \tag{24}$$

3. Finite element method

The weighted residual criterion for equation (4) and domain Ω oriented in Cartesian co-ordinate system has the following form [9]

$$\int_{\Omega_e}\left\{\sum_{k=1}^{3}\lambda_e\frac{\partial^2 T_e(x)}{\partial x_k^2} - k_e T_e(x) + Q_e\right\}w(x)\,\mathrm{d}\Omega_e = 0 \tag{25}$$

Using the Gauss-Green-Ostrogradski theorem, after a certain mathematical manipulations one has

$$\int_{\Omega_e}\left[\sum_{k=1}^{3}\lambda_e\frac{\partial^2 T_e(x)}{\partial x_k^2} - k_e T_e(x) + Q_e\right]w(x)\,\mathrm{d}\Omega_e = \int_{\Gamma}\lambda_e\frac{\partial T_e(x)}{\partial n_e}w(x)\,\mathrm{d}\Gamma \tag{26}$$

where $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_c$.

In order to solve the equation (26), the domains Ω_e, e = 1, 2 of biological tissue and tumor has been divided into N finite elements and the integrals in equation (26) have been substituted by the sum of integrals over the finite elements

$$\sum_{i=1}^{N}\int_{\Omega_i}\left[\sum_{k=1}^{3}\lambda_e\frac{\partial^2 T_e(x)}{\partial x_k^2}-k_e T_e(x)+Q_e\right]w(x)\,\mathrm{d}\Omega_i=\sum_{i=1}^{N}\int_{\Gamma_i}\lambda_e\frac{\partial T_e(x)}{\partial n_e}w(x)\,\mathrm{d}\Gamma_i \tag{27}$$

In this paper the 10-nodal tetrahedral finite elements have been used (Fig. 2). In order to transform the finite element Ω_i into the standardized tetrahedron the following substitution can be introduced

$$x_k=\eta_1 x_k^1+\eta_2 x_k^2+\eta_3 x_k^3+(1-\eta_1-\eta_2-\eta_3)x_k^4,\quad k=1,2,3 \tag{28}$$

where (x_1^1,x_2^1,x_3^1), (x_1^2,x_2^2,x_3^2), (x_1^3,x_2^3,x_3^3), (x_1^4,x_2^4,x_3^4) are the co-ordinates of the finite element nodes 1, 2, 3, 4 and $0\le\eta_1\le 1$, $0\le\eta_2\le 1-\eta_1$, $0\le\eta_3\le 1-\eta_1-\eta_2$.

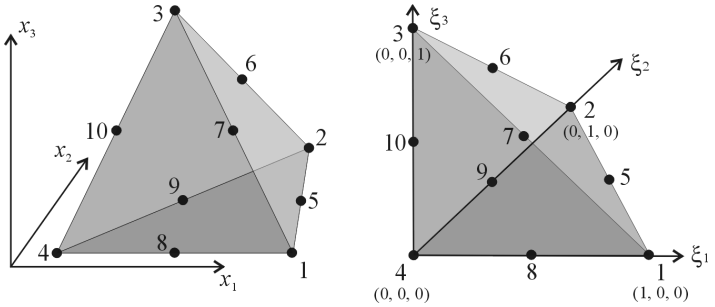

Fig. 2. 10-nodal tethrahedral element

The unknown function T is approximated in the following way

$$T=\sum_{l=1}^{10}N_l T_l^s \tag{29}$$

where T_k^s are the nodal values of temperature in the finite element considered, while:

$$\begin{aligned}
&N_1=\eta_1(2\eta_1-1),\quad N_2=\eta_2(2\eta_2-1),\quad N_3=\eta_3(2\eta_3-1)\\
&N_4=(1-\eta_1-\eta_2)(1-2\eta_1-2\eta_2),\quad N_5=4\eta_1\eta_2,\\
&N_6=4\eta_2\eta_3,\ N_7=4\eta_1\eta_3,\quad N_8=4\eta_1(1-\eta_1-\eta_2-\eta_3),\\
&N_9=4\eta_2(1-\eta_1-\eta_2-\eta_3),\quad N_{10}=4\eta_3(1-\eta_1-\eta_2-\eta_3)
\end{aligned} \tag{30}$$

are the shape functions. The weighting function w is defined as follows

$$w = \sum_{l=1}^{10} \beta_l N_l \tag{31}$$

where β_k are the unknown coefficients.
Finally, one obtains the following system of equations [6]

$$\mathbf{KT} = \mathbf{Z} + \mathbf{W} \tag{32}$$

where **K** is the conductivity matrix, **Z** is the heat source matrix, **W** is the matrix connected with boundary conditions.

4. Results of computations

The domain of biological tissue of dimensions 0.03×0.03×0.03 m has been considered. The radius of tumor region equals 0.0075 m, the position of tumor center (0.02, 0.02, 0.02) m. The following thermophysical parameters have been assumed λ_1 = 0.5 W/mK, λ_2 = 0.75 W/mK, k_1 = 1998.1 W/m^{3}K, k_2 = 7992.4 W/m^{3}K, Q_{m1} = = 420 W/m^3, Q_{m2} = 4200 W/m^3, blood temperature T_B = 37°C. On the arbitrary assumed internal boundary Γ_2 the temperature T_b = 37°C can be accepted.

Using the BEM the boundary $\Gamma_1 \cup \Gamma_2$ has been divided into 600 constant boundary elements, the boundary Γ_c has been divided into 600 constant element (Fig. 3). In the FEM application the boundary has been divided so as in BEM, and the interior is divided into 23055 tetrahedral elements.

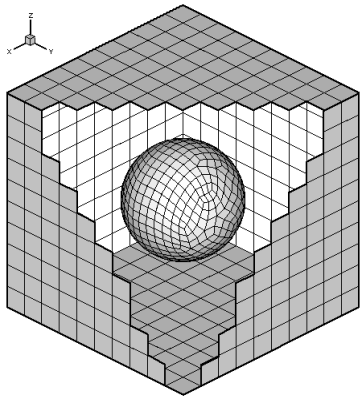

Fig. 3. Discretization of tumor and tissue

In the Table 1 the comparison of results obtained by MRBEM AND FEM are presented.

Table 1

Temperature at the internal points

No	x_1 [m]	x_2 [m]	x_3 [m]	MRBEM	FEM	error [%]
1	0.010330	0.019670	0.011457	37.1626	37.1649	0.006
2	0.003597	0.024055	0.024031	37.0508	37.0537	0.008
3	0.021575	0.011341	0.012704	37.2039	37.2117	0.021
4	0.021241	0.008778	0.023001	37.1835	37.1902	0.018
5	0.001857	0.014991	0.025455	37.0274	37.0300	0.007
6	0.008012	0.014990	0.026825	37.1058	37.1100	0.011

In Figure 4 the temperature distribution in the domain considered using MRBEM (Figure 4a) and FEM (Figure 4b) has been shown.

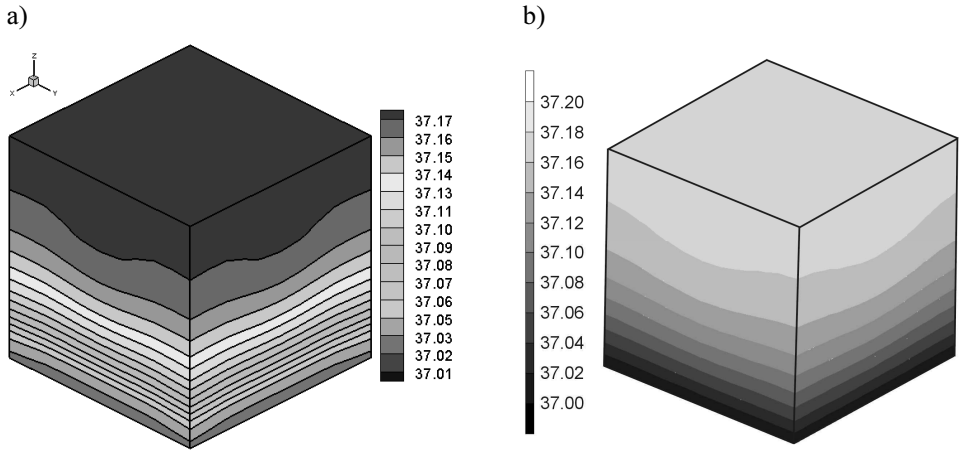

Fig. 4. Temperature of tissue with tumor: a) MRBEM, b) FEM

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