ESTIMATION OF EPIDERMIS AND DERMIS THICKNESSES ON THE BASIS OF SKIN SURFACE TEMPERATURE

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Abstract. Heat transfer in the skin tissue is treated as a multi-layer domain in which one can distinguish the epidermis, dermis and subcutaneous region is described by the system of Pennes equations and adequate boundary, initial and geometrical conditions. Many of the parameters used in the mathematical model are difficult to measure, e.g. epidermis or dermis thickness. In the paper the numerical algorithm of these geometrical parameters identification is presented in which the knowledge of skin surface temperature is assumed.

Introduction

The skin is treated as a multi-layer domain in which one can distinguish the epidermis Ω_1 of thickness L_1 [m], dermis Ω_2 of thickness $L_2 - L_1$ and subcutaneous region Ω_3 of thickness $L_3 - L_2$. The thermophisical parameters of these sub-domains are equal to λ_e [W/(mK)] (thermal conductivity) and c_e [J/(m^{3}K)] (specific heat per unit of volume), $e = 1, 2, 3$. The transient bioheat transfer in domain of skin is described by following system of equations [1, 2]

$$x \in \Omega_e: \quad c_e \frac{\partial T_e(x,t)}{\partial t} = \lambda_e \frac{\partial^2 T_e(x,t)}{\partial x^2} + k_e \left[T_B - T_e(x,t) \right] + Q_{me} \tag{1}$$

where $k_e = G_e c_B$ (G_e [(m^3 blood/s)/(m^3 tissue)] is the blood perfusion rate, c_B [J/(m^{3}K)] is the volumetric specific heat of blood), T_B is the arterial blood temperature and Q_{me} [W/m^3] is the metabolic heat source. It should be pointed out that for epidermis sub-domain ($e = 1$) $G_1 = 0$ and $Q_{m1} = 0$.

The system of equations (1) is supplemented by the following boundary conditions

- on the skin surface

$$x \in \Gamma_0: \quad q_1(x,t) = -\lambda_1 \frac{\partial T_1(x,t)}{\partial x} = q_S \tag{2}$$

- on the contact surfaces between sub - domains considered ($e = 1, 2$)

$$x \in \Gamma_{e,e+1}: \quad \begin{cases} -\lambda_e \dfrac{\partial T_e(x,t)}{\partial x} = -\lambda_{e+1} \dfrac{\partial T_{e+1}(x,t)}{\partial x} \\ T_e(x,t) = T_{e+1}(x,t) \end{cases} \tag{3}$$

- on the conventionally assumed internal boundary limiting the system

$$x \in \Gamma_3: \quad q_3(x,t) = -\lambda_3 \frac{\partial T_3(x,t)}{\partial x} = 0 \tag{4}$$

A quadratic initial temperature distribution between T_p = 32.5°C at the surface and T_c = 37°C at the base of the subcutaneous region ($x = L_3$) is assumed [1, 2]

$$t = 0: \quad T(x,0) = T_p + \frac{x(2L_3 - x)}{L_3^2}(T_c - T_p) \tag{5}$$

The direct problem described by equations (1)-(5) can be solved under the assumption that all thermal and geometrical parameters of skin tissue are given.

1. Shape sensitivity analysis

For 1D problem the following definition of material derivative is used [3, 4]

$$\frac{\mathrm{D}T_e}{\mathrm{D}b} = \frac{\partial T_e}{\partial b} + \frac{\partial T_e}{\partial x} v \tag{6}$$

where $v = v(x, b)$ is the velocity associated with design parameter b.

If the direct approach of sensitivity analysis is applied [3, 4], then the governing equations should be differentiated with respect to design parameter b.

Differentiation of equations (1) gives

$$c_e \frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\partial T_e}{\partial t}\right) = \lambda_e \frac{D}{\mathrm{D}b}\left(\frac{\partial^2 T_e}{\partial x^2}\right) - k_e \frac{\mathrm{D}T_e}{\mathrm{D}b} \tag{7}$$

Because [5]

$$\frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\partial^2 T_e}{\partial x^2}\right) = \frac{\partial^2}{\partial x^2}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b}\right) - 2\frac{\partial^2 T_e}{\partial x^2}\frac{\partial v}{\partial x} - \frac{\partial T_e}{\partial x}\frac{\partial^2 v}{\partial x^2} \tag{8}$$

and

$$\frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\partial T_e}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b}\right) \tag{9}$$

so the equations (7) can be written in the form

$$c_e \frac{\partial}{\partial t}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b}\right)=\lambda_e \frac{\partial^2}{\partial x^2}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b}\right)-2\lambda_e \frac{\partial^2 T_e}{\partial x^2}\frac{\partial v}{\partial x}-\lambda_e \frac{\partial T_e}{\partial x}\frac{\partial^2 v}{\partial x^2}-k_e \frac{\mathrm{D}T_e}{\mathrm{D}b} \tag{10}$$

or (c. f. equations (1))

$$\begin{gathered} c_e \frac{\partial}{\partial t}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b}\right)=\lambda_e \frac{\partial^2}{\partial x^2}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b}\right)-2\left[c_e \frac{\partial T_e}{\partial t}-k_e(T_B-T_e)-Q_{me}\right]\frac{\partial v}{\partial x}- \\ \lambda_e \frac{\partial T_e}{\partial x}\frac{\partial^2 v}{\partial x^2}-k_e \frac{\mathrm{D}T_e}{\mathrm{D}b} \end{gathered} \tag{11}$$

Differentiation of boundary condition (2) leads to the equation

$$x=0: \quad \frac{\mathrm{D}q_1}{\mathrm{D}b}=-\lambda_1 \frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\partial T_1}{\partial x}\right)=\frac{\mathrm{D}q_S}{\mathrm{D}b}=0 \tag{12}$$

Taking into account the formula [5]

$$\frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\partial T_e}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\mathrm{D}T_e}{\mathrm{D}b}\right)-\frac{\partial T_e}{\partial x}\frac{\partial v}{\partial x} \tag{13}$$

one has

$$x=0: \quad -\lambda_1 \frac{\partial}{\partial x}\left(\frac{\mathrm{D}T_1}{\mathrm{D}b}\right)+\lambda_1 \frac{\partial T_1}{\partial x}\frac{\partial v}{\partial x}=0 \tag{14}$$

We differentiate the continuity conditions (3)

$$x=L_e: \quad \begin{cases} -\lambda_e \dfrac{\mathrm{D}}{\mathrm{D}b}\left(\dfrac{\partial T_e}{\partial x}\right)=-\lambda_{e+1} \dfrac{\mathrm{D}}{\mathrm{D}b}\left(\dfrac{\partial T_{e+1}}{\partial x}\right) \\ \dfrac{\mathrm{D}T_e}{\mathrm{D}b}=\dfrac{\mathrm{D}T_{e+1}}{\mathrm{D}b} \end{cases}, \quad e=1,2 \tag{15}$$

and then (c.f. equation (13))

$$x=L_e: \quad \begin{cases} -\lambda_e \dfrac{\partial}{\partial x}\left(\dfrac{\mathrm{D}T_e}{\mathrm{D}b}\right)+\lambda_e \dfrac{\partial T_e}{\partial x}\dfrac{\partial v}{\partial x}=-\lambda_{e+1} \dfrac{\partial}{\partial x}\left(\dfrac{\mathrm{D}T_{e+1}}{\mathrm{D}b}\right)+\lambda_{e+1} \dfrac{\partial T_{e+1}}{\partial x}\dfrac{\partial v}{\partial x} \\ \dfrac{\mathrm{D}T_e}{\mathrm{D}b}=\dfrac{\mathrm{D}T_{e+1}}{\mathrm{D}b} \end{cases} \tag{16}$$

Differentiation of condition (4) gives

$$x = L_3 : \quad -\lambda_3 \frac{\mathrm{D}}{\mathrm{D}b}\left(\frac{\partial T_3}{\partial x}\right) = 0 \tag{17}$$

or

$$x = L_3 : \quad -\lambda_3 \frac{\partial}{\partial x}\left(\frac{\mathrm{D}T_3}{\mathrm{D}b}\right) + \lambda_3 \frac{\partial T_3}{\partial x}\frac{\partial v}{\partial x} = 0 \tag{18}$$

Finally, the initial condition (5) is differentiated

$$t = 0 : \quad \frac{\mathrm{D}T(x,0)}{\mathrm{D}b} = \frac{2(L_3 - x)}{L_3^2}(T_c - T_p)v \tag{19}$$

We assume that $b = L_1$ and [7]

$$v(x, L_1) = \begin{cases} \dfrac{x}{L_1}, & 0 \le x \le L_1 \\ \dfrac{L_2 - x}{L_2 - L_1}, & L_1 \le x \le L_2 \\ 0, & L_2 \le x \le L_3 \end{cases} \tag{20}$$

The equations (11) take a form

$$0 < x < L_1 : \quad c_1 \frac{\partial U_1(x,t)}{\partial t} = \lambda_1 \frac{\partial^2 U_1(x,t)}{\partial x^2} - \frac{2c_1}{L_1}\frac{\partial T_1(x,t)}{\partial t} \tag{21}$$

and

$$\begin{gathered} L_1 < x < L_2 : \quad c_2 \frac{\partial U_2(x,t)}{\partial t} = \lambda_2 \frac{\partial^2 U_2(x,t)}{\partial x^2} - k_2 U_2(x,t) + \\ \frac{2}{L_2 - L_1}\left[c_2 \frac{\partial T_2(x,t)}{\partial t} - k_2 [T_B - T_2(x,t)] - Q_{m2} \right] \end{gathered} \tag{22}$$

while

$$L_2 < x < L_3 : \quad c_3 \frac{\partial U_3(x,t)}{\partial t} = \lambda_3 \frac{\partial^2 U_3(x,t)}{\partial x^2} - k_3 U_3(x,t) \tag{23}$$

where $U_e = \mathrm{D}T_e / \mathrm{D}b$ is the sensitivity function.

The boundary conditions (16) can be written as follows

$$x = L_1: \quad \begin{cases} W_1(x,t) - \dfrac{1}{L_1} q_1(x,t) = W_2(x,t) + \dfrac{1}{L_2 - L_1} q_2(x,t) \\ U_1(x,t) = U_2(x,t) \end{cases} \tag{24}$$

and

$$x = L_2: \quad \begin{cases} W_2(x,t) + \dfrac{1}{L_2 - L_1} q_2(x,t) = W_3(x,t) \\ U_2(x,t) = U_3(x,t) \end{cases} \tag{25}$$

where: $q_e(x, t) = -\lambda_e \partial T_e(x, t) / \partial x$, $W_e(x, t) = -\lambda_e \partial U_e(x, t) / \partial x$.
The remaining boundary conditions are of the form

$$x = 0: \quad W_1(x,t) = -\frac{q_S}{L_1} \tag{26}$$

and

$$x = L_3: \quad W_3(x,t) = 0 \tag{27}$$

2. Inverse problem

We assume that the thicknesses of epidermis and dermis of skin tissue are unknown this means the value of L_1 must be found. Additionally, the time-dependent course of temperature on the skin surface is given

$$T_d^f = T_1(0, t^f), \quad f = 0, 1, 2, ..., F \tag{28}$$

In order to solve the inverse problem, the least squares criterion is applied [6]

$$S(L_1) = \sum_{f=1}^{F} \left(T^f - T_d^f \right)^2 \tag{29}$$

where $T^f = T_1(0, t^f)$ is the calculated temperature at the boundary point $x = 0$ for time t^f by using the current available estimate of unknown parameter L_1.

Differentiating the criterion (7) with respect to the unknown co-ordinate L_1 and using the necessary condition of optimum, one obtains

$$\frac{\mathrm{D}S}{\mathrm{D}L_1} = 2 \sum_{f=1}^{F} \left(T^f - T_d^f \right) \frac{DT^f}{DL_1} \bigg|_{L_1 = L_1^k} = 0 \tag{30}$$

where for $k = 0 : L_1^k$ is the arbitrary assumed value of L_1, while for $k > 0 : L_1^k$ results from the previous iteration, D(·)/DL_1 denotes the material derivative [3, 4]. Function T^f is expanded in a Taylor series about known value of L_1^k, this means

$$T^f = (T^f)^k + \left.\frac{DT^f}{DL_1}\right|_{L_1 = L_1^k} \left(L_1^{k+1} - L_1^k \right) \tag{31}$$

or

$$T^f = (T^f)^k + (U^f)^k \left(L_1^{k+1} - L_1^k \right) \tag{32}$$

where $(U^f)^k = \mathrm{DT^f}/\mathrm{D}L_1^k$ is the sensitivity function.
Putting (32) into (30) one has

$$\sum_{f=1}^{F} \left[(U^f)^k \right]^2 (L_1^{k+1} - L_1^k) = \sum_{f=1}^{F} (U^f)^k (T_d^f - (T^f)^k) \tag{33}$$

this means

$$L_1^{k+1} = L_1^k + \frac{\sum_{f=1}^{F} \left[T_d^f - (T^f)^k \right] \left(U^f \right)^k}{\sum_{f=1}^{F} \left[\left(U^f \right)^k \right]^2}, \quad k = 0, 2, ..., K \tag{34}$$

This equation allows to find the value of L_1^{k+1}. The iteration process is stopped when the assumed accuracy is achieved.

It should be pointed out that value L_1 corresponds to the epidermis thickness, while the value $L_2 - L_1$ corresponds to the dermis thickness. So, the identification of L_1 allows to determine both the epidermis and dermis thicknesses.

3. Results of computations

In numerical computations of direct problem the following values of parameters have been assumed [1, 2, 5]: λ_1 = 0.235 W/(mK), λ_2 = 0.445 W/(mK), λ_3 = = 0.185 W/(mK), c_1 = 4.3068 MJ/(m^{3}K), c_2 = 3.96 MJ/(m^{3}K), c_3 = 2.674 MJ/(m^{3}K), c_B = 3.9962 MJ/(m^{3}K), T_B = 37°C, G_1 = 0, G_e = 0.00125 (m^3 blood/s)/(m^3 tissue) for e = 2, 3, Q_{m1} = 0, Q_{me} = 245 W/m^3 for e = 2, 3. The thicknesses of successive skin layers: 0.1, 2 and 10 mm.

The basic problem and additional one resulting from the sensitivity analysis have been solved using the boundary element method [2, 7, 8]. The layers of skin have been divided into 10, 40 and 120 internal cells, time step: Δt = 0.05 s.

It is assumed that skin surface is subjected to the action of boundary heat flux $q_S = 6000$ W/m^2 (variant 1) and $q_S = 4000$ W/m^2 (variant 2). In Figure 1 the courses of skin surface for both cases are shown. On the basis of these values of temperature (c.f. equation (28)) the inverse problem has been solved. Figure 2 illustrates the courses of sensitivity function for $x = 0$ (skin surface) for two variants of computations and real thicknesses of epidermis and dermis layers.

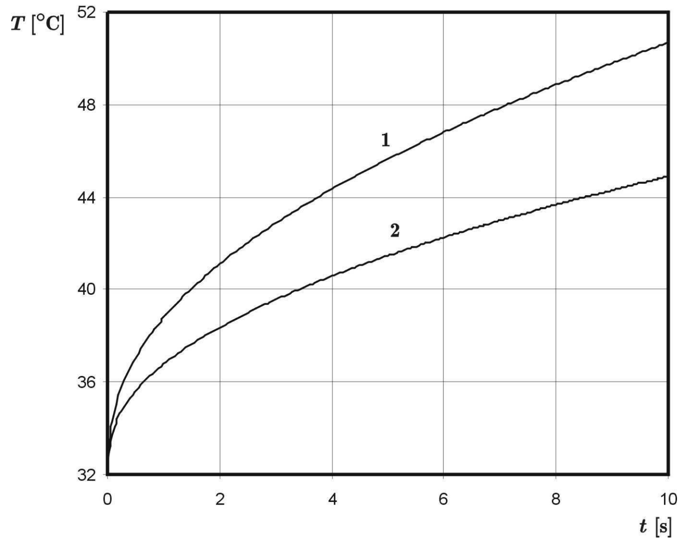

Fig. 1. Skin surface temperature (1 - q_S = 6000 W/m^2, 2 - q_S = 4000 W/m^2)

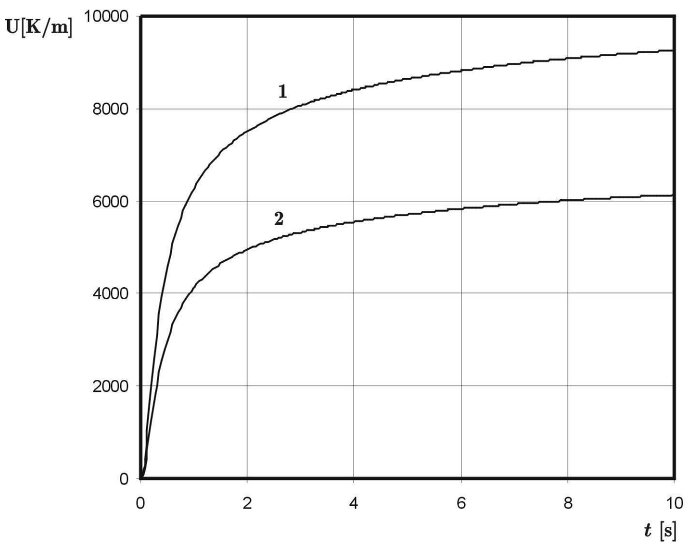

Fig. 2. Courses of sensitivity functions at the skin surface (1 - q_S = 6000 W/m^2, 2 - q_S = 4000 W/m^2)

The next figures concern the inverse problem solution. In Figures 3 and 5 the results of co-ordinates L_1 identification for different initial values of this parameter are shown, while Figures 4 and 6 illustrate the values of function S (c.f. equation (29)) for both variants of computations.

Summing up, the algorithm presented allows to determine the thicknesses of epidermis and dermis under the assumption that skin surface temperature is known.

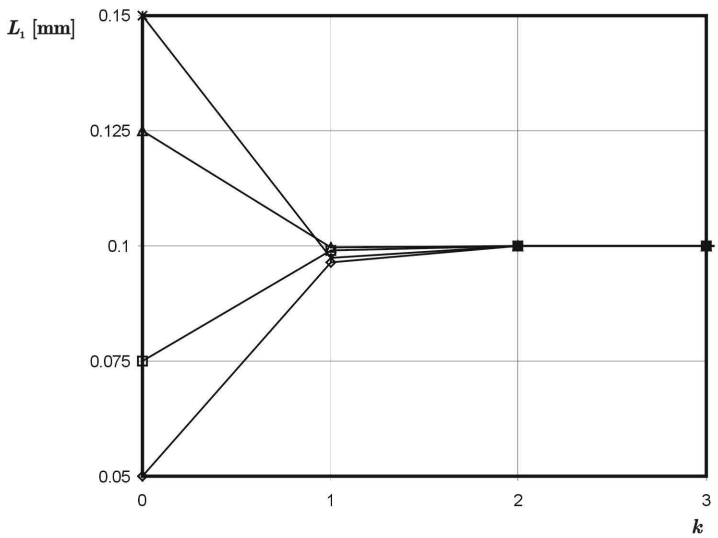

Fig. 3. Identification of value L_1 (q_S = 6000 W/m^2)

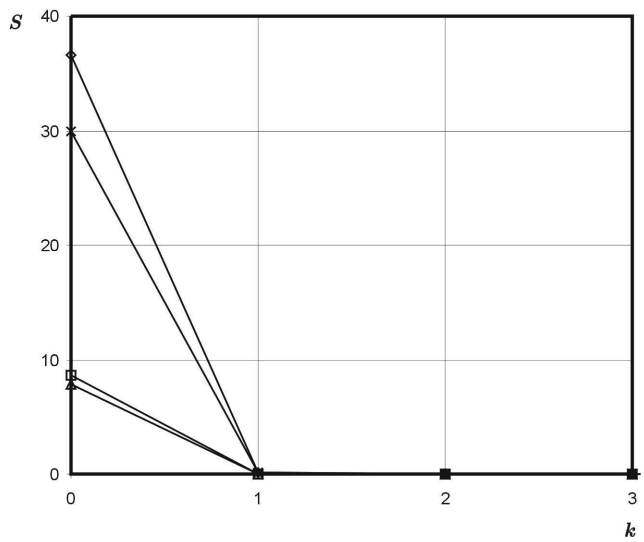

Fig. 4. The values of function S for successive iterations (q_S = 6000 W/m^2)

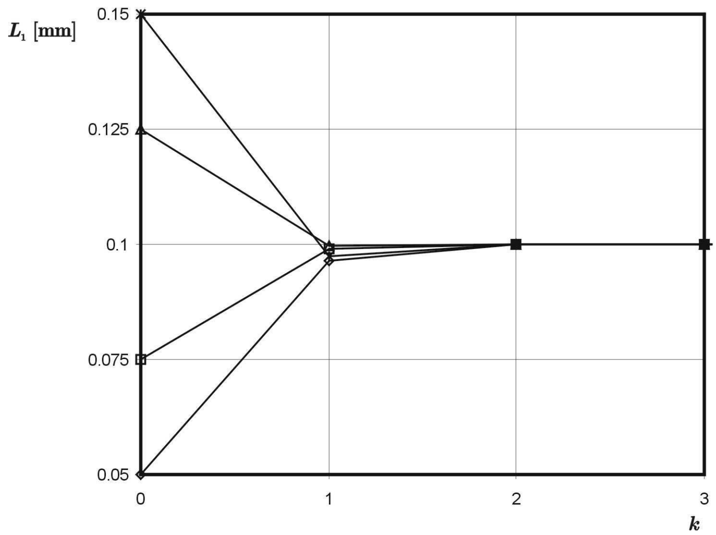

Fig. 5. Identification of value L_1 (q_S = 4000 W/m^2)

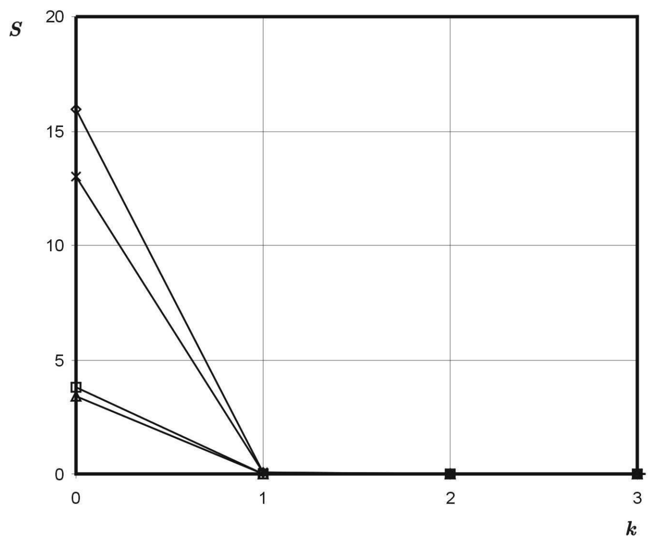

Fig. 6. The values of function S for successive iterations (q_S = 4000 W/m^2)

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