Please cite this article as:
Jarosław Kęsy, Zbigniew Domański, Jolanta Błaszczuk, Genetic algorithm approach to force chains in 2D granular packings, Scientific Research of the Institute of Mathematics and Computer Science, 2007, Volume 6, Issue 1, pages 99-102.
The website: http://www.amcm.pcz.pl/

GENETIC ALGORITHM APPROACH TO FORCE CHAINS IN 2D GRANULAR PACKINGS

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Abstract. We study the geometry of patterns formed by forces in 2D models of granular stacking using a genetic algorithm. As a fitness function we employ an entropy measure based on probability density functions of force-chain-arm lengths computed for different underlying lattice geometries.

Introduction

Granular materials represent an important class of materials used in technological processes. In general they are not regular solids or liquids [1, 2]. They resembles solids in the static state when the contact forces between grains are repulsive and balance on every particle. In ordinary solids and confined fluids, uniformly applied loads are distributed homogeneously throughout the material. This is not the case for granular materials. Inside a granular material forces are distributed in a very inhomogeneous way with small fraction of grains supporting large fraction of the internal forces [3]. These loaded grains form a structure, called the force network [4-6], experimentally observed in packings of photoelastic particles [7].

The geometrical properties of networks have attracted much attention due to progress in the fields of computer science, mathematical biology and statistical physics. One of an important question is how many pairs of points separated by a given number q of steps can be found in a bounded region of a regular lattice. Such number q is referred to as the so-called Manhattan distance. For a square lattice the Manhattan distance is defined as the sum of the horizontal and the vertical distances. Similarly, for other geometries we can define the Manhattan distance as the sum of the distances along directions parallel to the edges of an appropriate elementary cell. This contribution focuses on geometry but the knowledge of the number of Manhattan distances in a particular lattice enables us to analyze the distribution of contact forces in granular packing.

1. Formulation of the problem

Our motivation for studying the relation between forces and geometry comes from the fact that in a generic case of a regular packing of frictionless spherical grains the underlying physics is rather transparent and that regular packing mapped on an appropriate regular lattice can be resolved analytically.

Consider random geometrical points, i.e. points with uncorrelated positions, occupied vertices of a regular lattice. We address the following question: what is the distribution of distances between a given reference point and other randomly chosen point, where points are distributed uniformly. Assuming that these points are the end points of a zigzag distance we can obtain the end-to-end distribution of force-chain arms within a given lattice whereas the particular form of the zigzag line follows the Principle of Maximum Entropy [8, 9].

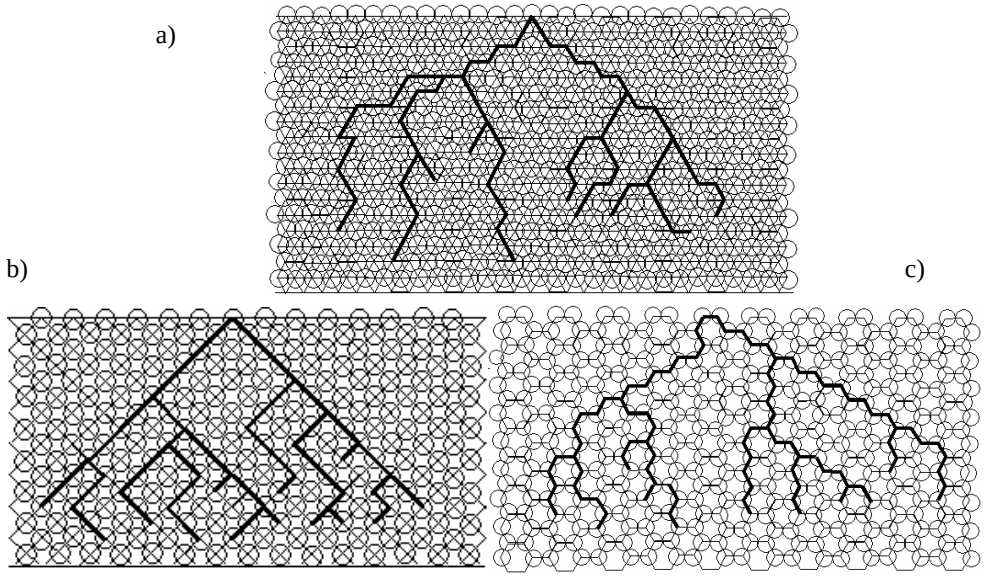

Fig. 1. Force chains in 2D regular tessellations: a) triangular, b) square and c) hexagonal lattices

For the triangular lattice presented in Figure 1 the number of Manhattan distances q is given by Equation (1):

$$\Delta_t(q) = \frac{3}{2} q(N-q)(N-q+1) \quad \text{for} \quad q = 1,2,\ldots,N-1 \tag{1}$$

where N is the linear extension of a bounded region measured in number of unit steps in one of three equivalent directions of the unit cell. Analogous, appropriate expressions for hexagonal $\Delta_h(q)$ (Eq. (2)), square $\Delta_s(q)$ (Eq. (3)) lattices are:

$$\Delta_h(q)=\begin{cases}\frac{3}{8}N^2q-\frac{3}{4}q(q-2)N+\frac{3}{8}q^3-\frac{3}{2}q^2-3 & for \quad even\ q\\ \frac{3}{8}N^2q-\frac{3}{4}q(q-2)N+\frac{3}{8}q^3-\frac{3}{2}q^2-\frac{3}{8}q-\frac{3}{2} & for \quad odd\ q\end{cases} \tag{2}$$

$$\Delta_s(q)=\begin{cases}2N(N-q)q+\frac{1}{3}(q-1)q(q+1) & for \quad q=1,2,...,N-1\\ \frac{1}{3}(2N-q-1)(2N-q)(2N-q+1) & for \quad q=N,N+1,2N-2\end{cases} \tag{3}$$

We have derived the probability density functions for the Manhattan distance within following tessellations of the plane: square (Fig. 2), triangular and hexagonal. We have also calculated the moments of these distributions and found that for the square lattice they diverge whereas for other lattices the moments asymptotically vanish.

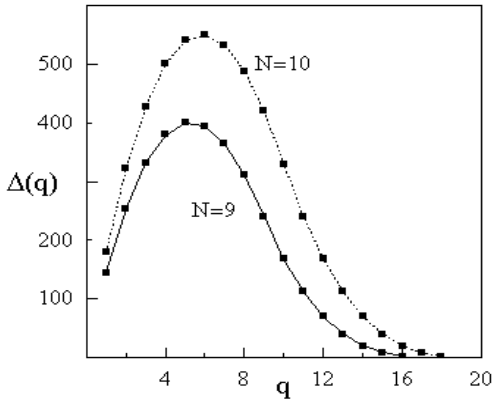

Fig. 2. Number of Manhattan distances for square lattice with N = 9 and N = 10

The probability density functions obtained for 2D tessellations give the probability weight of class q containing pairs of points with given distance q. Thus, they may contain valuable information related to the potential energy of granular systems. In physics, the potential energy of a body is, very often, the sum of the potential energies of its pairs of parts or particles. From a computational point of view, this means that we have to do a six-dimensional integral. But if the distribution of distance of the system is known then only a one-dimensional integral is necessary and so this increases the numerical precision.

2. Computational details

In this paper we consider 2D packing of spheres interacting in pairs via an arbitrary central force. We assume that the system is in equilibrium and that all the

equilibrium particle positions are given by one of the three regular planar tessellations (see Fig. 1). By associating a site of each sphere centre and a bond to each pair of spheres in contact we form a contact force network. When the number of contacts is greater than the rank of the rigidity matrix the network is said to be hyperstatic. For hyperstatic networks, the problem of finding the stresses from the equations for force equilibrium at each network site alone is underdetermined.

We assume that an information about particular repartition of inter-grain interactions over the network should fulfil the maximum entropy principles [8, 9]. Any possible force chain is considered as an ensemble of arms with given length measured is the sense of the Manhattan distance. So, we can compute the corresponding entropy using the probability weights $\Delta_t(q)$, $\Delta_s(q)$ and $\Delta_h(q)$. In order to compute the force chains formed due to a vertical force applied at the centre of the top layer (Fig. 1).

We employ a genetic algorithm using the entropy as a fitness function and the search strategy is based on selection and mutation only. Our mutation operation is constructed as follows. Any grain layer is crossed by at least two force network arms. For randomly chosen layer we pick two grains among grains crossed by arms of given force chain. These grains we consider as origins of two subnetworks. By interchanging their positions we change the given force chain. As a consequence of such mutation, the total length of arms does not change during evolution, which corresponds to the constant force applied to the system whereas the repartition of lengths of arms inside a new force chain is different from generation to generation.

In conclusion, the entropy maximum principle can be used as an selection tool yielding an valuable information about static properties of granular packing.

References

[1] Blumenfeld R., Stresses in isostatic granular systems and emergence of force chains, Phys. Rev. Letters 2004, 93, 10108301-10108304.

[2] de Gennes P.G., Granular matter: a tentative view, Rev. Mod. Phys. 1999, 71, S374-S382.

[3] Hinrichen H., Wolf D.E., (eds.), The Physics of Granular Media, Wiley-VCH Verlag GmbH&Co, KgaA, Weinheim 2004.

[4] Jaeger H.M., Nagel S.R., Behringer R.P., Granular solids, liquids and gases, Rev. Mod. Phys. 1996, 68, 1259-1273.

[5] Jaynes E.T., Information theory and statistical mechanics, Phys. Review 1957, 106, 4, 620-630.

[6] Jaynes E.T., The Maximum Entropy Formalism, R.D. Levine, M. Tribus, (eds.), MIT Press, Cambridge, USA 1979.

[7] Liu C. et al., Force fluctuations in bead packs, Science, New Series 1995, 269, 513-515.

[8] Snoeijer J.H. et al., Ensemble theory for force networks in hyperstatic granular matter, Phys. Review E 2004, 70, 61306, 1-16.

[9] Snoeijer J.H. et al., Packing geometry and statistics of force networks in granular media, Phys. Review E 2004, 70, 11301, 1-15.