ESTIMATION OF BOUNDARY HEAT FLUX DURING CAST IRON SOLIDIFICATION

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Abstract. In the paper the inverse problem consisting in estimation of boundary heat flux during cast iron solidification is presented. In order to solve the inverse problem formulated it is assumed that the cooling curves at selected set of points from the casting domain are given. The algorithm bases on the gradient method coupled with the finite differences method. In the final part of the paper the results of computations are shown.

1. Direct problem

The 1D casting-mould system is considered. The influence of the mould on the course of solidification process is substituted by the Neumann condition. Transient temperature field in casting domain determines the energy equation

$$0 < x < L: \; C(T)\frac{\partial T(x,t)}{\partial t} = \lambda \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

where $C(T)$ is the substitute thermal capacity [1, 2] of cast iron - Figure 1, λ is the mean value of thermal conductivity, T is the temperature, x is the spatial co-ordinate and t is the time.

In the case of cast iron solidification the following approximation of substitute thermal capacity can be taken into account (Fig. 1) [1, 3]

$$C(T) = \begin{cases} p_1 = c_L, & T > T_L \\ p_2 = \dfrac{c_L + c_S}{2} + \dfrac{Q_{aus1}}{T_L - T_A}, & T_A < T \le T_L \\ p_3 = \dfrac{c_L + c_S}{2} + \dfrac{Q_{aus2}}{T_A - T_E}, & T_E < T \le T_A \\ p_4 = \dfrac{c_L + c_S}{2} + \dfrac{Q_{eu}}{T_E - T_S}, & T_S < T \le T_E \\ p_5 = c_S, & T \le T_S \end{cases} \tag{2}$$

where T_L is the liquidus temperature, T_S is the solidus temperature, T_A, T_E correspond to the border temperatures, c_L, c_S are the volumetric specific heats of molten metal and solid state, respectively, $Q_{aus} = Q_{aus1} + Q_{aus2}$, Q_{eu} are the latent heats connected with the austenite and eutectic phases evolution, at the same time $Q = Q_{eu} + Q_{aus}$.

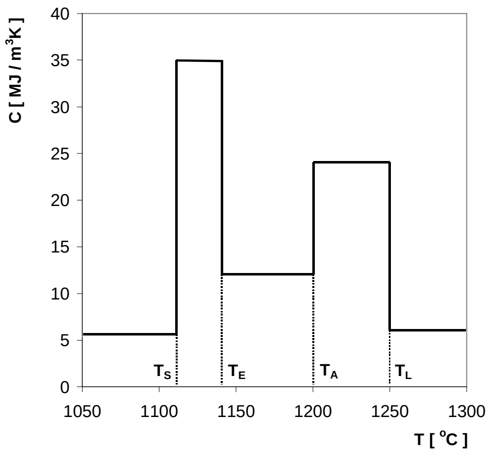

Fig. 1. Substitute thermal capacity of cast iron

For $x = 0$ (axis of symmetry) the no-flux condition

$$x = 0: \quad q(x, t) = \lambda \frac{\partial T(x, t)}{\partial x} = 0 \tag{3}$$

is accepted. For $x = L$ the time dependent boundary heat flux is given, namely

$$x = L: \quad q(x, t) = -\lambda \frac{\partial T(x, t)}{\partial x} = q_b(t) \tag{4}$$

For the moment $t = 0$ the initial temperature distribution is known

$$T(x, 0) = T_0(x) \tag{5}$$

2. Inverse problem

If the parameters appearing in governing equations are known then the direct problem is considered. If part of them is unknown then the inverse problem should be formulated. In particular, in this paper the problem of time dependent boundary heat flux $q_b(t)$ identification is presented. It is assumed, that the boundary heat flux can be expressed as follows

$$q_b(t) = a + \frac{b}{t} \tag{6}$$

where a, b are the unknown parameters.

In order to solve the inverse problem formulated the additional information concerning the cooling curves at the selected set of points from the domain considered must be given.

So, it is assumed that the values T_{di}^f at the sensors x_i from casting sub-domain for times t^f are known, namely

$$T_{di}^f = T_d\left(x_i,\ t^f\right), \quad i = 1,\ 2,\ \ldots,\ M, \quad f = 1,\ 2,\ \ldots,\ F \tag{7}$$

3. Gradient method of inverse problem solution

In order to solve the inverse problem the least squares criterion is applied [4, 5]

$$S(a, b) = \frac{1}{MF} \sum_{i=1}^{M} \sum_{f=1}^{F} \left(T_i^f - T_{di}^f\right)^2 \tag{8}$$

where T_{di}^f (c.f. equation (7)) and $T_i^f = T\left(x_i, t^f\right)$ are the measured and estimated temperatures, respectively, for the sensor x_i, i = 1, 2, ... , M and for time t^f.
The estimated temperatures are obtained from the solution of the direct problem (c.f. chapter 1) by using the current available estimate for the unknown parameters.

Differentiating the criterion (8) with respect to the unknown parameters a, b and using the necessary condition of optimum one obtains the following system of equations

$$\begin{cases} \dfrac{\partial S}{\partial a} = \dfrac{2}{MF} \displaystyle\sum_{i=1}^{M} \sum_{f=1}^{F} \left(T_i^f - T_{di}^f\right) \left(\frac{\partial T}{\partial a}\right)_i^f = 0 \\ \dfrac{\partial S}{\partial b} = \dfrac{2}{MF} \displaystyle\sum_{i=1}^{M} \sum_{f=1}^{F} \left(T_i^f - T_{di}^f\right) \left(\frac{\partial T}{\partial b}\right)_i^f = 0 \end{cases} \tag{9}$$

Function T_i^f is expanded in a Taylor series about known values of a, b, this means

$$T_i^f = \left(T_i^f\right)^k + \left[\left(\frac{\partial T}{\partial a}\right)_i^f\right]_{a=a^k} \left(a^{k+1} - a^k\right) + \left[\left(\frac{\partial T}{\partial b}\right)_i^f\right]_{b=b^k} \left(b^{k+1} - b^k\right) \tag{10}$$

where k is the number of iteration, a^0, b^0 are the arbitrary assumed values of a, b while a^k, b^k for $k > 0$ result from the previous iteration.

The dependence (10) can be written in the form

$$T_i^f = \left(T_i^f\right)^k + \left(U_{1i}^f\right)^k \left(a^{k+1} - a^k\right) + \left(U_{2i}^f\right)^k \left(b^{k+1} - b^k\right) \tag{11}$$

where

$$\left(U_{1i}^f\right)^k = \left[\left(\frac{\partial T}{\partial a}\right)_i^f\right]_{a=a^k}, \quad \left(U_{2i}^f\right)^k = \left[\left(\frac{\partial T}{\partial b}\right)_i^f\right]_{b=b^k} \tag{12}$$

are the sensitivity coefficients.
Putting (11) into (9) one obtains

$$\begin{cases} \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[\left(U_{1i}^f\right)^k\left(a^{k+1}-a^k\right)+\left(U_{2i}^f\right)^k\left(b^{k+1}-b^k\right)\right]\left(U_{1i}^f\right)^k = \\ \qquad \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[T_{di}^f-\left(T_i^f\right)^k\right]\left(U_{1i}^f\right)^k \\ \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[\left(U_{1i}^f\right)^k\left(a^{k+1}-a^k\right)+\left(U_{2i}^f\right)^k\left(b^{k+1}-b^k\right)\right]\left(U_{2i}^f\right)^k = \\ \qquad \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[T_{di}^f-\left(T_i^f\right)^k\right]\left(U_{2i}^f\right)^k \end{cases} \tag{13}$$

or

$$\begin{bmatrix} \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[\left(U_{1i}^f\right)^k\right]^2 & \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[\left(U_{1i}^f\right)^k\left(U_{2i}^f\right)^k\right] \\ \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[\left(U_{1i}^f\right)^k\left(U_{2i}^f\right)^k\right] & \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[\left(U_{2i}^f\right)^k\right]^2 \end{bmatrix} \begin{bmatrix} a^{k+1}-a^k \\ b^{k+1}-b^k \end{bmatrix} = \begin{bmatrix} \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[T_{di}^f-\left(T_i^f\right)^k\right]\left(U_{1i}^f\right)^k \\ \sum\limits_{i=1}^{M}\sum\limits_{f=1}^{F}\left[T_{di}^f-\left(T_i^f\right)^k\right]\left(U_{2i}^f\right)^k \end{bmatrix} \tag{14}$$

This system of equations allows to find the values of a^{k+1}, b^{k+1}. The iteration process is stopped when the assumed number of iterations K is achieved.

It should be pointed out that in order to obtain the sensitivity coefficients (12), the governing equations should be differentiated with respect to a and b.

Differentiation of equations (1), (3), (4), (5) with respect to a leads to the following additional boundary initial problem (c.f. condition (6))

$$
\begin{aligned}
&0 < x < L: && C(T)\frac{\partial U_1(x,t)}{\partial t} = \lambda \frac{\partial^2 U_1(x,t)}{\partial x^2} \\
&x = 0: && W_1(x,t) = \lambda \frac{\partial U_1(x,t)}{\partial x} = 0 \\
&x = L: && W_1(x,t) = -\lambda \frac{\partial U_1(x,t)}{\partial x} = 1 \\
&t = 0: && U_1(x,t) = 0
\end{aligned}
\tag{15}
$$

where $U_1(x, t) = \partial T(x, t)/\partial a$.

In similar way the governing equations are differentiated with respect to b and then

$$
\begin{aligned}
&0 < x < L: && C(T)\frac{\partial U_2(x,t)}{\partial t} = \lambda \frac{\partial^2 U_2(x,t)}{\partial x^2} \\
&x = 0: && W_2(x,t) = \lambda \frac{\partial U_2(x,t)}{\partial x} = 0 \\
&x = L: && W_2(x,t) = -\lambda \frac{\partial U_2(x,t)}{\partial x} = \frac{1}{t} \\
&t = 0: && U_2(x,t) = 0
\end{aligned}
\tag{16}
$$

where $U_2(x, t) = \partial T(x, t)/\partial b$.

So, for each time step the basic problem and two additional problems connected with the sensitivity functions should be solved. These problems have been solved by means of the finite differences method [1].

4. Results of computations

The casting of thickness $2L_1 = 0.02$ m has been considered. The following input data have been taken into account: $T_L = 1250^{\circ}$C, $T_A = 1200^{\circ}$C, $T_E = 1130^{\circ}$C, $T_S = 1110^{\circ}$C, $\lambda = 30$ W/mK, pouring temperature $T_0 = 1300^{\circ}$C. The substitute thermal capacity is defined by equation (2), where $p_1 = 5.88$ MJ/m^3 K, $p_2 = 24.384$, $p_3 = 11.32$, $p_4 = 34.75$, $p_5 = 5.4$.

In order to estimate the boundary heat flux (6) the courses of cooling curves (c.f. equation (5)) at the points $x_1 = 0$ m (axis of symmetry) and $x_2 = 3L/4$ have been taken into account (Fig. 2). They result from the direct problem solution under the assumption that $q_b(t) = 0.3 + 0.7/t$ MW/m^2 ($a = 0.3$, $b = 0.7$).

The basic problem and additional ones connected with the sensitivity functions have been solved using the explicit scheme of finite differences method [1] (time step $\Delta t = 0.05$, mesh step $h = 0.001$ m). In Figures 3 and 4 the courses of sensitivity functions U_1 and U_2 for real values of a, b at the points 1, 2 are shown.

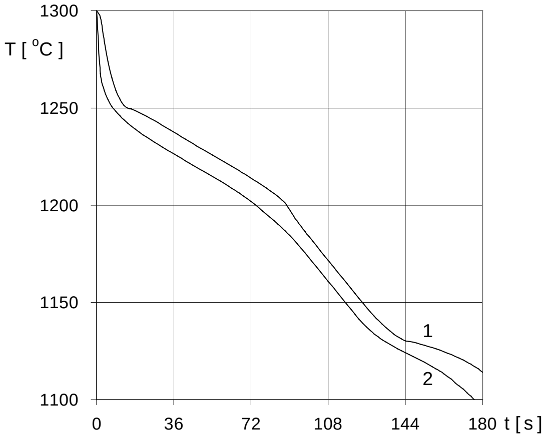

Fig. 2. Cooling curves

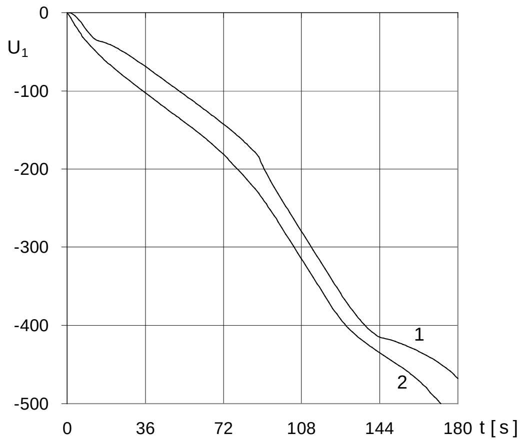

Fig. 3. Courses of sensitivity function U_1 at the points 1, 2

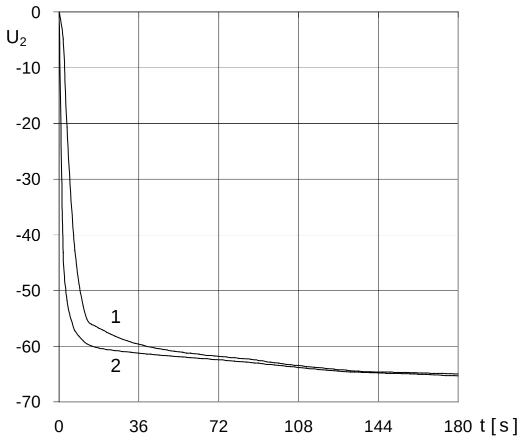

Fig. 4. Courses of sensitivity function U_2 at the points 1, 2

The inverse problem has been solved for zero initial values of unknown parameters *a*, *b*. Figure 5 illustrates the courses of iteration process. It is visible that the parameters are estimated correctly and after a few iterations the real values of *a*, *b* have been obtained.

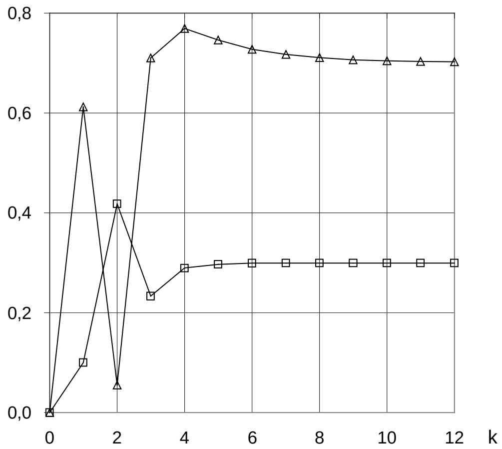

Fig. 5. Iteration process

Similar computations have been done for disturbed data - Figure 6. In this case also the proper estimation of time dependent boundary heat flux has been obtained.

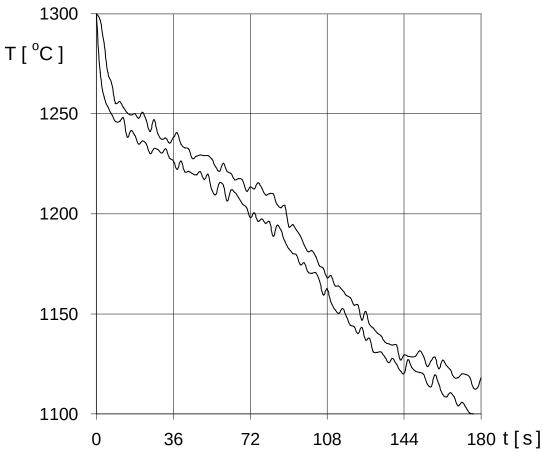

Fig. 6. Disturbed cooling curves

Acknowledgement

This work was funded by Grant No N N507 3592 33.

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