NUMERICAL SIMULATION OF BIOHEAT TRANSFER PROCESS IN THE HUMAN EYE USING FINITE ELEMENT METHOD

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Abstract. In this paper the finite element method is used for the numerical simulation of two dimensional transient bioheat transfer process in the human eye. The human eye is modelling as a composition of several homogeneous regions. On the outer surface the heat radiation is assumed, on the inner surface the Robin condition is accepted. In the final part of the paper the results of computations are shown.

1. Theoretical model of human eye

In Figure 1 can be seen that the eye is approximately a spherical organ. The back surface is covered with a thin membrane (retina), that is permeated with blood vessels and is connected with the brain by the optic nerve. Under the retina is a layer called the choroid which serves to nourish it. The anterior transparent surface of the eye is a cornea. The lens lies between the aqueous humour and vitreous humour. The aqueous and vitreous humours are transparent liquids with different concentrations of NaCl [1, 6, 7].

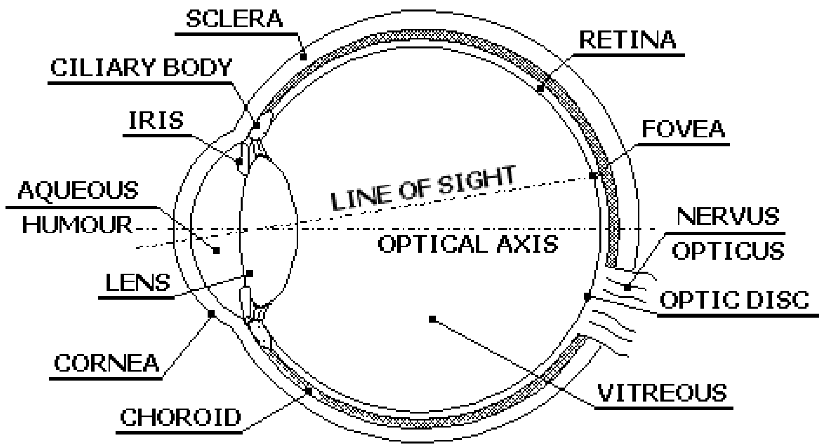

Fig. 1. Model of a human eye

2. A mathematical model of a human eye

A simplified model of a human eye is presented in Figure 2. Because in reality the retina and the choroid are very thin so they are modelled together with a sclera and optic nerve.

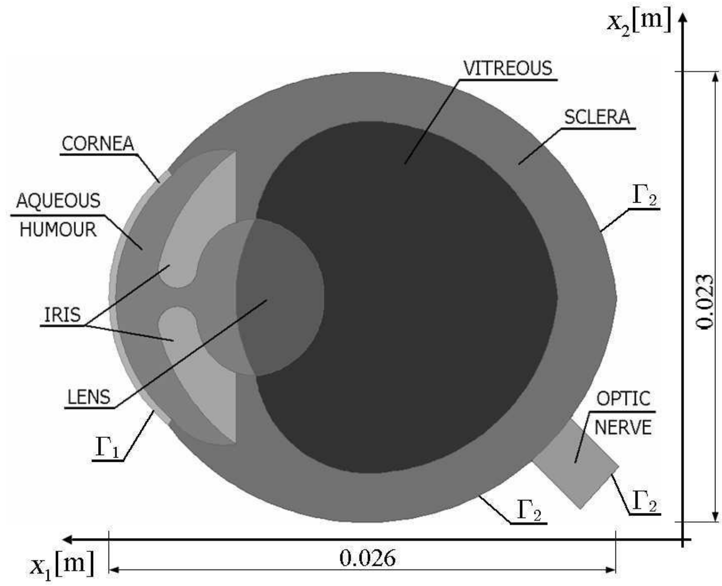

Fig. 2. Simplified model of a human eye

The temperature distribution in the eye can be described by the equation

$$x \in \Omega_e : \quad c_e \rho_e \frac{\partial T_e(x, t)}{\partial t} = \lambda_e \nabla^2 T_e(x, t) \tag{1}$$

where e = 1, 2, ..., 7 identifies the sub-domains of human eye, in particular the cornea, aqueous humour, iris, lens, vitreous, sclera and optic nerve, c_e [J/(kgK)], ρ_e [kg/m^3], λ_e [W/(mK)] are the specific heat, density and thermal conductivity, respectively, T_e denotes temperature, t is the time and $x = \{x_1, x_2\}$ are the geometrical co-ordinates.

Only the cornea is the region in eye which is exposed to the environment. Because the corneal surface temperature is greater than the ambient temperature, heat is extracted away from eye to environment by convection and radiation. Besides convection and radiation, the evaporation of tears increases the cooling rate to the corneal surface [2-4]. Thus on the cornea surface Γ_1 the following boundary condition can be accepted

$$x \in \Gamma_1: \qquad -\lambda_1 \frac{\partial T_1}{\partial n} = \alpha_a \left(T_1 - T_a\right) + \varepsilon\sigma\left(T_1^4 - T_a^4\right) + E \tag{2}$$

where λ_1 is a thermal conductivity of the cornea, $\partial T_1/\partial n$ is a normal derivative, α_a W/(m^{2}K) is a heat transfer coefficient between cornea and environment, T_a is a temperature of surrounding environment, ε is corneal emissivity, σ is the Stefan--Boltzmann constant and E is the loss of heat flux due to the evaporation of tears.

On the exterior boundary Γ_2 the Robin condition is accepted

$$x \in \Gamma_2: \qquad -\lambda_2 \frac{\partial T_2}{\partial n} = \alpha_b \left(T_1 - T_b\right) \tag{3}$$

where λ_2 is a thermal conductivity of the sclera, α_b [W/(m^{2}K)] is a heat transfer coefficient between sclera and blood vessels and T_b is a temperature of blood.

In the domain considered the initial temperature is assumed

$$x \in \Omega_e: \qquad T_e\left(x,\, 0\right) = T_0 \tag{4}$$

where T_0 is an initial temperature of domain.

On the surfaces between sub-domains of the human eye the ideal thermal contact is assumed

$$x \in \Gamma_c: \qquad \begin{cases} T^I\left(x,\, t\right) = T^{II}\left(x,\, t\right) = T\left(x,\, t\right) \\ -\lambda \dfrac{\partial T^I\left(x,\, t\right)}{\partial n} = \lambda \dfrac{\partial T^{II}\left(x,\, t\right)}{\partial n} = -\lambda \dfrac{\partial T\left(x,\, t\right)}{\partial n} \end{cases} \tag{5}$$

where I and II denote the sub-domains which are in the thermal contact.

Finally, the mathematical model of human eye is defined by equation (1) supplemented by conditions (2), (3), (4) and (5).

3. Results of computations

The domain of human eye of dimensions 0.026×0.023 m has been considered (c.f. Fig. 2). The computations have been done using the MSC MARC/MENTAT software. The thermophysical parameters of human eye have been collected in Table 1 [1, 5, 8-10].

On the stage of numerical simulation the following parameters have been assumed: blood temperature T_b = 37°C, environment temperature T_a = 25°C, heat transfer coefficients α_b = 65 W/(m^{2}K), α_a = 10 W/(m^{2}K), emissivity of cornea ε = = 0.975, loss of the heat flux E = 40 W/m^2, Stefan-Boltzmann constant σ = = $5.67\cdot10^{-8}$ W/(m^{2}K^4), initial temperature T_0 = 0°C and time step Δt = 1 s.

Table 1

Thermophysical parameters of a human eye

Domain	λ [W/(mK)]	*c* [J/(kgK)]	ρ [kg/m^3]
Cornea	0.58	1050	4178
Iris	1.0042	1000	3997
Aqueous humour	0.58	1000	3997
Lens	0.40	1050	3000
Sclera	1.0042	1000	3997
Vitreous	0.603	1000	4178
Optic nerve	1.0042	1000	3997

Using the FEM the interior is divided into 8092 6-nodal triangular elements c.f. Figure 3.

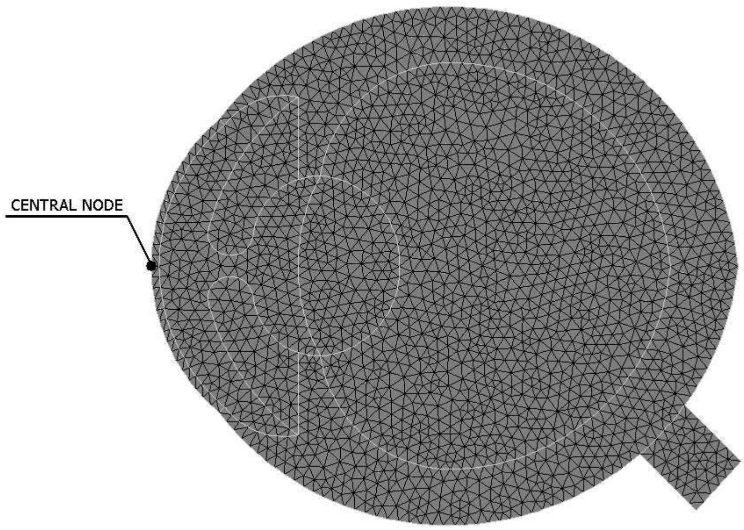

Fig. 3. Discretization of a human eye

In Figure 4 the temperature distribution in a human eye for time 10, 100 and 2700 s is presented. In Figure 5 the heating curve at the central node of corneal surface (marked in Figure 3) is shown.

Summing up, the finite element method allows to obtain a good results of temperature distribution of a human eye.

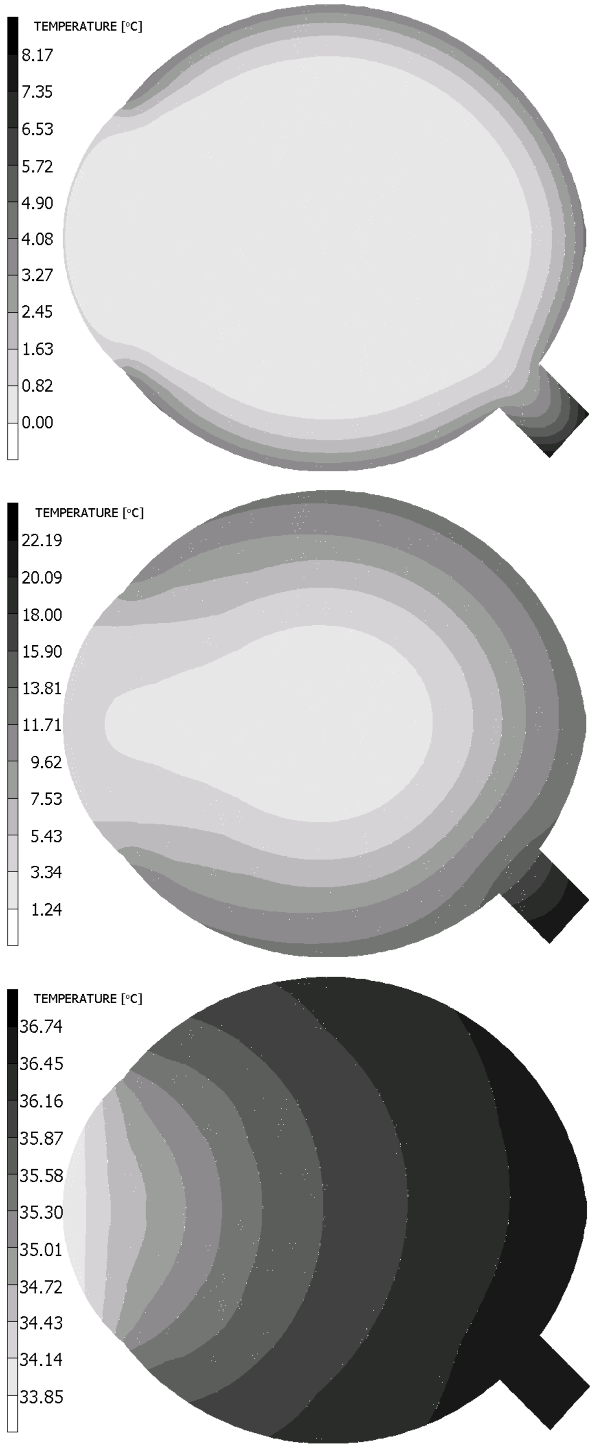

Fig. 4. Temperature distribution of human eye for time *t* = 10, 100 and 2700 s

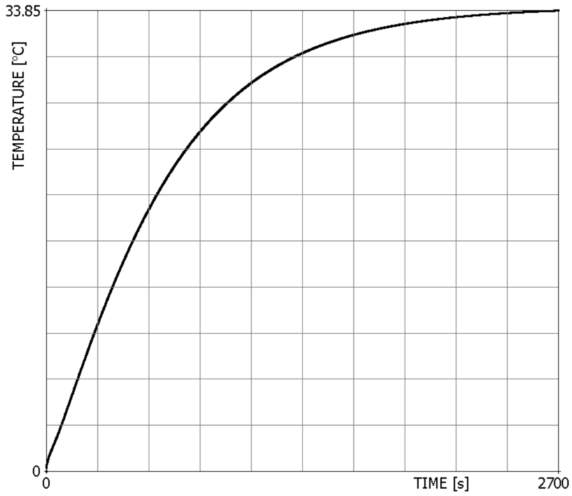

Fig. 5. Temperature of central node on the corneal surface

References

[1] Amara E.H., Numerical investigations on thermal effects of laser ocular media interaction, International Journal of Heat and Mass Transfer 1995, 38, 2479-2488.

[2] Ooi E.H., Ang W.T., Ng E.Y.K., Bioheat transfer in the human eye: A boundary element approach, Engineering Analysis with Boundary Elements 2007, 31, 494-500.

[3] Craig J.P., Singh I., Tomlinson A., Morgan P.B., Efron N., The role of tear physiology in ocular surface temperature, Eye 2000, 14, 635-641.

[4] Emery A.F., Kramar P.O., Guy A.W., Lin J.C., Microwave inducted temperature rises in rabbit eyes in cataract research, Journal of Heat Transfer 1975, 97, 123-128.

[5] Guy A.W., Lin J.C., Kramar P.O., Emery A.F., Effect of 2450-MHz radiation on the rabbit eye, IEEE Trans Microwave Theory and Techniques 1975, 23, 392-498.

[6] Lagendijk J.J.W., A mathematical model to calculate temperature distributions in human and rabbit eyes during hyperthermic treatment, Physics in Medicine and Biology 1982, 27, 1301--1311.

[7] Morgan P.B., Soh M.P., Efron N., Tullo A.B., Potential applications of ocular thermography, Optometry and Vision Science 1993, 70, 568-576.

[8] Okuno T., Thermal effect on infra-red radiation on the eye: a study based on the model, Annals of Occupational Hygiene 1991, 35, 1-12.

[9] Ng E.Y.K., Ooi E.H., FEM simulation of the eye structure with bioheat analysis, Computer Methods and Programs in Biomedicine 2006, 82, 268-276.

[10] Scott J.A., A finite element method of heat transport in the human eye, Physics in Medicine and Biology 1988, 33, 227-241.