THE CHARACTERISTIC OF THE FUZZY FLOW MODEL AMONG OPERATING MODULES

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Abstract. In this paper we refer to models characterizing successive use of means in indirect modules. Intermodular flow balance is characterized by Leontief's model. On its basis it is possible to estimate the initial value, indirect operating levels and the final value. In existing publications we deal with deterministic knowledge relating to initial, final and transferred values. In this paper we will take the mentioned parameters as fuzzy parameters. The problem is set to find a simple and fast method to select optimal parameters from fuzziness ranges for which the assumed criteria will be fulfilled. One of the criteria can be the minimal initial value, the maximal final value or the minimal diffusion of operation.

Introduction. Input data, their fuzziness and optimal selection criteria

Leontief's model has next figure:

$$\left(\begin{array}{c|c|c} X_1 & x_{1,1},\ x_{1,2},\ \dots,\ x_{1,n} & x_1 \\ X_2 & x_{2,1},\ x_{2,2},\ \dots,\ x_{2,n} & x_2 \\ & & \\ X_n & x_{n,1},\ x_{n,2},\ \dots,\ x_{n,n} & x_n \end{array}\right)$$

where:

- X_i - initial value in *i*-th row vector (department)
- x_i - final value in *i*-th row vector (department)
- $x_{i,j}$ - value level which is transferred from *i*-th department into *j*-th department after an operation.

The assumption of fuzziness admissibility of all parameters fully corresponds with real situations and does not limit their use as elements of criterial functions (1).
The sum of the flows and the final value in each row vector gives the initial value:

$$X_i = \sum_{j=1}^{n} x_{i,j} + x_i \quad \text{, where: } i = 1, \dots, n$$

The domains of all the parameters are reduced to the interval [0, 1] by the normalization of all the parameters $a_{i,j} = \frac{x_{i,j}}{X_j}$ (for i,j, ..., n).

$$\left(\begin{array}{l|llll|l} [\underline{X}_1, \overline{X}_1] & [\underline{x}_{1,1}, \overline{x}_{1,1}], & [\underline{x}_{1,2}, \overline{x}_{1,2}], & \ldots, & [\underline{x}_{1,n}, \overline{x}_{1,n}] & [\underline{x}_1, \overline{x}_1] \\ [\underline{X}_2, \overline{X}_2] & [\underline{x}_{2,1}, \overline{x}_{2,1}], & [\underline{x}_{2,1}, \overline{x}_{2,1}], & \ldots, & [\underline{x}_{2,n}, \overline{x}_{2,n}] & [\underline{x}_2, \overline{x}_2] \\ & & & & & \\ [\underline{X}_n, \overline{X}_n] & [\underline{x}_{n,1}, \overline{x}_{n,1}], & [\underline{x}_{n,1}, \overline{x}_{n,1}], & \ldots, & [\underline{x}_{n,n}, \overline{x}_{n,n}] & [\underline{x}_n, \overline{x}_n] \end{array}\right) \tag{1}$$

where: $\underline{x}, \overline{x}$ - lower and upper limits of fuzziness intervals.
The following relationship combines the normalized parameters:

$$\begin{pmatrix} 1-[\underline{a}_{1,1}, \overline{a}_{1,1}], & -[\underline{a}_{1,2}, \overline{a}_{1,2}], & \ldots, & -[\underline{a}_{1,n}, \overline{a}_{1,n}] \\ -[\underline{a}_{2,1}, \overline{a}_{2,1}], & 1-[\underline{a}_{2,1}, \overline{a}_{2,1}], & \ldots, & -[\underline{a}_{2,n}, \overline{a}_{2,n}] \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ -[\underline{a}_{n,1}, \overline{a}_{n,1}], & -[\underline{a}_{n,1}, \overline{a}_{n,1}], & \ldots, & 1-[\underline{a}_{n,n}, \overline{a}_{n,n}] \end{pmatrix} * \begin{pmatrix} [\underline{X}_1, \overline{X}_1] \\ [\underline{X}_2, \overline{X}_2] \\ \ldots\ldots \\ [\underline{X}_n, \overline{X}_n] \end{pmatrix} = \begin{pmatrix} [\underline{x}_1, \overline{x}_1] \\ [\underline{x}_2, \overline{x}_2] \\ \ldots\ldots \\ [\underline{x}_n, \overline{x}_n] \end{pmatrix} \tag{2}$$

or shorter $(I - A^{\sim})^{-1}X^{\sim} = x^{\sim}$, where ~ defines the fuzziness of the variable preceding the tilde character.
Optimization criteria can be determined with reference to all groups of parameters, i.e. to the flow, initial and final values (A, X and x). The following forms of criteria can be proposed by way of example:

1) $\sum_{i=1}^{n} \mathbf{X_i} \rightarrow \mathbf{min}$ $(X > 0, x > 0)$

2) $\sum_{i=1}^{n} \mathbf{x_i} \rightarrow \mathbf{max}$ $(X > 0, x > 0)$

3) $\sum_{i=1}^{n} \mathbf{x_i} \rightarrow \mathbf{min}$ $(X > 0, x > 0)$

4) $\mathbf{k - w_1 * X_1 + w_2 * X_2 + \ldots + w_n * X_n \rightarrow min > 0}$
k - set threshold, w_i - comparability weights $(X > 0, x > 0)$

5) $\sum_{i=1}^{n} \sum_{j=1}^{n} (x_{i,j} - x_{sri})^2 \rightarrow \min, (X > 0, x > 0)$

6) $\sum_{i=1}^{n} \mathbf{X_i - x_i} \rightarrow \mathbf{min}$ $(X > 0, x > 0)$

7) $\sum_{i=1}^{n}$ $\mathbf{X_i - x_i} \rightarrow \mathbf{max}$ ($X > 0$, $x > 0$) (3)

The solution to the optimization problem consists in such a selection of extracriterial parameters from the interval of their fuzziness so as to reach the extreme value of the selected criterion. An additional problem is to use a convenient and simple tool enabling quick selection of the parameter value.

1. Pragmatic methodology of linear system sensitivity research

Taking the linear model as a black box, it is possible to realize simple and practical methodology to create stimuli and to watch the reactions of the system. In this way, it is possible to sublime not only the trend of changes of the values of the criterial functions, but also their quantity (intensity). The quantity equivalent of changes for unitary stimuli will be defined as the system sensitivity with reference to the selected criterion (Fig. 1).

Tables 1, 2, 3, 4 and 5 illustrate examples of sensitivity research results of the linear system described by the equation: $(I - A)^{-1}X = x$.

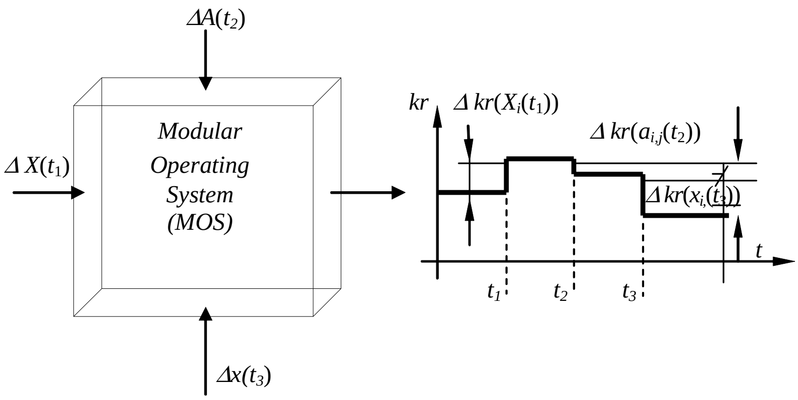


Fig. 1. Examples of linear system reactions to unitary parametric stimuli (system sensitivity - Δ kr($X_i(t_1)$)/ $\Delta X_i(t_1)$; Δ kr($a_{i,j}(t_2)$)/ $\Delta a_{i,j}(t_2)$; Δ kr($x_{i,}(t_3)$)/ $\Delta x_i(t_3)$): kr - criterial function, t - time

Table 1

Data: flow parameters ($a_{i,j}$)

0.079271	0.05468	0.03283	0.04865	0.017398	0.088359	0.098272	0.049494
0.063538	0.099182	0.086565	0.071737	0.049968	0.058966	0.022166	0.009676
0.027025	0.052863	0.084839	0.023439	0.015265	0.037701	0.079416	0.066732
0.029994	0.093699	0.038473	0.061769	0.052398	0.018673	0.008841	0.049292
0.087473	0.0187	0.073874	0.035554	0.901808	0.065076	0.029653	0.04505
0.095586	0.053098	0.096826	0.050359	0.09702	0.052725	0.031653	0.081507
0.066474	0.099999	0.010938	0.046925	0.075412	0.044783	0.034593	0.062712
0.058768	0.014358	0.03107	0.063745	0.057969	0.034424	0.001552	0.063796

Table 2

Matrix (*I-A*)

0.920729	-0.05468	-0.03283	-0.04865	-0.0174	-0.08836	-0.09827	-0.04949
-0.06354	0.900818	-0.08656	-0.07174	-0.04997	-0.05897	-0.02217	-0.00968
-0.02702	-0.05286	0.915161	-0.02344	-0.01526	-0.0377	-0.07942	-0.06673
-0.02999	-0.0937	-0.03847	0.938231	-0.0524	-0.01867	-0.00884	-0.04929
-0.08747	-0.0187	-0.07387	-0.03555	0.098192	-0.06508	-0.02965	-0.04505
-0.09559	-0.0531	-0.09683	-0.05036	-0.09702	0.947275	-0.03165	-0.08151
-0.06647	-0.1	-0.01094	-0.04693	-0.07541	-0.04478	0.965407	-0.06271
-0.05877	-0.01436	-0.03107	-0.06374	-0.05797	-0.03442	-0.00155	0.936204

Table 3

Inverse matrix $(I\text{-}A)^{-1}$

1.226822	0.149139	0.158816	0.140062	0.818092	0.202849	0.17468	0.153821
0.245669	1.210072	0.252382	0.181637	1.165464	0.202394	0.117866	0.134645
0.140751	0.126791	1.185971	0.095033	0.645335	0.126482	0.139879	0.149725
0.188764	0.185287	0.180994	1.157548	1.073073	0.142641	0.086846	0.155613
1.747202	0.771986	1.511477	0.914398	14.21711	1.344627	0.810852	1.111727
0.369556	0.200809	0.337435	0.208745	1.873313	1.26673	0.171403	0.268562
0.288642	0.222187	0.204197	0.177183	1.506438	0.217483	1.142475	0.209392
0.220555	0.100297	0.171859	0.158135	1.1154	0.159944	0.081732	1.174488

Table 4

Vector of final values *x*

52.31	85.09634	14.93371	9.937257	83.54432	53.29013	46.56676	45.28311

Table 5

Vector of initial values *X*

158.2292	183.6484	136.5524	165.8932	1173.496	245.7217	207.6041	166.5023

Taking $(\sum_{i=1}^{n} X_i \rightarrow min)$ as the criterion of the initial value and assuming that the flow values between the columns $A(a_{i,j})$ are independent parameters, the test of sensitivity of the operating linear system on the successively changing parameters $a_{i,j}$ ($a_{i,j} = a_{i,j}+\Delta$, $\Delta = 0.01$) will be carried out. Table 6 and the diagram in Figure 2 show the test results.

Table 6

Changes in the criterion (*X*) values under the influence of increments of flow parameters

7.8406111	10.70385	4.911181	6.57422	69.23556	12.78598	10.61108	8.117144
5.1962212	7.241181	3.289847	4.404328	45.92936	8.551646	7.104294	5.431635
7.0125398	9.677544	4.486956	5.943011	62.44313	11.55554	9.584914	7.334786
5.311651	7.32644	3.362165	4.546865	47.02139	8.743104	7.259496	5.556016
39.525422	54.68685	24.98653	33.57602	401.4149	65.71419	54.37717	41.46061
6.4198056	8.851208	4.062328	5.436191	57.04328	10.67367	8.772048	6.711052
4.7755954	6.580563	3.023162	4.042782	42.21719	7.855079	6.588309	4.989749
5.882081	8.108308	3.724762	4.983948	52.16784	9.686518	8.040596	6.215109

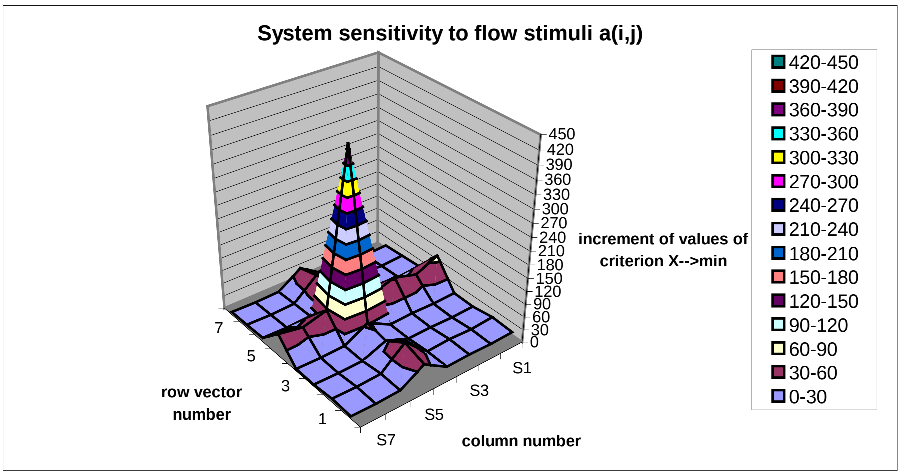
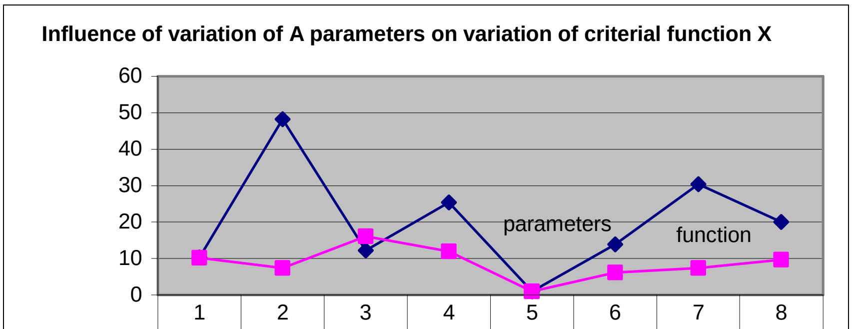


Fig. 2. Influence of changes of flow parameters on value of criterial function *X*

Supplementary information is the system reaction to the changes in individual parameters presented in the form of a_normalized (the ratio of the maximal value ($max = a_{5,5}$ and $X(a_{5,5})$) to the current value of the parameter or function).



	1	2	3	4	5	6	7	8
Serie1	10,3096	48,225	12,2074	25,3645	1	13,8578	30,412	20,0179
Serie2	10,1559	7,34025	16,0653	11,9554	1	6,1085	7,38205	9,68184

parameter number increased with set increment of 0.01

Fig. 3. Dynamics of changes of *X* criterion values with reference to flow changes

Setting the real levels of fuzziness for the discussed case, it is possible to determine the degree of changes in criterial function distribution (data in Table 7).

If for each row vector $i = 1,2,...,n$ the condition $\sum_{\substack{j=1 \\ j \neq i}}^{n} a_{i,j} = 1 - a_{i,i}$ is satisfied, then the matrix determinant (I-A) is equal to zero ($det(I\text{-}A) = 0$). Also, for every column

$j = 1,2,...,n$ in the situation $\sum_{\substack{i=1 \\ i \neq j}}^{n} a_{i,j} = 1 - a_{j,j}$ the determinant $(I\text{-}A)$ is equal zero. An increase of elements in the matrix, where the determinant is equal to zero, leads to negative values of the elements in the inverse matrix $(I\text{-}A)^{-1}$ and to negative values of the elements of the matrix of the initial values X. If for all the parameters $a_{i,j} = \underline{a}_{i,j} = \underline{x}_{i,j}/X_i$ the determinant of the matrix $(I\text{-}A)$ is positive and, at the same time, the determinant is negative for $a_{i,j} = \overline{a}_{i,j} = \overline{x}_{i,j}/X_i$, then the values of the operating parameters are at the limit of admissibility and these parameters or their fuzziness limits should be decreased.

Table 7

Values of upper and lower fuzziness of flow parameters *A*

0.006693	0.013591	0.012378	0.0119839	0.010038	0.007729	0.016887	0.014764	upper
0.017728	0.016887	0.009193	0.0166004	0.019338	0.025886	0.00831	0.022613	
0.027132	0.027323	0.027482	0.0291776	0.029546	0.017162	0.01432	0.007332	
0.006978	0.010075	0.017241	0.0114093	0.006797	0.01087	0.021651	0.000355	
0.019216	0.016347	0.020549	0.0070539	0.012547	0.01798	0.01008	0.005147	
0.027849	0.005026	0.012109	0.008699	0.029374	0.011905	0.016754	0.022861	
0.027081	0.004448	0.014431	0.0095296	0.026302	0.02252	0.016982	0.0136	
0.019089	0.011974	0.003666	0.0137452	0.02844	0.014924	0.021373	0.00418	

-0.0192	-0.03575	-0.00044	-0.0461	-0.03919	-0.00254	-0.02458	-0.04117	lower
-0.00286	-0.04683	-0.04711	-0.00423	-0.02019	-0.02585	-0.03031	-0.04635	
-0.02079	-0.01703	-0.02809	-0.00218	-0.031	-0.02215	-0.03344	-0.03462	
-0.01598	-0.02999	-0.04535	-0.00308	-0.00597	-0.03986	-0.00231	-0.03365	
-0.01476	-0.03903	-0.02405	-0.04731	-0.04747	-0.00539	-0.02496	-0.00123	
-0.00986	-0.04581	-0.00429	-0.03382	-0.0034	-0.04453	-0.02728	-0.01039	
-0.04114	-0.01176	-0.03564	-0.02378	-0.01348	-0.01887	-0.01385	-0.01795	
-0.04106	-0.02398	-0.04097	-0.04963	-0.02397	-0.0264	-0.00361	-0.02464	

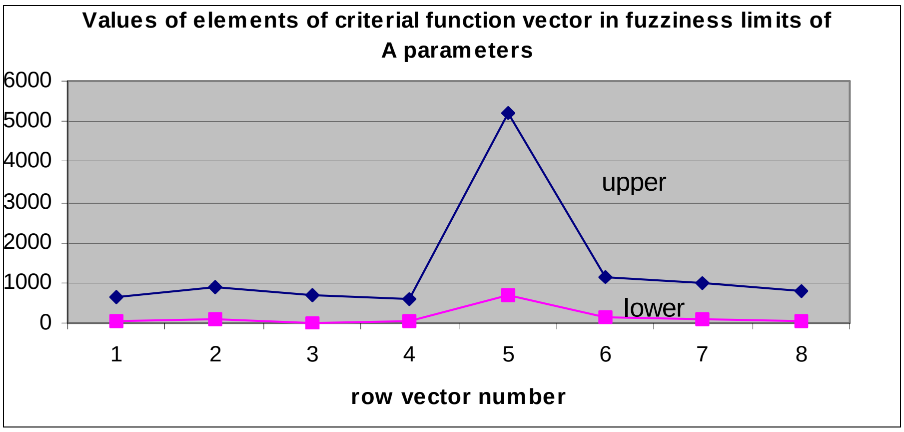


Fig. 4. Ceilings of values of criterial functions for flow extreme values in fuzziness limits

An attempt to solve this problem is made in the work [1], where the upper range of criterial function fuzziness was determined, which is to some extent a *post factum* reaction, since it does not lead to the analysis of the circumstance exceeding the determinant zero point. The aim of the present paper is to develop pragmatics to prevent such a situation. For example, for the data in Table 8 the determinant value of matrix (I-A) is similar to zero.

Table 8

Values of flow factors A, for which $det(I\text{-}A)\approx 0$

0.079271	0.05468	0.03283	0.04865	0.017398	0.088358	0.098272	0.58054
0.063538	0.099182	0.086565	0.071737	0.049968	0.058966	0.022166	0.547878
0.027025	0.052863	0.0848390	0.023439	0.015265	0.037701	0.079416	0.679452
0.029994	0.093699	0.038473	0.0617690	0.052398	0.018673	0.008841	0.696153
0.004563	0.0187	0.001232	0.035554	0.9018080	0.00321	0.029653	0.00528
0.095586	0.053098	0.096826	0.050359	0.09702	0.0527250	0.031653	0.522733
0.066474	0.099999	0.010938	0.046925	0.075412	0.044783	0.0345930	0.620876
0.058768	0.014358	0.03107	0.063745	0.057969	0.034424	0.67587	0.0637960

Table 9

Corrections of the flow factor $a_{i,j}$ for investigation of determinant zero zone

-0.00001	-0.0001	-0.001	-0.01	-0.1	0.1	0.01	0.001	0.0001
1.68E+08	1 526 5092	1 511 319	149 560.1	13 518.95	-16 709.08	-152 720.3	-1 511 487	-14 965 962

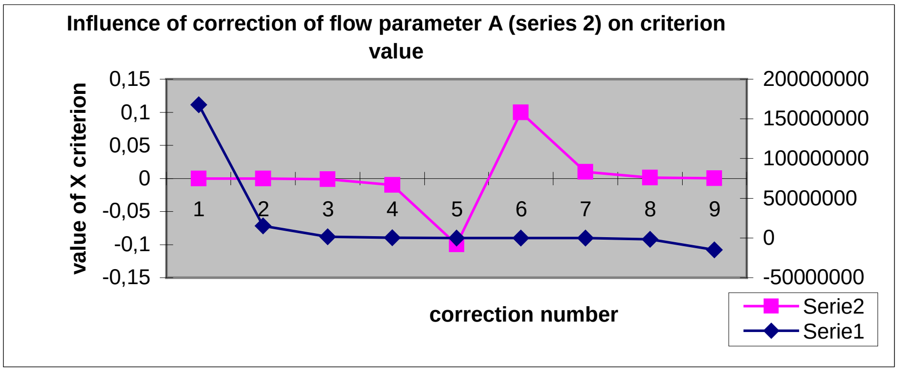


Fig. 5. Behaviour of the criterial function near a determinant zero value

To develop the pragmatics of procedure for the determination of optimal values of fuzzy parameters one predetermines possible situations which will be subjected to analysis:

1. The values of the matrix determinants of lower and upper fuzziness limits of flow factors $\underline{A}$ and $\overline{A}$ are greater than zero: det($\underline{A}$)>*0*, det($\overline{A}$)>*0* and the values of lower and upper fuzziness limits of final quantities x_i are positive $\underline{x}_i > 0$,

$\overline{x}_i > 0 \rightarrow$ in this situation the solution concerning the criterial function value (final quantity X) is a fuzzy interval with a positive range of values:

$$[\underline{X}, \overline{X}] = [((\mathrm{I} - \underline{A})^{-1}\underline{x}), ((\mathrm{I} - \overline{A})^{-1}\overline{x})], \quad \underline{X}_i > 0, \quad \overline{X}_i > 0$$

$\rightarrow$ if the criterion has the form $\sum_{i=1}^{n} X_j \rightarrow min$ and if it is satisfied, then the optimal solution is $X_{min} = ((I\text{-}\underline{A})^{-1}\underline{x})$.

2. The value of the matrix determinant of lower fuzziness limits of flow factors is positive $(\underline{A}) > 0$, and the determinant value of upper fuzziness limits is negative $\det(\overline{A}) < 0$ as well as the values of lower and upper fuzziness limits of the final quantities x_i are positive $\underline{x}_i > 0$, $\overline{x}_i > 0 \rightarrow$ in this situation the lower fuzziness limit of the criterial function is, as in the previous case, equal to $\underline{X} =$ $= ((\mathrm{I}\text{-}\underline{A})^{-1}\underline{x})$, and one has to limit the upper fuzziness limit of the criterion to the values for which $\det(\overline{A}) > 0$, using the following pragmatics:
   a) selection of the row vector (or row vectors) for which

$$\hat{i} = \{\ \mathrm{i} \in [1,...,\mathrm{n}] \mid \max_{i=1,n} (\sum_{\substack{j=1 \\ j \neq i}}^{n} a_{i,j} - (1 - a_{i,i}))\} \tag{4}$$

   b) proportional correction of the upper fuzziness limits in the $\hat{i}$ -th row vector to the nearest summary fuzziness in the remaining row vectors

$$\hat{\hat{i}} = \{\ \mathrm{i} \in [1,...,\mathrm{n}] \backslash [\,\hat{i}\,] \mid \max_{i=[1,n] \backslash \hat{i}} (\sum_{\substack{j=1 \\ j \neq i}}^{n} a_{i,j} - (1 - a_{i,i}))\}$$

   thus, the correction factor amounts to

$$\delta = (\sum_{\substack{j=1 \\ j \neq \hat{i}}}^{n} a_{\hat{i},j} - (1 - a_{\hat{i},\hat{i}}))/(\sum_{\substack{j=1 \\ j \neq \hat{\hat{i}}}}^{n} a_{\hat{\hat{i}},j} - (1 - a_{\hat{\hat{i}},\hat{\hat{i}}})) \tag{5}$$

   if there is more than one such row vector (with the same summary fuzziness), then all the row vectors should be subjected to fuzziness correction (it relates to a decrease in the upper fuzziness limits) (Fig. 6)
   c) calculation of the determinant value for the corrected upper limits of the flow factors matrix $det(I - \hat{\overline{A}} / \delta)$, where $\hat{\overline{A}}$ is the matrix of the corrected

flow factors, and check whether the determinant is positive → if not - one iteratively returns to point a).

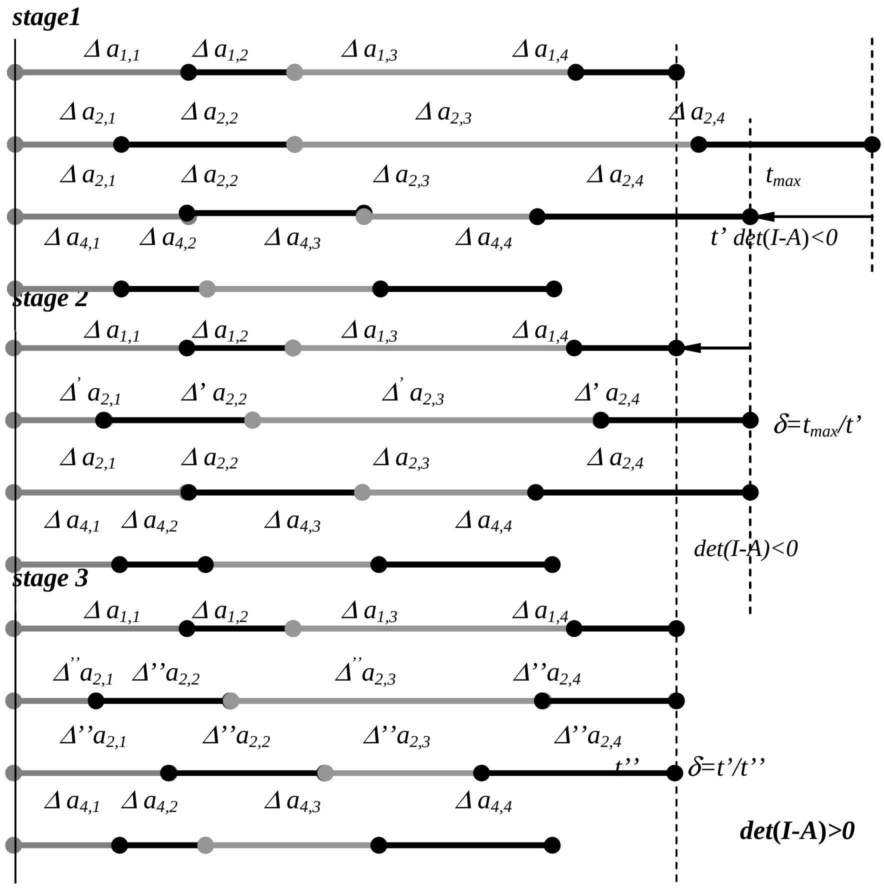


Fig. 6. The pragmatics method of correction of upper fuzziness limits of flow factors till the moment when the determinant becomes positive

Three stages of the analysis of the summary values of the fuzziness limits for each row vector were presented in the above example. In the first stage the longest value of summary fuzziness occurs in the second row vector, in the second stage the longest fuzzinesses are noticed in the second and the third row vectors. The matrix determinant (*I-A*) becomes positive after the second stage. In the first and the second stages all fuzzy sets in the longest fuzziness sequences get proportionally shortened to the level of successive fuzziness's as for the length. It is possible to describe many more similar pragmatics. For example, it is possible to reduce all upper limits of fuzziness using a constant, negative increment up to the moment of positive determinant obtainment, but it is not the main goal of this paper. And even, simpler pragmatic is the estimation of the global index of the fuzziness upper limit exceeding δg for all row vectors and columns causing a transition to the negative zone of the determinant of the matrix (*I-A*):

$$\delta g = \frac{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq 1}}^{n} (\overline{a}_{i,j})}{\sum_{i=1}^{n} (1 - \overline{a}_{i,i})} \tag{6}$$

and then the proportional correction (shortening) of all the intervals $[\underline{a}_{i,j}, \overline{a}_{i,j}]$ (and as a matter of fact - their upper limits of fuzziness) to the level $[\underline{a}_{i,j}, \underline{a}_{i,j} + (\overline{a}_{i,j} - \underline{a}_{i,j}) / \delta g]$. Plausibility (in the Dempster-Shafer (Dms_Sh) formulation) [2] gives the following canonical form of optimization limitations of flow fuzziness ranges:

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \overline{a}_{i,j} < \sum_{i=1}^{n} 1 - \overline{a}_{i,i} \;\; , \;\; 1 \geq a_{i,j} \geq 0 \, , \;\; i,j = 1,..., n \tag{7}$$

Thus, the beliavable limitations (in the Dms_Sh formulation) for the optimization of fuzziness intervals will have the following canonical form:

$$\begin{cases} \sum_{\substack{j=1 \\ j \neq 1}}^{n} \overline{a}_{1,j} < 1 - \overline{a}_{1,1} \\ \sum_{\substack{j=1 \\ j \neq 2}}^{n} \overline{a}_{2,j} < 1 - \overline{a}_{2,2} \\ \cdots\cdots\cdots\cdots \\ \sum_{\substack{j=1 \\ j \neq n}}^{n} \overline{a}_{n,j} < 1 - \overline{a}_{n,n} \\ 1 \geq a_{i,j} \geq 0 \, , \;\; i,j=1,...,n \end{cases} \tag{8}$$

Evidently, it is assumed that if the limitations for the upper limits of the fuzziness interval $\overline{a}$ are satisfied then they will be also fulfilled for the lower limits $\underline{a}$. In such a situation when the upper limit is subjected to correction one will be able to claim admissible (conditional) fuzziness for the sake of imposed limitations. But, if it turned out that the corrected limit was smaller than the lower fuzziness limitation (by using other pragmatics than those characterized above (6)-(8)) then the fuzziness would become inadmissible and the lower limit would already be a deterministic value and it will be corrected to the level corresponding with the positive value of the determinant *det*(*I-A*).

2. Generalization of Leontief's operating model

It is assumed in the characterised operating model that $1 \geq a_{i,j} \geq 0$, $i,j=1,...,n$. If making these limitations less strict admitting both negative values of the flow factor and the possibility of exceeding the unit then real situations adequate to the proposed models should be characterized. As the modelling results show, negative values of the flow factor do not lead to negative values of the determinant (*I*-*A*). The conversion of a positive value of the flow factor into a negative value leads to a decrease in the initial value *X*. A physical sense of negative values of the operating factors can be validated, for example, by a refund of investment, an interest on the account or marketing activity effects. Flow factors exceeding the unit value leads to negative initial values of *X* (in one, several or in all row vectors). It can be physically well-founded by using greater quantities of resources than those which one has. For example, it can represent a coming into debt zone. Thus, if the limitations concerning the flow factors $1 \geq a_{i,j} \geq 0$ are rejected then the limitations concerning the upper limits of fuzziness (in plausibility and beliavability variants) will have the following forms:

plausibility criterion

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \overline{a}_{i,j} \neq \sum_{i=1}^{n} 1 - \overline{a}_{i,i}$$

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \underline{a}_{i,j} \neq \sum_{i=1}^{n} 1 - \underline{a}_{i,i}, \quad i,j = 1,...,n \tag{9}$$

beliavability criterion

$$\begin{cases} \sum\limits_{\substack{j=1 \\ j \neq 1}}^{n} \overline{a}_{1,j} \neq 1 - \overline{a}_{1,1} \\ \sum\limits_{\substack{j=1 \\ j \neq 2}}^{n} \overline{a}_{2,j} \neq 1 - \overline{a}_{2,2} \\ \dots\dots\dots\dots \\ \sum\limits_{\substack{j=1 \\ j \neq n}}^{n} \overline{a}_{n,j} \neq 1 - \overline{a}_{n,n} \end{cases} \qquad \begin{cases} \sum\limits_{\substack{j=1 \\ j \neq 1}}^{n} \underline{a}_{1,j} \neq 1 - \underline{a}_{1,1} \\ \sum\limits_{\substack{j=1 \\ j \neq 2}}^{n} \underline{a}_{2,j} \neq 1 - \underline{a}_{2,2} \\ \dots\dots\dots\dots \\ \sum\limits_{\substack{j=1 \\ j \neq n}}^{n} \underline{a}_{n,j} \neq 1 - \underline{a}_{n,n} \end{cases} \tag{10}$$

$i,j=1,...,n$

Using the convention proposed by the Dms_Sh theory the plausibility by satisfying the conditions $1 \geq a_{i,j} \geq 0$, *i,j=1,...,n* can be estimated as:

$$\text{Pl(Constr)} = \left(\sum_{i=1}^{n}(1-\underline{a}i,i) - \sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\underline{a}_{\text{i,j}}\right) / \sum_{i=1}^{n}(1-\underline{a}i,i) \tag{11}$$

Beliavability takes a smaller value than plausibility (*Bel(Constr) < Pl(Constr))*, since it makes the difference between plausibility and doubts (doubt function).

$$\text{Bel(Constr)} = \prod_{i=1}^{n}\left(\left(1-\underline{a}i,i - \sum_{\substack{j=1\\j\neq i}}^{n}\underline{a}_{\text{i,j}}\right)/\left(1-\underline{a}i,i\right)\right) \tag{12}$$

However, doubts can have a twofold character - they can be plausible or unplausible (Fig. 7). The convention accepted here is consistent with St. Augustine's philosophy which goes "He who believes does not doubt". However, the convention is not consistent with the thesis that there is no faith without doubt. Doubts are fully well-founded in our example (since they are true).

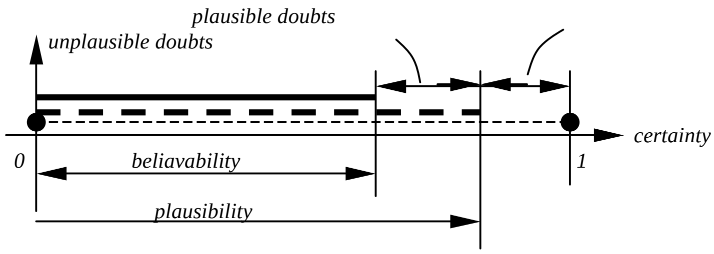


Fig. 7. A philosophy inspired Dempster-Shafer theory

If the limitations $1 \geq a_{i,j} \geq 0$, *i,j* = 1,...,*n* (generalized model) are rejected, then plausibility as well as beliavability become equal to one ($\lim_{n\to\infty}(n-1)/n = 1$).

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