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EVAPORATION EFFECT IN DOMAIN OF TISSUE SUBJECTED TO A STRONG EXTERNAL HEAT SOURCE

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Abstract. Thermal processes proceeding in domain of biological tissue subjected to a strong external heat flux can cause the water evaporation effect (the concentration of liquid in the tissue is taken to be 79%). The phase change is assumed to occur between 98 and 102°C. This rather artificial range results from the mathematical model basing on a concept of fixed domain approach which is used in this paper. The high temperature of tissue can be an effect of accidental burns or conscious activities connected with the hyperthermia treatments. From the mathematical point of view the problem is described by the well known Pennes equation with additional term controlling the evaporation process and a set of boundary-initial conditions. On a stage of numerical simulation the finite difference method is used. The examples of computations are also presented.

1. Governing equations

Thermal processes proceeding in domain of living tissue are described by the following partial differential equation (Pennes equation)

$$c(T)\frac{\partial T(x,t)}{\partial t}=\nabla\left[\lambda(T)\nabla T(x,t)\right]+Q_b(T)+Q_{met}(T)+Q_v(T) \tag{1}$$

where c [J/(m^{3}K], λ [W/(mK)] are the volumetric specific heat and thermal conductivity of material, Q_b, Q_{met}, Q_v are the perfusion heat source, metabolic heat source and internal heat source controlling the evaporation process, T, x, t denote the temperature, spatial co-ordinates and time.

According to the concept called “a fixed domain approach” [2, 3] the last term of equation (1) can be connected with the first one and then one obtains

$$C(T)\frac{\partial T(x,t)}{\partial t}=\nabla\left[\lambda(T)\nabla T(x,t)\right]+Q_b(T)+Q_{met}(T) \tag{2}$$

where $C(T)$ is called "a substitute thermal capacity" [2-4]. This parameter must fulfil the following condition

$$\int_{T_1}^{T_2} C(\mu)\,\mathrm{d}\mu = \int_{T_1}^{T_2} c(\mu)\,\mathrm{d}\mu + z_w L \tag{3}$$

where T_1, T_2 is the range of temperatures in which the evaporation process proceeds (here we assume $T_1 = 98$°C, $T_2 = 102$°C), L is the latent heat of evaporation, $z_w = 0{,}79$ [1] is the volumetric fraction of liquid in tissue domain.

The substitute thermal capacity is approximated by a broken line shown in Figure 1. Figure 2 illustrates the course of tissue thermal conductivity [1].

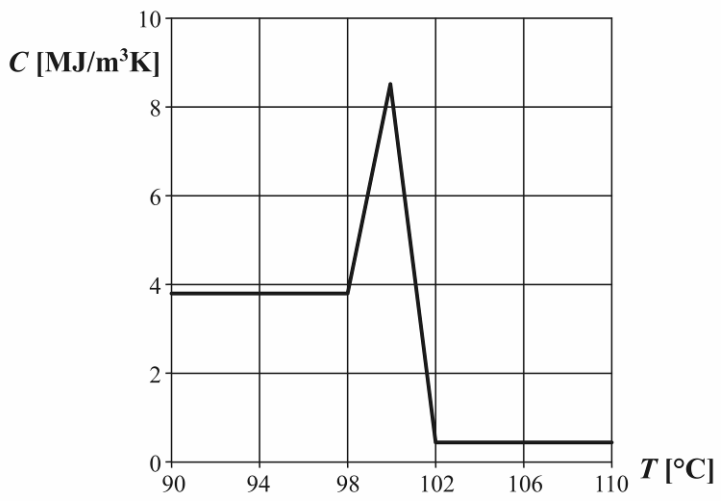

Fig. 1. Substitute thermal capacity

For the assumed input data one has

$$C(T) = \begin{cases} 3.8, & T < 98 \\ 3.8 + 2.339515(T - 98), & 98 \le T \le 100 \\ 0.44 - 4.019515(T - 102), & 100 \le T \le 102 \\ 0.44, & T > 100 \end{cases} \tag{4}$$

and

$$C(T) = \begin{cases} 0.52, & T < 98 \\ 0.52 - 0.107(T - 98), & 98 \le T \le 102 \\ 0.092, & T > 102 \end{cases} \tag{5}$$

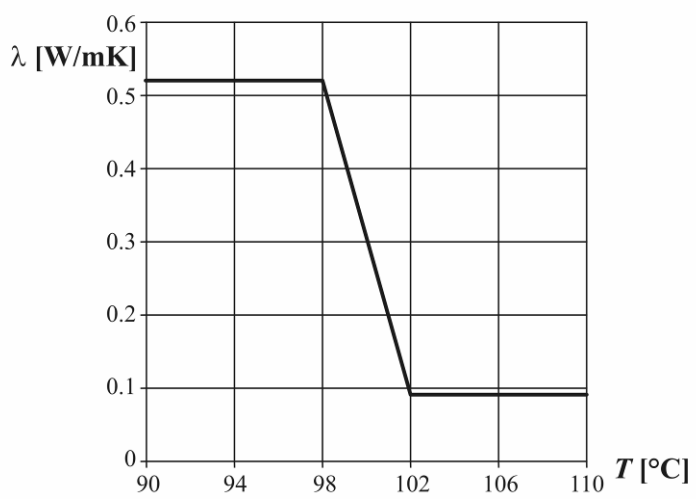

Fig. 2. Thermal conductivity

The perfusion heat source is equal to

$$Q_b(T) = c_B W_B(T)\left[T_B - T(x,t)\right] \tag{6}$$

where $c_B = 3900\,\mathrm{J/(kgK)}$ is the specific heat of blood, $W_B(T)$ [kg/(m^{3}s)] is the volumetric perfusion coefficient, $T_B = 37°\mathrm{C}$ is the blood temperature.

In numerous works (e.g. [4-6]) connected with numerical modelling of bio-heat transfer one assumed that the value of W_B and also the metabolic heat source Q_{met} are independent of temperature (and others thermophysical parameters). In a case of non-homogeneous domains (e.g. muscle and fat) the parameters discussed are taken to be different for successive sub-domains [7, 8], but constant. In this paper on the basis of information presented in [9], the temperature-dependent values of W_B and Q_{met} are taken into account. In particular, for the input data assumed we have (Fig. 3)

$$W_B = \begin{cases} 1.159, & T \le 42.5 \\ 1.159\left[1 + 9.6(T - 42.5)\right], & 42.5 < T < 45 \\ 28.975, & T \ge 45 \end{cases} \tag{7}$$

and (Fig. 4)

$$Q_{met}(T) = 1091\left[1 + 0.1(T - 37)\right] \tag{8}$$

For instance, if $T = 45\,°\mathrm{C}$ then the capacity of metabolic heat source equals $Q_{met} = 1963.8\,\mathrm{W/m^3}$.

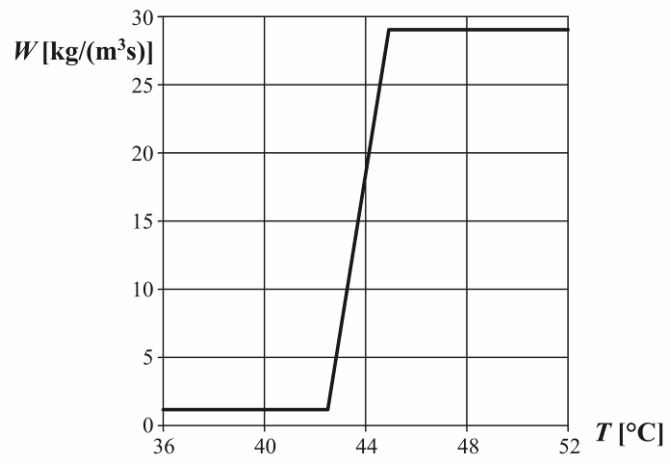

Fig. 3. Perfusion coefficient

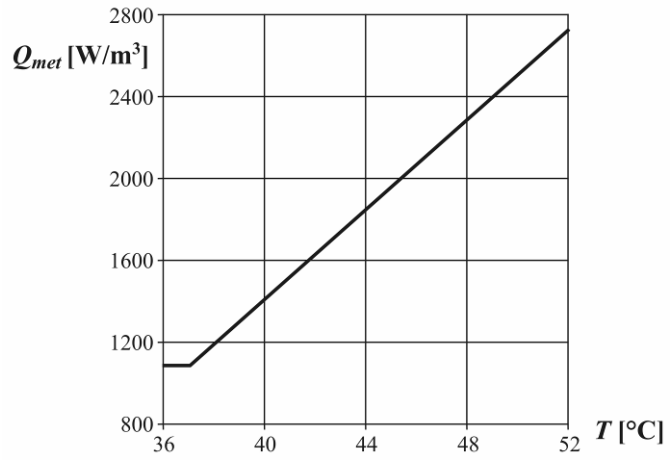

Fig. 4. Metabolic heat source

The bio-heat transfer equation must be supplemented by the boundary and initial conditions. On the skin surface subjected to an external heat source the knowledge of heat flux q and its exposure time t_e is accepted. In a case of 1D task this condition takes a form

$$x=0, \quad t<t_e: \quad q(x,t)=-\lambda\frac{\partial T(x,t)}{\partial x}, \quad t\geq t_e: \quad q(x,t)=0 \tag{9}$$

For $x=G$ (conventionally assumed internal boundary of domain) the non-flux condition can be accepted, while for $t=0$: $T(x,\,0)=37$°C.

2. Example of computations

The 1D problem is considered. The tissue layer G = 2 cm is subjected to the external heat flux equals 16 kW/m^2, while t_e = 100 s. The explicit scheme of FDM for non-linear parabolic equations has been applied [2]. The number of internal nodes n = 100, time step Δt = 0.005 s. In Figure 5 the temperature profiles for times 5, 10, 15, 20 and 30 s are shown (curves 1, 2, 3, 4, 5, respectively). The next Figure illustrates the heating curves at the points x_i = 0.1, 0.5, 0.9, 1.3 and 1.7 mm (1, 2, 3, 4, 5, respectively).

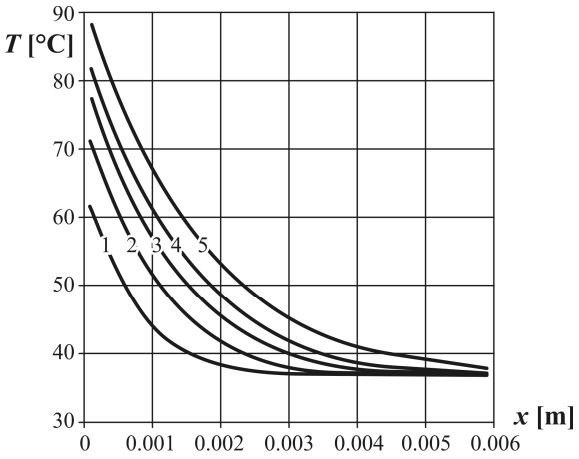

Fig. 5. Temperature profiles

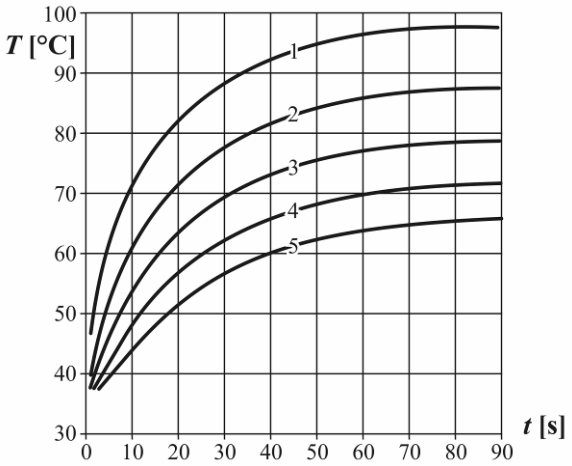

Fig. 6. Cooling curves

In comparison with the solutions for which the evaporation effect has not been taken into account the temperatures reach the essentially lower values. It results from the big value of substitute thermal capacity controlling the evaporation process. So, in the case of numerical modelling of tissue heating, especially when the intensity of external heat flux is big, the evaporation effect should be taken into account.

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