NUMERICAL MODELLING OF TEMPERATURE FIELD IN THE TISSUE WITH A TUMOR SUBJECTED TO THE ACTION OF TWO EXTERNAL ELECTRODES

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Abstract. The domain of tissue with a tumor subjected to the action of electrodes located on the skin surface is considered. External electric field causes the heat generation in the domain analyzed. The distribution of electric potential is described by the system of Laplace's equations, while the temperature field is described by the system of Pennes' equations. On the contact surface between healthy tissue and tumor region the ideal electric and ideal thermal contacts are assumed. To assure the optimum conditions of tumor destruction the magnetic nanoparticles are introduced to the tumor region. The aim of investigations is to determine the temperature field in the domain considered for different size and positions of external electrodes, in other words to choose such electrodes which assure the cancer destruction. To solve the coupled problem connected with the biological tissue heating the boundary element method is used. In the final part of the paper the examples of computations are shown.

1. Governing equations

The potential $\varphi_e(x, y)$ inside the healthy tissue ($e = 1$) and tumor region ($e = 2$) (Fig. 1) is described by the system of Laplace's equations

$$(x,\, y) \in \Omega_e : \quad \varepsilon_e \nabla^2 \varphi_e(x,\, y) = 0 \tag{1}$$

where $\varepsilon_e\ [\mathrm{C^2/(Nm^2)}]$ is the dielectric permittivity of sub-domain Ω_e. At the interface Γ_c of the tumor and healthy tissue the ideal electric contact is assumed

$$(x,\, y) \in \Gamma_c : \quad \begin{cases} \varphi_1(x,\, y) = \varphi_2(x,\, y) \\ -\varepsilon_1 \dfrac{\partial \varphi_1(x,\, y)}{\partial n} = -\varepsilon_2 \dfrac{\partial \varphi_2(x,\, y)}{\partial n} \end{cases} \tag{2}$$

On the external surface of tissue being in a contact with the electrodes the following condition is given

$$
\begin{aligned}
(x, y) \in \Gamma_1: \quad & \varphi_1(x, y) = U \\
(x, y) \in \Gamma_2: \quad & \varphi_2(x, y) = -U
\end{aligned}
\tag{3}
$$

where U [V] is the electric potential of the electrode relative to the ground. On the remaining external boundary of tissue the ideal electric isolation is assumed: $-\varepsilon_1 \partial \varphi_1(x, y) / \partial n = 0$.

The electric field inside the tissue is determined by equation

$$
\mathbf{E}_e(x, y) = -\nabla \varphi_e(x, y)
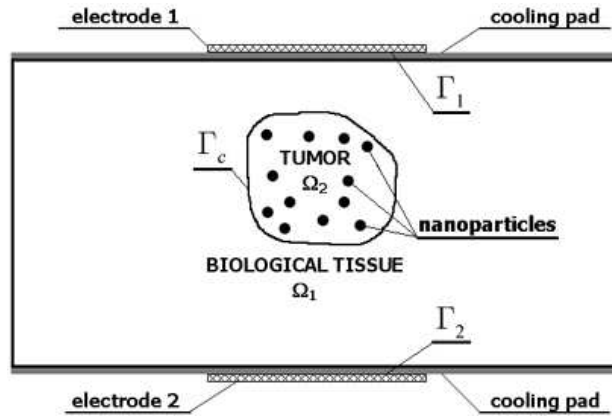\tag{4}
$$

Fig. 1. Action of electric field on the tissue with a tumor - hyperthermia system

The temperature field in the healthy tissue and the tumor region with embedded magnetic nanoparticles is described by the system of Pennes' equations [1, 2]

$$
\lambda_e \nabla^2 T_e(x, y) + k_e \left[T_B - T_e(x, y) \right] + Q_{met\,e} + Q_e(x, y) = 0
\tag{5}
$$

where $e = 1$, $e = 2$ correspond to the healthy tissue and tumor region, respectively, T_e denotes temperature, λ_e [W/(mK)] is the thermal conductivity, $k_e = G_{Be} c_B$ (G_{Be} [1/s] is the perfusion coefficient, c_B [J/(m^{3}K)] is the volumetric specific heat of blood), T_B is the supplying arterial blood temperature, $Q_{met\,e}$ [W/m^3] is the metabolic heat source, $Q_e(x, y)$ [W/m^3] is the heat source connected with the electromagnetic field action.

It should be pointed out that the thermal conductivity λ_2 of tumor region with nanoparticles can be calculated as follows: $1 / \lambda_2 = (1 - \Theta) / \lambda_2' + \Theta / \lambda_3$, where λ_2', λ_3 are the thermal conductivities of tumor and nanoparticles, respectively and $\Theta = n\pi r^2$ is the concentration of particles (n is the number of particles, r is the radius of particle).

Source function Q_1 [W/m³] connected with the electromagnetic dissipated power in healthy tissue depends on the conductivity σ_1 [S/m] and the electric field $\mathbf{E}_1$ [1]

$$Q_1(x, y) = \frac{\sigma_1 |\mathbf{E}_1(x, y)|}{2} \tag{6}$$

The tumor region with embedded magnetic particles is treated as a composite and due to the assumed homogeneity of Ω_2 the mean value of electrical conductivity σ_2 of this sub-domain can be approximated as: $1/\sigma_2 = (1 - \Theta)/\sigma_2' + \Theta/\sigma_3$, where σ_2', σ_3 are the electrical conductivities of tumor and particles, respectively.

Under the assumption that P_t is the tumor area, for $(x, y) \in \Omega_2$ one has

$$Q_2(x, y) = \frac{\Theta}{P_t} P_{SPM}(x, y) + \frac{P_t - \Theta}{P_t} \frac{\sigma_2 |\mathbf{E}_2(x, y)|}{2} \tag{7}$$

where P_{SPM} is the heat generation connected with the superparamagnetism (SPM) [1].

At the contact surface Γ_c between the tumor and healthy tissue the ideal contact is assumed

$$(x, y) \in \Gamma_c : \quad \begin{cases} T_1(x, y) = T_2(x, y) \\ -\lambda_1 \dfrac{\partial T_1(x, y)}{\partial n} = -\lambda_2 \dfrac{\partial T_2(x, y)}{\partial n} \end{cases} \tag{8}$$

On the upper and lower surfaces of healthy tissue domain (skin surface) the Robin condition (convection) is assumed

$$-\lambda_1 \frac{\partial T_1(x, y)}{\partial n} = \alpha_w \left[T_1(x, y) - T_w \right] \tag{9}$$

where α_w [W/(m²K)] is the heat transfer coefficient between the skin surface and the cooling water, T_w is the cooling water temperature. On the remaining boundaries the adiabatic condition $-\lambda_1 \partial T_1 / \partial n = 0$ can be taken into account. This condition results from the consideration that at the positions far from the center of the domain the temperature field is almost not affected by the external heating [1].

2. Boundary element method

To solve the equations describing the potential of electric field and the temperature field in the domain considered the boundary element method has been applied [3, 4].

The boundary integral equations corresponding to the equations (1) can be expressed as

$$B_e(\xi, \eta)\varphi_e(\xi, \eta) + \int_\Gamma \psi_e(x, y)\varphi_e^*(\xi, \eta, x, y)d\Gamma =$$
$$\int_\Gamma \varphi_e(x, y)\psi_e^*(\xi, \eta, x, y)d\Gamma, \quad e = 1, 2 \tag{10}$$

where (ξ, η) is the observation point, the coefficient $B_e(\xi, \eta)$ is dependent on the location of source point (ξ, η), $\psi_e(x, y) = -\varepsilon_e \partial\varphi_e(x, y)/\partial n$. For domain Ω_1 the boundary Γ corresponds to the external and internal boundary of healthy tissue, for domain Ω_2 the boundary Γ denotes Γ_c. Fundamental solutions of the problem discussed have the following form

$$\varphi_e^*(\xi, \eta, x, y) = \frac{1}{2\pi\varepsilon_e}\ln\frac{1}{r} \tag{11}$$

where r is the distance between points (ξ, η) and (x, y). Differentiating the function $\varphi_e^*(\xi, \eta, x, y)$ with respect to the outward normal $\mathbf{n} = [\cos\alpha, \cos\beta]$ the function $\psi_e^*(\xi, \eta, x, y)$ is obtained

$$\psi_e^*(\xi, \eta, x, y) = -\varepsilon_e \frac{\partial\varphi_e^*(\xi, \eta, x, y)}{\partial n} = \frac{d}{4\pi r^2} \tag{12}$$

where

$$d = (x - \xi)\cos\alpha + (y - \eta)\cos\beta \tag{13}$$

To solve the system of equations (10) the external and internal boundaries should be divided into boundary elements. Here the constant boundary elements have been taken into account as shown in Figure 2. Next, the integrals appearing in equations (10) are substituted by the sum of integrals over the boundary elements. Introducing the boundary conditions, finally one obtains the system of algebraic equations from which the 'missing' boundary values are determined. Last stage of computations consists in the determination of potentials $\varphi_e(x, y)$ at the internal points from healthy tissue and tumor region, separately.

The Pennes equations (5) can be written in the form

$$(x, y) \in \Omega_e : \quad \lambda_e \nabla^2 T_e(x, y) - k_e T_e(x, y) + Q^e(x, y) = 0 \tag{14}$$

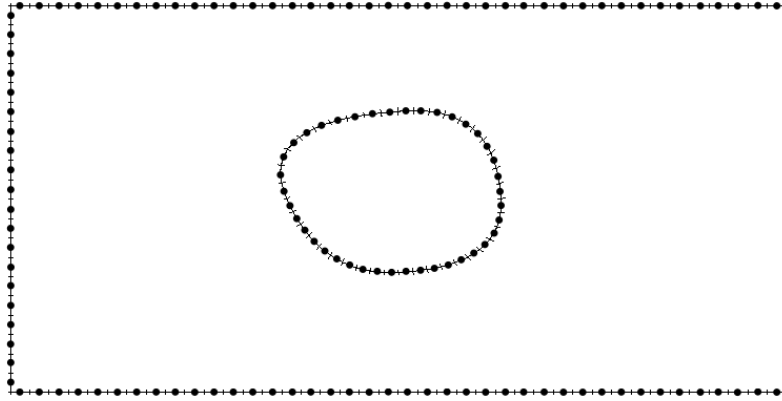where $k_e = G_{Be}c_B$, $Q^e(x, y) = k_e T_B + Q_{mete} + Q_e(x, y)$.

Fig. 2. Discretization of tissue-tumor boundaries

The boundary integral equations corresponding to the equations (14) can be written as follows [4]

$$B_e(\xi, \eta)T_e(\xi, \eta) + \int_\Gamma q_e(x, y)T_e^*(\xi, \eta, x, y)d\Gamma =$$

$$\int_\Gamma T_e(x, y)q_e^*(\xi, \eta, x, y)d\Gamma + \int_\Omega T_e^*(\xi, \eta, x, y)Q^e(x, y)d\Omega_e, \quad e = 1,2 \tag{15}$$

where

$$T_e^*(\xi, \eta, x, y) = \frac{1}{2\pi\lambda_e}K_0\left(r\sqrt{\frac{k_e}{\lambda_e}}\right) \tag{16}$$

is a fundamental solution and

$$q_e^*(\xi, \eta, x, y) = -\lambda_e\frac{\partial T_e^*(\xi, \eta, x, y)}{\partial n} = \frac{d}{2\pi r}\sqrt{\frac{k_e}{\lambda_e}}K_1\left(r\sqrt{\frac{k_e}{\lambda_e}}\right) \tag{17}$$

while $q_e(x, y) = -\lambda_e\partial T_e(x, y)/\partial n$. In the formulas (16), (17) $K_0(\cdot)$, $K_1(\cdot)$ are the modified Bessel's functions of second kind, zero and first order, respectively. To solve the equations (15), not only the boundary but also the interior of the sub-domains considered should be discretized. It should be pointed out that the mesh shown in Figure 3 has been obtained using the commercial package MSC Patran/ Nastran and next for the nodes obtained the boundary element method has been applied.
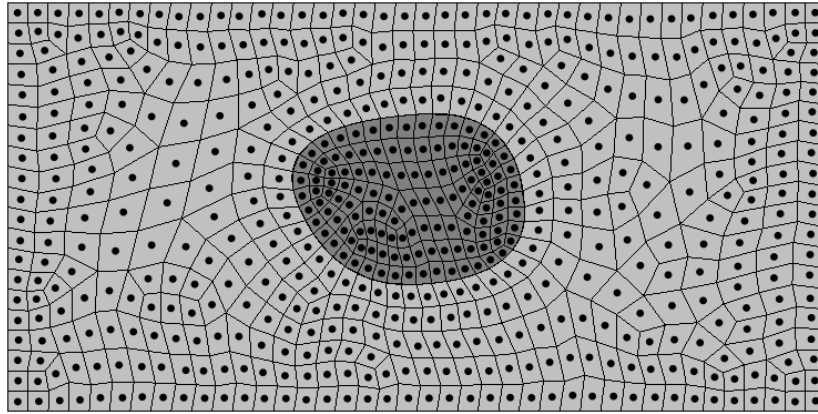
Fig. 3. Discretization of tissue-tumor domains

3. Results of computations

The rectangular domain of dimensions 0.08 m×0.04 m has been considered. The external boundary of the tissue has been divided into 120 constant boundary elements, the interface Γ_c of the tumor and tissue has been divided into 40 boundary elements (Fig. 2). To solve the Pennes equation in the interiors of Ω_1 and Ω_2, 461 and 129 nodes (internal cells) have been distinguished (Fig. 3).

The different heating areas are collected in Table 1 and shown in Figure 4. In the Table 2 the maximum temperatures of tumor region for all variants are collected.

Table 1. Size and location of external electrodes

	x [m]	y [m]
variant 1	0.034 – 0.044 0.034 – 0.044	0 0.04
variant 2	0.026 – 0.052 0.026 – 0.052	0 0.04
variant 3	0.02 – 0.058 0.02 – 0.058	0 0.04
variant 4	0.01 – 0.03 0.01 – 0.03	0 0.04

Table 2. Maximum temperature in tumor region

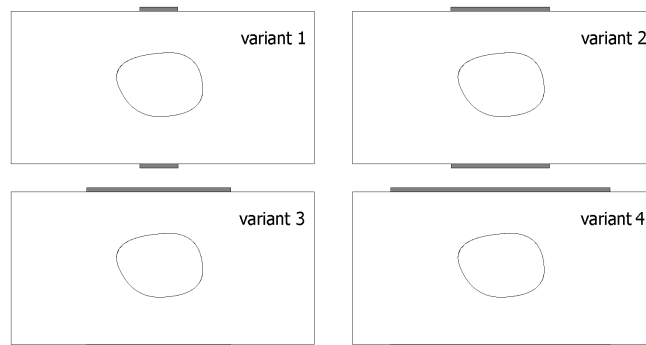|	variant 1	variant 2	variant 3	variant 4
T [°C]	40.58	46.16	48.34	49.84

Fig. 4. Size and location of external electrodes

The voltage applied on external electrodes (upper and lower) equals 6 and −6 V, respectively. For biological tissue and tumor region the following parameters have been assumed: thermal conductivities $\lambda_1 = 0.5$ W/(mK), $\lambda_2 = 0.6$ W/(mK), perfusion coefficients $G_{B1} = 0.0005$ 1/s, $G_{B2} = 0.002$ 1/s, metabolic heat sources $Q_{met1} = 420$ W/m³, $Q_{met2} = 4200$ W/m³, blood temperature $T_B = 37°C$ [2]. For nanoparticles the following parameters have been assumed: thermal conductivity $\lambda_3 = 40$ W/(mK), electrical conductivity $\sigma_3 = 25000$ S, in tumor region $n = 10^8$ nanoparticles with radiuses $r = 10^{-8}$ (iron oxide Fe_3O_4) are embedded.
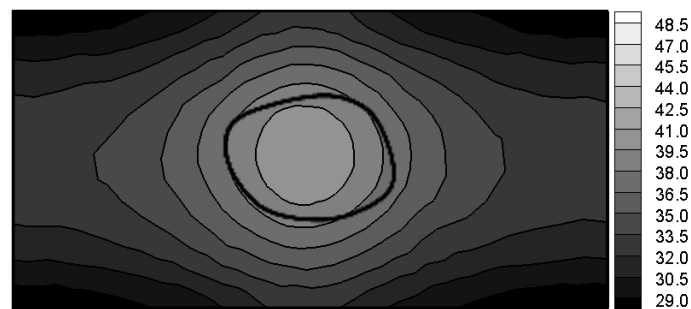

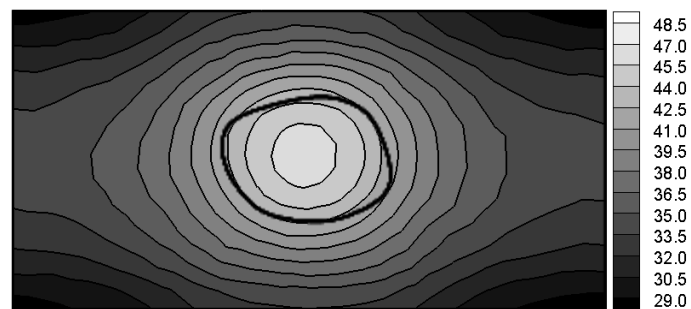Fig. 5. Temperature distribution for variant 1

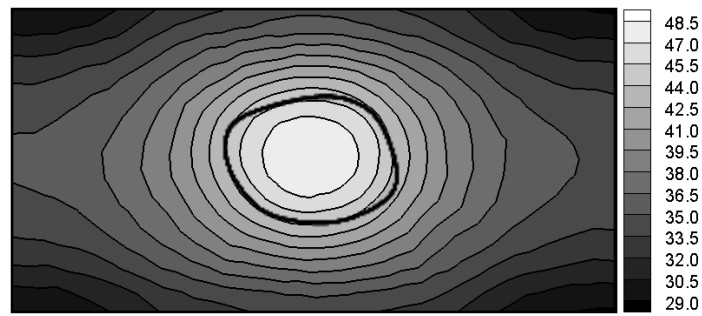Fig. 6. Temperature distribution for variant 2
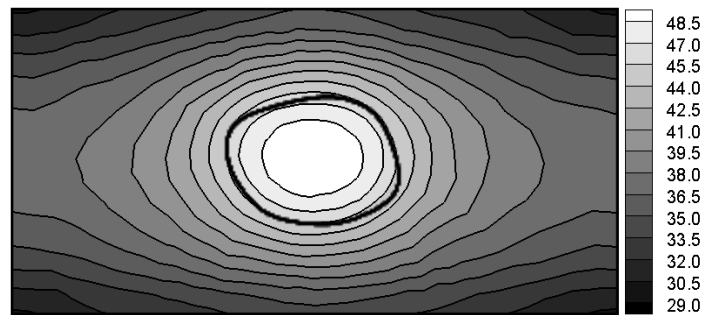
Fig. 7. Temperature distribution for variant 3

Fig. 8. Temperature distribution for variant 4

Conclusions

The computations presented in this paper show that the location and size of external electrodes have big influence on the temperature distribution in the tissue with a tumor. The electrodes assumed in the variant 1 are too small for obtaining the hyperthermia state, but electrodes assumed in the variant 4 are too big, and then not only the tumor will be destroyed but also part of the healthy tissue. Optimum size of electrodes corresponds to the variant 2.

Acknowledgement

References

[1] Lv Y.G., Deng Z.S., Liu J., 3D numerical study on the induced heating effects of embedded micro/nanoparticles on human body subject to external medical electromagnetic field, IEEE Transactions on Nanobioscience 2005, 4, 4, 284-294.

[2] Majchrzak E., Dziatkiewicz G., Paruch M., The modelling of heating a tissue subjected to external electromagnetic field, Acta of Bioengineering and Biomechanics 2008, 10, 2, 29-37.

[3] Majchrzak E., Mochnacki B., Analysis of thermal processes occurring in tissue with a tumor region using the BEM, Journal of Theoretical and Applied Mechanics 2002, 1, 40, 101-112.

[4] Majchrzak E., Boundary element method in heat transfer, Publication of Czestochowa University of Technology, Czestochowa 2001 (in Polish).