OPTIMISATION OF STOCHASTIC MODEL OF LOCAL COMPUTER NETWORK WITH METHOD OF BRANCH AND BOUNDS

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Abstract. In article the stochastic model of local computer network (LCN) in queueing network (QN) with one-line systems which are functioning in condition of high load is investigated. Optimization problem of finding optimal channel capacity which minimizes customer's fee for communication service during given time interval is solved. For finding of average number of calls which serve and wait in server queues the method of multidimensional generating functions was used.

Introduction

In connection with intensive development and embedding of information-computer networks (ICN) in different spheres of production and science activity the necessity of creation the theory of analysis and synthesis of such networks appears. Queueing theory makes it possible to build adequate mathematical models of ICN and to solve a set of problems which are connected with finding of qualitative characteristics of ICN functioning such as efficiency, capacity factor of their systems and etc. In present work the network model of LCN which is functioning in the condition of high load is observed [1].

1. Optimisation problem

Let us consider the functioning of LCN. LCN users via switches of levels and corpses and then via root switch connect to specialized servers such as proxy-server, web-server, mail server, database server, monitoring server, NetWare server. Calls of external users from Internet to resources of LCN servers: web-server, mail server, proxy-server, and calls of internal users to Internet resources create the total load in communication lines of four Internet-providers, which have specified order channel capacity and which in its turn is defined by agreement between LCN administration and providers. Finding of optimal channel capacity is an urgent problem. There are two cases can be observed: 1) if high channel capacity is ordered but network idles then LCN administration incurs unreasonable high loss and coasts of LCN resources should be reduced; 2) if low channel capacity is ordered and network is overload then users waiting time of service startup is big and users incurs unreasonable high loss with low loss for LNC resources therefore coasts of LCN resources should be enhanced. Functioning of network services can be described with help of one-line queueing systems (QS) which have input flow of messages and waiting queue. Under message we will understand scope of generated sessions of one LCN user which appear in access to Internet-resources.

Distribution law of intervals between arrival time of messages describes the type of input flow of messages. Service of each message consists in file transfer of defined size V_i with transfer rate G_i, which equals to server capacity. The set of random variables $\eta_i = \frac{V_i}{G_i}$ generates the set of service time and their distribution law describes the service type.

Let us suppose that LCN functions in condition of high load, i.e. there are users calls to servers at every time moment. In practice it can be explained that LCN users can leave calls for file downloading of external Internet-resources in their working places during time they absence or create loading in servers by means of the remote access.

Optimization problem for finding of optimal ordered channel capacity of LCN which minimizes service fee of Internet-provider during concrete time interval T will take form:

$$\begin{cases} S_{net}(G,T) = \dfrac{C(G)}{G} V_{\inf}(G,T) = \dfrac{C(G)}{G} \displaystyle\int_0^T \sum_{j=1}^{N_{ser}} MV_{ij} Mv_j(G,t)dt \to \min_G \\ g_1 \le G \le g_2 \end{cases} \tag{1}$$

where: $S_{net}(G,T)$ - service fee of provider, $V_{\inf}(G,T)$ - information scope which transits via dedicated channel to Internet at time interval $[0, T]$, *C(G)* - provider fee for communication service which is function of capacity of dedicate channel G Mb/s and corresponds the maximum quantity of megabytes in a second which dedicated channel is able to transmit, $\frac{C(G)}{G}$ - coast of traffic unit, V_{ij} - amount in bytes of *i*-th call in *j*-th server, $v_j(G,t)$ - number of calls transmitting via *j*-th server at the moment *t*, N_{ser} - server number.

For calculation of criteria in problem (1) variables V_{ij} are defined by observation by means of LNC monitoring and average number of calls $Mv_j(G,t)$ can be calculated with help of method of multidimensional generating functions which was worked up earlier [1]. The integral can be found by means of trapezium method for example. Let us describe more detail how it can be done.

2. Application of method of multidimensional generating functions

Let us examine open exponential QN of arbitrary topology with one type messages which consists of n one-line QS $S_1, S_2,\ldots,S_n$. Under the network state at the moment t we will understand vector $k(t)=(k,t)=(k_1,k_2,\ldots,k_n,t)$, where k_i - message number in system S_i, $i=\overline{1,n}$. Poisson flow of messages of the rate $\lambda(t)$ enters the network, service times in network systems are distributed by exponential law with parameter $\mu_i(G,t)$, $i=\overline{1,n}$, which depends on time and channel capacity. Let p_{ij} - probability of message transmit after service in the system S_i to the system S_j, $i,j=\overline{0,n}$, under the system S_0 we will understand the environment; $P(k,t)$ - probability that network is in state (k,t).

Let us denote n-dementional generating function as $\Psi_n(z,t)$, where $z=(z_1,z_2,\ldots,z_n)$:

$$\Psi_n(z,t)=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\ldots\sum_{k_n=0}^{\infty}P(k_1,k_2,\ldots,k_n,t)z_1^{k_1}z_2^{k_2}\ldots z_n^{k_n}=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\ldots\sum_{k_n=0}^{\infty}P(k,t)\prod_{i=1}^{n}z_i^{k_i}$$

and

$$\Lambda(t)=\int\lambda(t)dt,\;\; M_i(t,G)=\int\mu_i(t,G)dt,\;\; i=\overline{1,n}$$

$\alpha=(\alpha_1,\alpha_2,\ldots,\alpha_n,0)$, $\alpha_i>0$ - network state at the initial moment. In [1] expression for generating function $\Psi_n(z,t)$ was obtained and it is of the form

$$\Psi_n(z,t)=a_0(t)\sum_{l_1=0}^{\infty}\ldots\sum_{l_n=0}^{\infty}\sum_{q_1=0}^{\infty}\ldots\sum_{q_n=0}^{\infty}\sum_{r_1=0}^{\infty}\ldots\sum_{r_n=0}^{\infty}\left(\Lambda(t)-\Lambda(0)\right)^{\sum_{i=1}^{n}l_i}\times$$

$$\times\prod_{i=1}^{n}\left[\frac{p_{0i}^{l_i}p_{i0}^{q_i}\left(\prod_{\substack{j=1,\\p_{ij}\neq 0}}^{n}p_{ij}\right)^{r_i}}{l_i!q_i!r_i!}\left(M_i(G,t)-M_i(G,0)\right)^{q_i+r_i}z_i^{\alpha_i+l_i-q_i-r_i+R}\right]$$

where

$$R=\sum_{i=1}^{n}r_i,\;\; a_0(G,t)=\exp\left\{-\left(\Lambda(t)-\Lambda(0)\right)-\sum_{i=1}^{n}\left(M_i(G,t)-M_i(G,0)\right)\right\}$$

The average number of messages in the system S_j can be calculated with formula

$$Mv_j(t,G) = \left.\frac{\partial \Psi_n(z,t)}{\partial z_j}\right|_{z=(1,1,\dots,1)} = a_0(G,t)\sum_{l_1=0}^{\infty}\dots\sum_{l_n=0}^{\infty}\sum_{q_1=0}^{\infty}\dots\sum_{q_n=0}^{\infty}\sum_{r_1=0}^{\infty}\dots\sum_{r_n=0}^{\infty}\left(\alpha_j + l_j - q_j - r_j + R\right)\times$$

$$\times\left(\Lambda(t)-\Lambda(0)\right)^{\sum_{i=1}^{n} l_i}\prod_{i=1}^{n}\left[\frac{p_{0i}^{l_i}p_{i0}^{q_i}\left(\prod_{\substack{c=1,\\ p_{ic}\neq 0}}^{n} p_{ic}\right)^{r_i}}{l_i!q_i!r_i!}\left(M_i(G,t)-M_i(G,0)\right)^{q_i+r_i}\right], j=\overline{1,n}$$

If we make change of variables $k_j = \alpha_j + l_j - q_j - r_j + R$ in the last expression and consider that network systems function in condition of high load (there are non empty queues in these systems at every moment), i.e. $\alpha_j - q_j - r_j + R \geq 1$ and consequently $q_j \leq \alpha_j - r_j + R - 1$, we obtain final formula for average calls in the *j*-th server:

$$Mv_j(G,t) = a_0(G,t)\sum_{k_1=1}^{\infty}\dots\sum_{k_{j-1}=1}^{\infty}\sum_{k_j=1}^{\infty}k_j\sum_{k_{j+1}=1}^{\infty}\dots\sum_{k_n=1}^{\infty}\sum_{r_1=0}^{\infty}\dots\sum_{r_n=0}^{\infty}\sum_{q_1=0}^{\alpha_1-r_1+R-1}\dots\sum_{q_n=0}^{\alpha_n-r_n+R-1}\times$$

$$\times\prod_{i=1}^{n}\left[\frac{\left((\Lambda(t)-\Lambda(0))p_{0i}\right)^{k_i-\alpha_i+q_i+r_i-R}p_{i0}^{q_i}\left(\prod_{\substack{j=1,\\ p_{ij}\neq 0}}^{n} p_{ij}\right)^{r_i}}{\left(k_i-\alpha_i+q_i+r_i-R\right)!q_i!r_i!}\left(M_i(G,t)-M_i(G,t)\right)^{q_i+r_i}\right]$$

Optimization problem (1) is problem of one-variable *G* function minimization at the segment which has complicated form under the complexity of expression for average number of messages in server. In practice it is known that parameter *G* Mb/s possesses the integer value so we will consider problem (1) as problem of integer programming. It can be solved by means of exhaustion method or by means of combinatorial methods of discrete programming which allow to replace exchaustive search by partial enumeration and to reduce calculation time. In present article we will use method of branch and bounds for problem solving.

3. Algorithm of finding of optimal channel capacity by means of method of branch and bounds

1. Optimization problem L_0 which correspond to problem (1) nonmetering requirement that G should be integral variable is solved and solution $G^{(0)}$ is found. Before it was said that objective function of problem (1) has complicated form, so to find its minimum by equating its derivative to zero doesn't seem to be possible. For finding solution of problem L_0 the method of golden section was used. It allows to reduce the segment of localization to a given value δ corresponding to error of finding solution. If problem L_0 doesn't have solution so initial problem has no integer solution and calculations are completed.
2. Lower bound of objective function is calculated: $\xi^{(0)} = S(G^{(0)}, T)$. If solution $G^{(0)}$ is integer so it is supposed that $G^* = G^{(0)}$, $S^*_{net}(G^{(0)}, t) = \xi^{(0)}$ and calculations are completed, where G^* - solution of problem (1). If solution $G^{(0)}$ is not an integer for speeding up the process of solution we introduce upper bound of objective function $S_{net}(G, T)$ which is designated as Θ and which is defined the next way:

$$\Theta^{(v)} = \begin{cases} \Theta^{(v-1)}, & \text{if problem } L_v \text{ has no solution} \\ & \text{or } G^{(v)} \text{ isn't integer} \\ \min\{\Theta^{(v-1)}, \xi^{(v)}\}, & \text{if } G^{(v)} \text{ is integer} \end{cases}$$

   where v - number of node of problem graph, from which branching is carried on, and which corresponds to problem L_v, $v = 0, 1, 2\ldots$. At zero iteration it is supposed that $\Theta^{(0)} = +\infty$. Then we suppose $k = 1$ and pass to item 3.
3. The v-th node from the node set I (set of nodes of problem graph from which branching is possible) which satisfies the condition

$$\xi^{(v)} = \min_{i \in I} \xi^{(i)}$$

   is chosen.
4. From chosen node v the branching by variable G is carried out. Then we compose problems L_{2k-1} and L_{2k}, which follow from problem L_v, $v = 0, 1, 2\ldots$, by addition of new restriction:

$$L_{2k-1} : \begin{cases} L_v, \\ G \le [G^{(v)}]; \end{cases} \quad L_{2k} : \begin{cases} L_v, \\ G \ge [G^{(v)}] + 1 \end{cases}$$

5. Problems $L_j, j = 2k-1, 2k$, are solved by means of method of golden section nonmetering condition that G is integer. If problem L_j has no solution then it is supposed $\xi^{(j)} = +\infty$, $\Theta^{(j)} = \Theta^{(j-1)}$, $j = 2k-1, 2k$, and we pass (with j = 2k) to item 7.
6. We find $G^{(j)}$ and calculate $\xi^{(j)} = S(G^{(j)}, T)$, $j = 2k-1, 2k$. If solution $G^{(j)}$ is integer then we suppose $\Theta^{(j)} = \min\{\Theta^{(j)}, \xi^{(j)}\}$ and pass (with j = 2k) to item 7. If solution $G^{(j)}$ isn't integer then it is supposed $\Theta^{(j)} = \Theta^{(j-1)}$ and we pass (with j = 2k) to item 7. Node v, from which brunching was carried out, is deleted from set I.
7. The rest of nodes from I are checked and if condition $\xi^{(i)} \geq \Theta^{(2k)}$, $i \in I$, is satisfied so branching is completed.
8. Condition of conclusion of calculations $I = \varnothing$ is checked. If it executes then it is supposed $S^*(G,T) = \Theta^{(2k)}$, $G^* = G^{(v)}$, where $G^{(v)}$ is defined from condition $S(G^{\{v\}},T) = \Theta^{(2k)}$, and calculations are completed. If condition doesn't execute then we suppose $k = k+1$ and pass to item 3.

4. Numerical examples

Example 1. Let us consider model of LCN. The main loading to external communication channel of Internet-provider is made up at the expense of working of two servers: proxy-server and mail server. Let us denote them as systems S_1 and S_2. System S_0 corresponds to LCN users, system S_3 - communication channel. Since web-server, database server and NetWare-server are for access to domestic resources of LCN then loading in these servers influences on loading of external communication channels insignificantly and correspondingly on value of coast criterion in (1). So problem (1) will be solved for $N_{ser} = 2$. We will consider case when $\lambda(t) = \lambda t$, $\mu_1(G,t) = \mu_i[\cos(\omega_i t)+1]G,\ i=\overline{1,3}$. So $\Lambda(t) = 0.5\lambda t^2$, $\Lambda(0)=0$, $M_i(G,t) = \mu_i\left[\frac{\sin(\omega_i t)}{\omega_i}+t\right]G$, $M_i(G,0) = 0,\ \ i=\overline{1,3},\ \ a_0(G,t) = \exp\left\{-0.5t^2 - \sum_{i=1}^{3}\mu_i\left[\frac{\sin(\omega_i t)}{\omega_i}+t\right]G\right\}$. Probabilities of message transitions between network systems are $p_{01} = 0.6$, $p_{02} = 0.4$, $p_{10} = p_{13} =$ $= p_{20} = p_{31} = p_{23} = p_{32} = 0.5$, reminder - $p_{ij} = 0,\ i,j = \overline{0,3}$. Let fee to provider for communication service is given by function $C(G) = aG^2 + bG + c$, then $\frac{C(G)}{G} =$ $= aG + b + \frac{c}{G}$. Problem (1) in this case takes on form

$$\left\{\begin{array}{l}
S_{net}(G,T)=\dfrac{(aG^2+bG+c)}{G}\displaystyle\int_0^T e^{-\frac{\lambda}{2}t^2-\sum_{i=1}^{3}\mu_i\left[\frac{\sin(\omega_i t)}{\omega_i}+t\right]G}\times \\
\times\displaystyle\sum_{j=1}^{2}MV_{ij}\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\sum_{k_3=1}^{\infty}k_j\sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty}\sum_{q_1=0}^{\alpha_1-r_1+R-1}\sum_{q_2=0}^{\alpha_2-r_2+R-1}\times \\
\times\displaystyle\sum_{q_3=0}^{\alpha_3-r_3+R-1}\prod_{m=1}^{3}\left[\frac{\left(0.5\lambda t^2 p_{0m}\right)^{k_m-\alpha_m+q_m+r_m-R}p_{m0}^{q_m}\left(\prod_{\substack{c=1,\\ p_{mc}\neq 0}}^{3}p_{mc}\right)^{r_{im}}}{(k_m-\alpha_m+q_m+r_m-R)!q_m!r_m!}\times\right. \\
\times\left(G\mu_m\left(\dfrac{\sin(\omega_m t)}{\omega_m}+t\right)\right)^{q_m+r_m}dt\to\min\limits_{G}, \\
g_1\le G\le g_2,
\end{array}\right. \qquad (2)$$

Let $a=300$, $b=1000$, $c=400000$; $\lambda=600$, $\mu_1=4.8\cdot 10^{-2}$, $\mu_2=1.38\cdot 10^{-2}$, $\mu_3=0.87\cdot 10^{-2}$; minimum and maximum values of channel capacity correspondingly equals $g_1=10$ Mb/s and $g_2=360$ Mb/s, $T=1$.

Follow the circumscribed algorithm at the first stage the problem (2) is solved nonmetering the condition that parameter G must be integer. Using method of golden section we find solution $G^{(0)}=36.4$ of problem (2) accurate within $\delta=0.01$. Calculating time of problem (2) with computer will increase if accuracy of found solution will be increased. Then we pass to item 2. of algorithm and calculate the lower bound of function $\xi^{(0)}=S(G^{(0)},T)=1833495.94$. Since solution $G^{(0)}$ is not integer so for speeding up the process of solution we find upper bound of objective function $\Theta^{(0)}=+\infty$, suppose $k=1$ and pass to item 3 of algorithm. Set of nodes contains just one zero node $I=\{0\}$.

We choose 0-th node for brunching, since it is singular, and pass to item 4 of algorithm.

Then we carry out branching by variable G, compose problems L_1 and L_2:

$$L_1:\begin{cases}S_{net}(G,T)\to\min\limits_{G},\\ 10\le G\le 36;\end{cases}\qquad L_2:\begin{cases}S_{net}(G,T)\to\min\limits_{G},\\ 37\le G\le 360,\end{cases}$$

and pass to item 5 of algorithm.

We solve problems L_j, $j = 1,2$, by means of method of golden section nonmetering that variable G must be integer and pass to item 6.

Then we find $G^{(1)} = 35.9$, $\xi^{(1)} = S(G^{(1)}, T) = 1833533.25$ and $G^{(2)} = 37.1$, $\xi^{(2)} = S(G^{(2)}, T) = 1834749.18$. Since $G^{(j)}$, $j = 1,2$, are not integer so we calculate upper bounds for nodes 1 and 2: $\Theta^{(1)} = \Theta^{(2)} = \Theta^{(0)} = +\infty$. Branching is possible from set of nodes $I = \{1,2\}$. Pass to item 7 of algorithm.

Then we check nodes from set *I*, since $\xi^{(i)} < \Theta^{(2k)}$, $i \in I$, so brunching from these nodes is not completed, and pass to item 8.

At the next step we check condition of ending of calculations. Since $I \neq \varnothing$, then we suppose $k = k + 1$ and pass to item 3. Choose the next node for brunching such that $\xi^{(3)} = \min_{i \in \{1,2\}} \xi^{(i)}$.

Then follow the algorithm we will find the solution of problem (2) $G^* = G^{(3)} =$ = 35 Mb/s and value of coast criterion $S_{net}(G^*, T) = \xi^{(3)} = 1833518.79$.

Notice that the same solution was found by means of exhaustive search but timetable is less. Plot of dependence of coast criterion of optimization problem (2) from channel capacity G is presented in Figure 1.

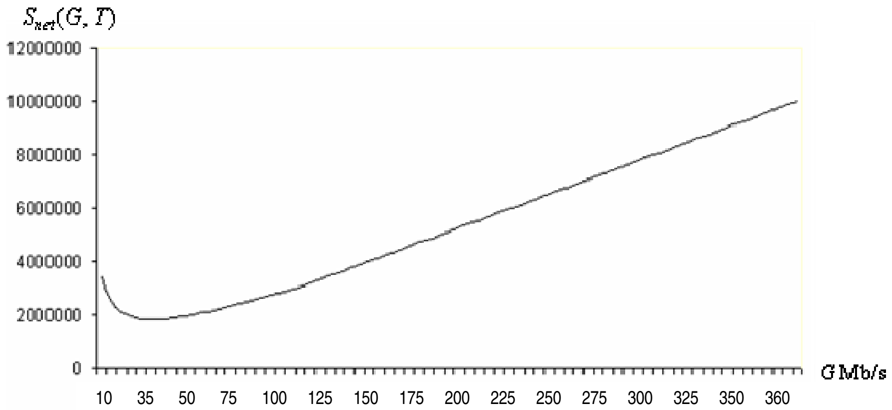

Fig. 1. Dependence of the provider service fee from channel capacity

Example 2. Let us consider queueing network from the first example in case when $\lambda(t) = \lambda t$, $\mu_1(G,t) = \mu_i G$, $i = \overline{1,3}$. Then $\Lambda(t) = 0.5\lambda t^2$, $\Lambda(0) = 0$, $M_i(G,t) = \mu_i tG$, $M_i(G,0) = 0$, $i = \overline{1,3}$, $a_0(G,t) = \exp\left\{-0.5t^2 - \sum_{i=1}^{3} \mu_i t G\right\}$. Probabilities of transitions between network systems - $p_{01} = 0.45$, $p_{02} = 0.55$, $p_{10} = 0.6$, $p_{13} = 0.4$, $p_{20} = p_{31} = 0.7$, $p_{23} = p_{32} = 0.3$, reminders - $p_{ij} = 0$, $i, j = \overline{0,3}$. Fee to provider for communication service is also given by quadratic function. Objective function of problem (1) in this case takes form:

$$S_{net}(G,T)=\left(aG+b+\frac{c}{G}\right)\int_0^T e^{\frac{\lambda}{2}t^2+\sum_{i=1}^{3}\mu_i Gt}\times$$

$$\times\sum_{j=1}^{2}MV_{ij}\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\sum_{k_3=1}^{\infty}k_j\sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty}\sum_{q_1=0}^{\alpha_1-r_1+R-1}\sum_{q_2=0}^{\alpha_2-r_2+R-1}\sum_{q_3=0}^{\alpha_3-r_3+R-1}\times \tag{3}$$

$$\times\prod_{m=1}^{3}\left[\frac{\left(\lambda tp_{0m}\right)^{k_m-\alpha_m+q_m+r_m-R}p_{m0}^{q_m}\left(\prod_{\substack{c=1,\\ p_{mc}\neq 0}}^{3}p_{mc}\right)^{r_m}}{\left(k_m-\alpha_m+q_m+r_m-R\right)!q_m!r_m!}\left(\mu_m Gt\right)^{q_m+r_m}\right]dt$$

Let $a=4000$, $b=300000$; $\lambda=200$, $\mu_1=5\cdot10^{-2}$, $\mu_2=2\cdot10^{-2}$, $\mu_3=4\cdot10^{-2}$; $g_1=10$ Mb/s and $g_2=100$ Mb/s, $T=1$.

Objective function (3) reaches its minimum at the left bound of the segment, i.e. $G^*=10$ Mb/s, value of coast criterion $S_{net}\left(G^*,T\right)=2391151.81$. This solution can be recieved by the means of the method of brunche and bounds at the first iteration. So results of observed examples show that in case when solution of optimization problem is at the bound of the definitional domain using of method of branch and bounds is more effective, in other cases - method of exhaustive search. It was proved by another observed examples. Plot of function $S_{net}(G,T)$ is presented in Figure 2.

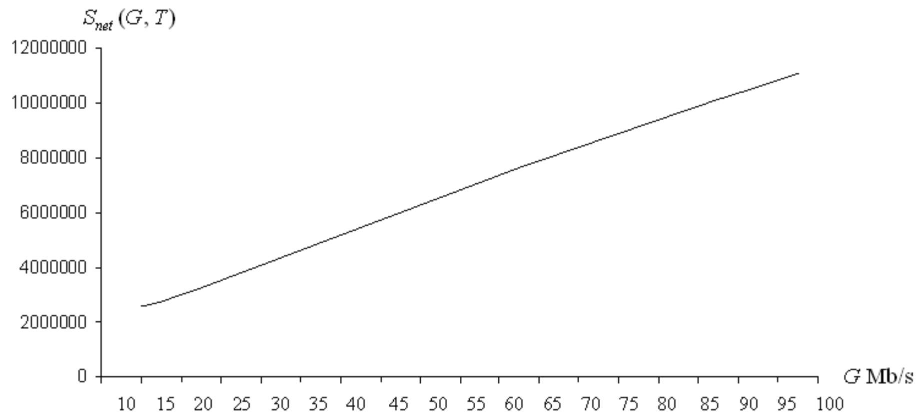

Fig. 2. Plot of the function $S_{net}(G,T)$

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[1] Koluzaeva E., Matalytski M., Finding of state probabilities of models of information-computer networks with the dependent from time parameters of arrival and service which are functioning in the condition of high load, Westnik GrSU 2008, 3, 22-29 (in Russian).