Please cite this article as:
Stanisław J. Matysiak, Dariusz M. Perkowski, On heat conduction in a periodically layered space with a vertical cylindrical hole, Scientific Research of the Institute of Mathematics and Computer Science, 2010, Volume 9, Issue 1, pages 131-145.
The website: http://www.amcm.pcz.pl/

Scientific Research of the Institute of Mathematics and Computer Science

ON HEAT CONDUCTION IN A PERIODICALLY LAYERED SPACE WITH A VERTICAL CYLINDRICAL HOLE

***Stanisław J. Matysiak*[1]*, Dariusz M. Perkowski*[2]**

[1]*Institute of Hydrogeology and Engineering Geology, Faculty of Geology*
University of Warsaw, Poland
[2]*Faculty of Mechanical Engineering, Białystok University of Technology, Poland*
s.j.matysiak@uw.edu.pl, d.perkowski@pb.edu.pl

Abstract. The paper deals with the stationary problems of heat conduction in a periodically layered half-space with cylindrical hole. The lateral surface of the hole is assumed to be kept at zero temperature or be thermally isolated. The boundary plane with circular hole is heated by a given temperature. The problems are solved within the framework of the homogenized model with microlocal parameters [1, 2]. The influences of geometrical and thermal parameters of the composite constituents on temperature and heat flux distributions are investigated.

Introduction

The analysis of temperature and heat flux distributions in laminated composites has been a subject of increasing importance, due to the expanding use of such materials in advanced engineering applications. Moreover, many rocks and soils are stratified and clearly piece-wise homogeneous. These are various metamorphic rods with fabrics having parallel arrangements of flat minerals (shale/sandstone, slate/ sandstone, varved clay, flotation wastes).

This paper is devoted to the analysis of axisymmetrical stationary problems of heat conduction for a periodic two-layered half-space with a cylindrical hole. The hole is located perpendicular to the layering and two cases of boundary conditions on the lateral surface of hole is considered: (1°) zero temperature (Problem 1), (2°) zero radial component of heat flux (Problem 2). Moreover, the perfect thermal bonding between the layers is taken into account. The boundary plane with a circle cut-out is assumed to be kept in a given temperature. The considered problem is determined within the framework of the classical descriptions by partial differential equations with discontinuous and rapidly oscillating coefficients. The compliance of continuity conditions on interfaces is complicated for analytical and numerical approaches, so the problem will be solved by using the approximated model with microlocal parameters [1, 2]. In the case of periodically two-layered composites the governing equations of the homogenized model are expressed by unknown macro-temperature connected with averaged temperature and certain extra unknown called the microlocal parameter. It is important that the continuity conditions on interfaces are fulfilled within the homogenized model. This model has been applied in many thermal and mechanical problems of periodically stratified composites, (see, for a partial review, [3, 4]). The considered boundary value problems will be solved by using the Weber-Orr integral transforms (see, [5-8]). The exact solutions within the framework of the homogenized model obtained in integral forms will be analyzed numerically and the results will be presented in the form of figures. Especially, the influence of geometrical and thermal properties of the composite constituents on the temperature and heat flux distributions will be investigated.

1. Formulations and solutions of the problems

Consider a rigid, periodically layered half-space with a cylindrical hole normal to the layering. The constituents of the body are assumed to be isotropic and homogeneous heat conductors. Let a be the radius of hole, l_1 l_2 be the thicknesses of the subsequent layers, $l = l_1+l_2$ be the thickness of the fundamental lamina (the repeated unit). Let K_1, K_2 denote the coefficients of heat conductivities of the subsequent layers. Let (r, ϕ, z) be the cylindrical coordinate system with the axis z being the symmetry axis of the hole, and $z=0,\ r \geq a$ represents the upper boundary surface of the body (Fig. 1).

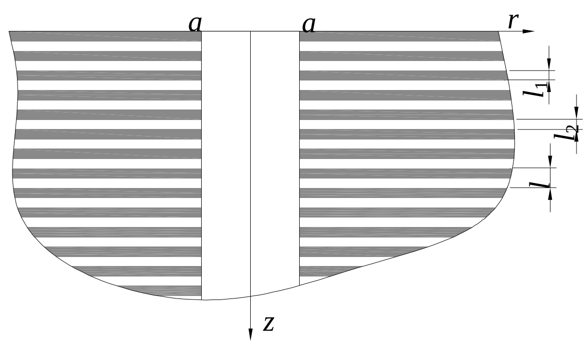


Fig. 1 The scheme of stratified half-space

The boundary $z=0,\ r \geq a$ is assumed to be kept at given temperature $\theta_0(r)$, and the lateral surface of the hole is kept at zero temperature (Problem 1) or is thermally insulated (Problem 2). The ideal thermal contact between the layers being constituents of the composite is taken into account. The mentioned above continuity conditions on the interfaces lead to some difficulty in analytical and numerical approaches. For this reason the homogenized model with microlocal parameters is suitable to the approximated formulation of the considered problem. We briefly recall only the governing relations of this model for the axisymmetric case (for a more detailed treatment see the following papers [1, 2, 9-12]). The temperature $T(r,z)$ and the temperature gradient is approximated as follows [12]:

$$T(r,z) = \theta(r,z) + h(z)\gamma(r,z) \approx \theta(r,z)$$
$$\frac{\partial T(r,z)}{\partial r} \approx \frac{\partial \theta(r,z)}{\partial r}, \quad \frac{\partial T(r,z)}{\partial z} \approx \frac{\partial \theta(r,z)}{\partial z} + h'(z)\gamma(r,z) \tag{1}$$

where $\theta(r,z)$ is an unknown function (called the macro-temperature), $\gamma(r,z)$ stands for the unknown thermal micro-parameter, and $h(z)$ is a given l periodic function taken in the form

$$h(z) = \begin{cases} z - 0.5l_1, & \text{for } 0 \le z \le l_1 \\ \dfrac{-\eta z}{1-\eta} - 0.5l_1 + \dfrac{l_1}{1-\eta}, & \text{for } l_1 \le z \le l \end{cases}$$
$$h(z+l) = h(z) \tag{2}$$

where

$$\eta = \frac{l_1}{l} \tag{3}$$

The governing equations of the homogenized model with microlocal parameters for the stationary axially symmetric case take the form [5]:

$$\tilde{K}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial z}\right) + [K]\frac{\partial\gamma}{\partial z} = 0$$
$$\hat{K}\gamma = -[K]\frac{\partial\theta}{\partial z} \tag{4}$$

where

$$\tilde{K} = \eta K_1 + (1-\eta)K_2, \; [K] = \eta(K_1 - K_2), \quad \hat{K} = \eta K_1 + \frac{\eta^2}{1-\eta^2}K_2 \tag{5}$$

By using the algebraic equation $(4)_2$ the microlocal parameter γ can be eliminated, what it leads to the equation for unknown macro-temperature:

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \hat{K}^{-1}K^*\frac{\partial^2\theta}{\partial z^z} = 0 \tag{6}$$

where

$$K^* = \tilde{K} - \frac{[K]^2}{\hat{K}} = \frac{K_1 K_2}{(1-\eta)K_1 + \eta K_2} \tag{7}$$

The heat flux vector $\mathbf{q}^{(j)}(r,z),\ j=1,2$ in the layer of j-th kind has the form:

$$\mathbf{q}^{(j)}(r,z) = \left[q_r^{(j)}(r,z), q_z^{(j)}(r,z) \right],\ q_r^{(j)}(r,z) = K_j \frac{\partial \theta}{\partial r},\quad q_z^{(j)}(r,z) = -K^* \frac{\partial \theta}{\partial z} \tag{8}$$

The equation (6) together with (8) stand for the governing relations of the stationary axisymmetrical heat conduction problems formulated within the framework of the homogenized model. From (8) it is seen that the continuity conditions on interfaces are satisfied.

The considered problems are described by the following boundary conditions and the regularity condition in infinity:

Problem (1)

$$\theta(r,\,0) = \theta_0(r),\ \text{for}\ r > a \tag{9}$$

$$\theta(a,z) = 0, \qquad \text{for z} > 0 \tag{10}$$

$$\lim_{z\to\infty} \theta(r,z) = 0, \qquad \text{for}\ \ r \ge 0 \tag{11}$$

and

Problem (2)

$$\theta(r,\,0) = \theta_0(r),\ \text{for}\ \ r > a \tag{12}$$

$$\frac{\partial \theta}{\partial r}(a,z) = 0, \qquad \text{for}\ \text{z} > 0 \tag{13}$$

$$\lim_{z\to\infty} \theta(r,z) = 0, \qquad \text{for}\ \ r \ge 0 \tag{14}$$

where $\theta_0(r)$ is given function satisfies the condition $\left| \int_a^\infty \sqrt{r}\,\theta_0(r)\,\mathrm{dr} \right| < \infty.$

To solve the above formulated problems. the Weber-Orr integral transforms will be employed [5-8]. For this aim introduce the following notations

$$\begin{aligned}\bar{\theta}(\xi,z)&=W_{00}\left[\theta(r,z);r\to\xi\right]=\int_a^\infty r\,C_{00}(\xi r,\xi a)\theta(r,z)\,\mathrm{dr},\\ \theta^*(\xi,z)&=W_{01}\left[\theta(r,z);r\to\xi\right]=\int_a^\infty r\,C_{01}(\xi r,\xi a)\theta(r,z)\,\mathrm{dr}\end{aligned} \tag{15}$$

where

$$C_{\mu\nu}(\xi r,\xi a)=J_\mu(\xi r)Y_\nu(\xi a)-J_\nu(\xi a)Y_\mu(\xi r),\quad \mu=0,\quad \nu=0;1 \tag{16}$$

and

$J_\mu(\cdot), Y_\mu(\cdot)$ are Bessel's and Neumann's functions, respectively.

The inverse transforms take the form [5-8]:

$$\theta(r,z)=W_{00}^{-1}\left[\bar{\theta}(\xi,z);\xi\to r\right]=\int_0^\infty \xi\frac{C_{00}(\xi r,\xi a)}{J_0^2(\xi a)+Y_0^2(\xi a)}\bar{\theta}(\xi,z)\,\mathrm{d}\xi \tag{17}$$

and

$$\theta(r,z)=W_{01}^{-1}\left[\theta^*(\xi,z);\xi\to r\right]=\int_0^\infty \xi\frac{C_{01}(\xi r,\xi a)}{J_1^2(\xi a)+Y_1^2(\xi a)}\theta^*(\xi,z)\,\mathrm{d}\xi \tag{18}$$

The important relations from the point of view of the considered problems are the following expressions [5-8]:

$$\begin{aligned}W_{00}\left[B_0\theta(r,z);\ r\to\xi\right]&=-\frac{2}{\pi}\theta(a,z)-\xi^2W_{00}\left[\theta(r,z);\ r\to\xi\right],\\ W_{01}\left[B_0\theta(r,z);\ r\to\xi\right]&=-\frac{2}{\pi\xi}\frac{\partial\theta(a,z)}{\partial r}-\xi^2W_{01}\left[\theta(r,z);\ r\to\xi\right]\end{aligned} \tag{19}$$

where

$$B_0\theta(r,z)\equiv\frac{\partial^2\theta(r,z)}{\partial r^2}+\frac{1}{r}\frac{\partial\theta(r,z)}{\partial r} \tag{20}$$

The equations (19) and boundary conditions (10) and (13) lead to the application of Weber-Orr transform $W_{00}[\cdot]$ in the case of Problem 1 and transform $W_{01}[\cdot]$ in the case of Problem 2, respectively. Form (19), (10), (13) and (6) it follows that

$$\left(\tilde{K}^{-1}K^*\frac{\partial^2}{\partial z^2}-\xi^2\right)\left(\bar{\theta}(\xi,z),\theta^*(\xi,z)\right)=0 \tag{21}$$

By using equation (21), boundary conditions (9), (11) and relation (17) we arrive at the solution to Problem 1 in the form:

$$\begin{aligned}\theta(r,z) &= W_{00}^{-1}\left[\bar{\theta}(\xi,z);\ \xi\to r\right]= \\ &= \int_0^\infty \frac{C_{00}(\xi r,\,\xi a)}{J_0^2(\xi a)+Y_0^2(\xi a)}\ \xi\bar{\theta}_0\ (\xi)\exp\left[-\sqrt{\frac{\tilde{K}}{K^*}}\ \xi z\right]\mathrm{d}\xi\end{aligned} \tag{22}$$

where

$$\bar{\theta}_0(\xi)=W_{00}\left[\theta_0(r);\ r\to\xi\right]=\int_a^\infty C_{00}(\xi r,\xi a)\,r\theta_0(r)\,\mathrm{dr} \tag{23}$$

From equations (22), (8) and the following relation [8]:

$$\frac{\mathrm{d}}{\mathrm{dx}}\left[x^{\mu}C_{\mu\nu}(\xi x,\xi a)\right]=\xi x^{\mu}C_{(\mu+1)\nu}(\xi x,\,\xi a) \tag{24}$$

we obtain the components of heat fluxes in the layer of j-th kind as follows

$$\begin{aligned} q_z^{(j)}(r,z) &= -\sqrt{\tilde{K}K^*}\int_0^\infty \frac{C_{00}(\xi r,\xi a)}{J_0^2(\xi a)+Y_0^2(\xi a)}\xi^2\bar{\theta}_0(\xi)\exp\left[-\sqrt{\frac{\tilde{K}}{K^*}}\ \xi z\right]\mathrm{d}\xi, \\ q_r^{(j)}(r,z) &= K_j\int_0^\infty \frac{C_{10}(\xi r,\xi a)}{J_0^2(\xi a)+Y_0^2(\xi a)}\xi^2\bar{\theta}_0(\xi)\exp\left[-\sqrt{\frac{\tilde{K}}{K^*}}\ \xi z\right]\mathrm{d}\xi, \quad j=1,2\end{aligned} \tag{25}$$

In the case of Problem 2 the solution takes the following form

$$\theta(r,z)=\int_0^\infty \frac{C_{01}(\xi r,\,\xi a)}{J_1^2(\xi a)+Y_1^2(\xi a)}\ \xi\theta_0^*(\xi)\exp\left[-\sqrt{\frac{\tilde{K}}{K^*}}\ \xi z\right]\mathrm{d}\xi \tag{26}$$

where

$$\theta_0^*(\xi)=W_{01}\left[\theta_0(r);\ r\to\xi\right]=\int_a^\infty r\,C_{01}(\xi r,\xi a)\theta_0(r)\,\mathrm{dr} \tag{27}$$

Using equations (26), (8) and (24) we obtain:

$$\begin{aligned} q_z^{(j)}(r,z) &= -\sqrt{\tilde{K}K^*}\int_0^\infty \frac{C_{01}(\xi r,\,\xi a)}{J_1^2(\xi a)+Y_1^2(\xi a)}\xi^2\theta_0^*(\xi)\exp\left[-\sqrt{\frac{\tilde{K}}{K^*}}\ \xi z\right]\mathrm{d}\xi, \\ q_r^{(j)}(r,z) &= K_j\int_0^\infty \frac{C_{11}(\xi r,\,\xi a)}{J_1^2(\xi a)+Y_1^2(\xi a)}\xi^2\theta_0^*(\xi)\exp\left[-\sqrt{\frac{\tilde{K}}{K^*}}\ \xi z\right]\mathrm{d}\xi, \quad j=1,2\end{aligned} \tag{28}$$

The equations (22) and (25) stands for the solution to Problem 1 in the integral forms. The equations (26) and (28) are the solution of Problem 2. The obtained results for some case of boundary temperature will be analyzed numerically.

2. Special cases and numerical results

As a special case the distributions of temperature and heat fluxes in the periodically layered half-space with cylindrical hole caused by a constant temperature ϑ_0 on the ring $b \le r \le c,\ \ z = 0$ will be considered. So, the function $\theta_0(r)$ in boundary conditions (9) and (12) is assumed in the form

$$\theta_0(r) = \vartheta_0\, H(r-b)\, H(c-r) \tag{29}$$

where ϑ_0 is given constant, $H(\cdot)$ is the Heaviside step function, and b, c are given constants such that $a < b < c$.

The function $\theta_0(r)$ defined by (29) will be taken into account in the solutions to Problem 1 and Problem 2.

Problem 1

Substituting (29) into (23) and using (16) as well as the following relations [13]:

$$\int_0^1 x^{\nu+1} J_\nu(ax)\mathrm{dx} = a^{-1} J_{\nu+1}(a), \quad \int_0^1 x^{\nu+1} Y_\nu(ax)\mathrm{dx} = a^{-1} Y_{\nu+1}(a), \quad \mathrm{Re}\,\nu > -1 \tag{30}$$

we obtain that

$$\bar{\theta}_0(\xi) = \frac{\vartheta_0}{\xi}\left\{Y_0(a\xi)\left[cJ_1(\xi c) - b\,J_1(\xi b)\right] - J_0(\xi a)\left[cY_1(\xi c) - bY_1(\xi b)\right]\right\} \tag{31}$$

The solution of Problem 1 for the considered case is determined by equations (22), (25) and (31) in the integral form. The integrals will be calculated numerically. For this aim the following dimensionless variables and constants are introduced:

$$\breve{r} = \frac{r}{a}, \quad \breve{z} = \frac{z}{a}, \quad \breve{\xi} = \frac{\xi}{a}, \ \breve{b} = \frac{b}{a}, \quad \breve{c} = \frac{c}{a}, \ \ \delta = \frac{l}{a} \tag{32}$$

Using (32) and (31) form equations (22) and (25) it follows that

$$\begin{aligned}\theta(\breve{r},\breve{z})/\vartheta_0 = \int_0^\infty \frac{C_{00}\left(\breve{\xi}\breve{r},\breve{\xi}\right)}{J_0^2\left(\breve{\xi}\right) + Y_0^2\left(\breve{\xi}\right)}\Big[& Y_0\left(\breve{\xi}\right)\left(-\breve{b}J_1\left(\breve{b}\breve{\xi}\right) + \breve{c}J_1\left(\breve{c}\breve{\xi}\right)\right) - \\ & J_0\left(\breve{\xi}\right)\left(-\breve{b}Y_1\left(\breve{b}\breve{\xi}\right) + \breve{c}Y_1\left(\breve{c}\breve{\xi}\right)\right)\Big]\exp\left(-\sqrt{\frac{\tilde{K}}{K^*}}\,\breve{\xi}\,\breve{z}\right)\mathrm{d}\breve{\xi},\end{aligned} \tag{33}$$

$$q_z^{(j)}\left(\breve{r},\breve{z}\right)/\left(\vartheta_0 K^*\right)=\sqrt{\frac{\tilde{K}}{K^*}}\int_0^{\infty}\frac{C_{00}\left(\breve{\xi}\,\breve{r},\breve{\xi}\right)}{J_0^2\left(\breve{\xi}\right)+Y_0^2\left(\breve{\xi}\right)}\cdot\xi\left[Y_0\left(\breve{\xi}\right)\left(-\breve{b}J_1\left(\breve{b}\breve{\xi}\right)+\breve{c}J_1\left(\breve{c}\breve{\xi}\right)\right)-\right.$$
$$\left.J_0\left(\breve{\xi}\right)\left(-\breve{b}Y_1\left(\breve{b}\breve{\xi}\right)+\breve{c}Y_1\left(\breve{c}\breve{\xi}\right)\right)\right]\exp\left(-\sqrt{\frac{\tilde{K}}{K^*}}\;\breve{\xi}\,\breve{z}\right)\mathrm{d}\breve{\xi}, \tag{34}$$

$$q_z^{(j)}\left(\breve{r},\breve{z}\right)/\left(\vartheta_0 K^*\right)=\frac{K_j}{K^*}\int_0^{\infty}\frac{C_{10}\left(\breve{\xi}\,\breve{r},\breve{\xi}\right)}{J_0^2\left(\breve{\xi}\right)+Y_0^2\left(\breve{\xi}\right)}\cdot\xi\left[Y_0\left(\breve{\xi}\right)\left(-\breve{b}J_1\left(\breve{b}\breve{\xi}\right)+\breve{c}J_1\left(\breve{c}\breve{\xi}\right)\right)-\right.$$
$$\left.J_0\left(\breve{\xi}\right)\left(-\breve{b}Y_1\left(\breve{b}\breve{\xi}\right)+\breve{c}Y_1\left(\breve{c}\breve{\xi}\right)\right)\right]\exp\left(-\sqrt{\frac{\tilde{K}}{K^*}}\;\breve{\xi}\,\breve{z}\right)\mathrm{d}\breve{\xi}, \tag{35}$$

The integrals in equations (33)-(35) were calculated numerically and the results are presented in the forms of figures.

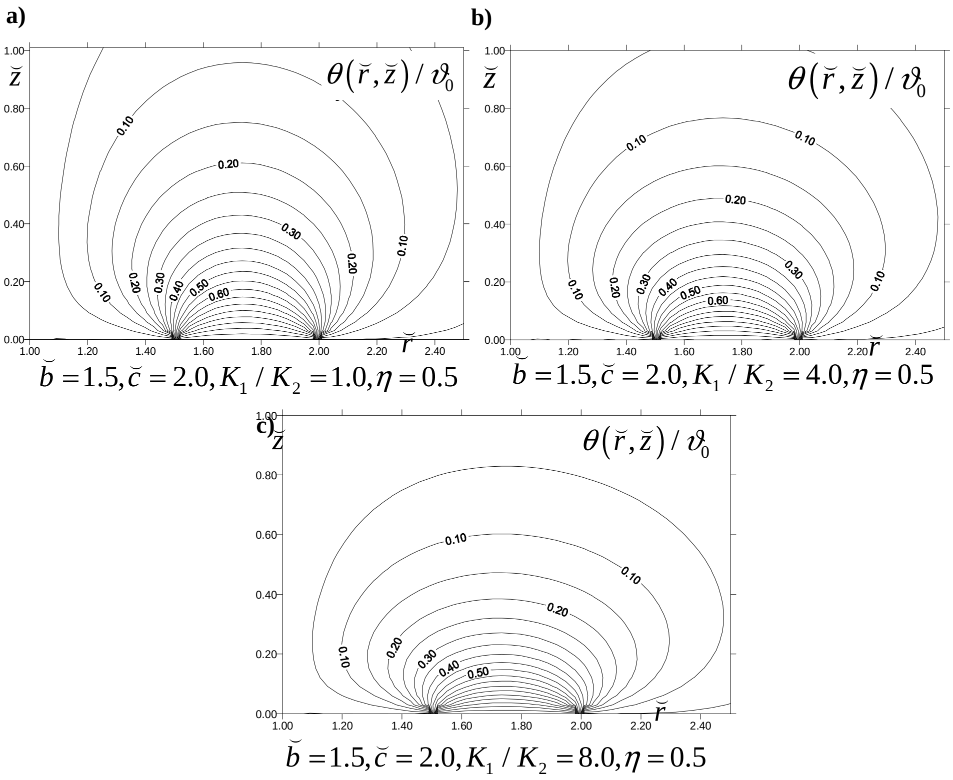


Fig. 2. The dimensionless temperature θ/ϑ_0 for Problem (1) and $\breve{b}=1.5$, $\breve{c}=2.0$, $\eta=0.5$; Fig. 2a for $K_1/K_2=1.0$ Fig. 2b for $K_1/K_2=4.0$, Fig. 2c for $K_1/K_2=8.0$

The dimensionless temperature θ / ϑ_0 is presented in Figures 2a, b, c (the isothermal curves) for $\breve{b} = 1.5$, $\breve{c} = 2.0$, $\eta = 0.5$. Figure 2a presents the isothermal curves for $K_1 / K_2 = 1.0$ (for the homogeneous half-space), Figure 2b for $K_1 / K_2 = 4.0$, Figure 2c for $K_1 / K_2 = 8.0$. It can be observed that the greater values of the dimensionless temperature under the heated section (*b*, *c*) are for the homogeneous body and they decrease together with increase of the ratio K_1 / K_2.

The dimensionless radial heat fluxes $q_r^{(j)} / \left(\vartheta_0 K^*\right)$ as functions of $\breve{z}$ are presented in Figures 3a, b, c for $\breve{b} = 1.5$, $\breve{c} = 2.0$, $\eta = 0.5$ and $\breve{r} = 1.75$. The case of homogeneous body is shown in Figure 3a $\left(K_1 / K_2 = 1.0\right)$, Figure 3b presents the radial heat fluxes for $K_1/K_2 = 4.0$, Figure 3c for $K_1/K_2 = 8.0$. For the layered structure of the body the radial heat fluxes $q_r^{(j)} / \left(\vartheta_0 K^*\right)$, $j = 1,2$ are discontinuous on the interfaces. The values of jumps increases with increase of the ratio K_1 / K_2. The upper curves represent the radial heat fluxes in the layer of first kind $\left(j = 1\right)$, the lower for $j = 2$.

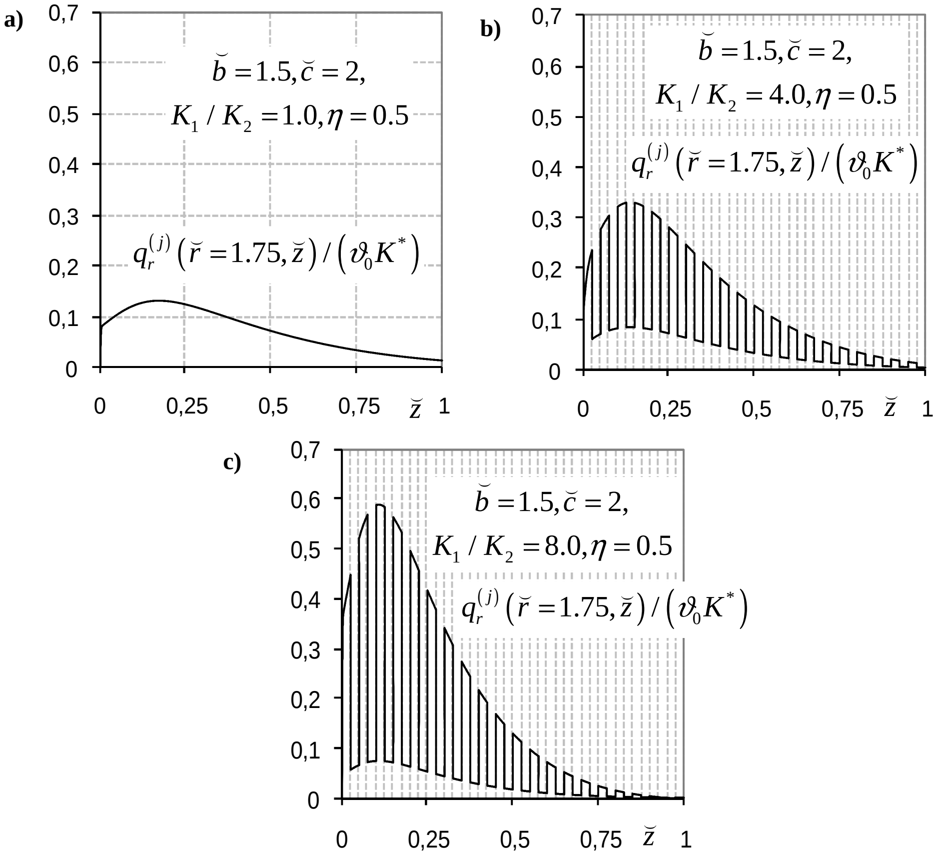


Fig. 3. The dimensionless radial heat flux $q_r^{(j)} / \left(\vartheta_0 K^*\right)$ for Problem (1) and $\breve{b} = 1.5$, $\breve{c} = 2.0$, $\eta = 0.5$; Fig. 3a for , $K_1 / K_2 = 1.0$ Fig. 3b for $K_1/K_2 = 4.0$, Fig. 3c for $K_1/K_2 = 8.0$

The dimensionless heat fluxes $q_z^{(j)}/\left(\vartheta_0 K^*\right)$ as functions of dimensionless radius $\breve{r}$ for $\breve{b}=1.5$, $\breve{c}=2.0$, $\eta=0.5$ and for cases of the ratio $K_1/K_2=1.0;\,4.0;\,8.0$, and the dimensionless depths $\breve{z}=\delta;\,2\delta;\,5\delta;\,10\delta$. It is seen that the normal to the layering component of heat flux changes its sing under the tips of heated area.

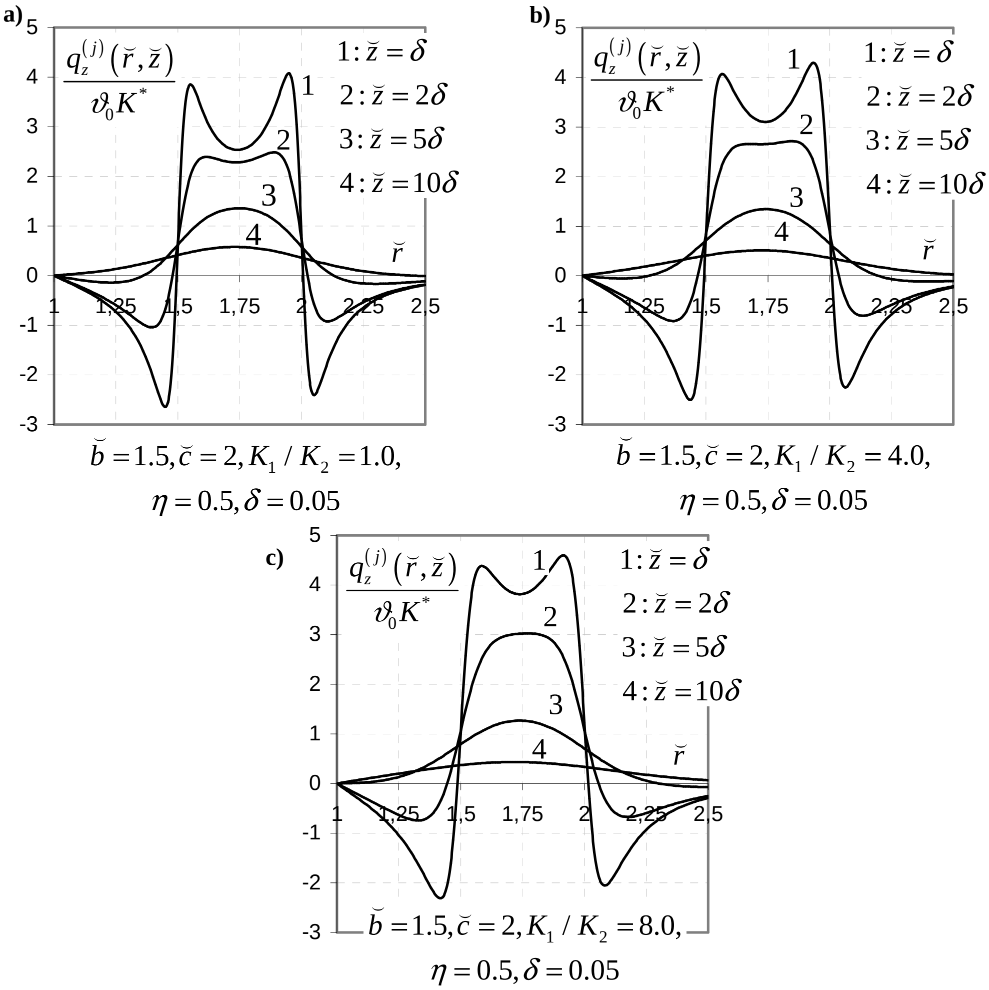


Fig. 4. The dimensionless heat flux $q_z^{(j)}/\left(\vartheta_0 K^*\right)$ θ/ϑ_0 for Problem (1) and $\breve{b}=1.5$, $\breve{c}=2.0$, $\eta=0.5$, $\delta=0.05$, Fig. 4a for , $K_1/K_2=1.0$ Fig. 4b for $K_1/K_2=4.0$, Fig. 4c for $K_1/K_2=8.0$, and the dimensionless depths $\breve{z}=\delta;\,2\delta;\,5\delta;\,10\delta$

Problem 2

Let the boundary temperature $\theta_0(r)$ be given by equation (29). By using (27), (29) and (30) we obtain

$$\theta_0^*(\xi) = \vartheta_0 \frac{1}{\xi}\left[Y_1(\xi a)\left(-bJ_1(b\xi)+cJ_1(c\xi)\right) - J_1(a\xi)\left(-bY_1(b\xi)+cY_1(c\xi)\right)\right] \tag{36}$$

Substituting (36) into (26) and (28) and using (32) it follows that

$$\begin{aligned}\theta(\breve{r},\breve{z})/\vartheta_0 = \int_0^\infty \frac{C_{01}(\breve{\xi}\breve{r},\breve{\xi})}{J_1^2(\breve{\xi})+Y_1^2(\breve{\xi})}\Big[& Y_1(\breve{\xi})\left(-\breve{b}J_1(\breve{b}\breve{\xi})+\breve{c}J_1(\breve{c}\breve{\xi})\right)- \\ & J_1(\breve{\xi})\left(-\breve{b}Y_1(\breve{b}\breve{\xi})+\breve{c}Y_1(\breve{c}\breve{\xi})\right)\Big]\exp\left(-\sqrt{\frac{\tilde{K}}{K^*}}\,\breve{\xi}\,\breve{z}\right)\mathrm{d}\breve{\xi},\end{aligned} \tag{37}$$

$$\begin{aligned}q_z^{(j)}(\breve{r},\breve{z})/\left(\vartheta_0 K^*\right) = \sqrt{\frac{\tilde{K}}{K^*}}\int_0^\infty \frac{C_{01}(\breve{\xi}\breve{r},\breve{\xi})}{J_1^2(\breve{\xi})+Y_1^2(\breve{\xi})}\cdot\breve{\xi}\Big[& Y_1(\breve{\xi})\left(-\breve{b}J_1(\breve{b}\breve{\xi})+\breve{c}J_1(\breve{c}\breve{\xi})\right)- \\ & J_1(\breve{\xi})\left(-\breve{b}Y_1(\breve{b}\breve{\xi})+\breve{c}Y_1(\breve{c}\breve{\xi})\right)\Big]\exp\left(-\sqrt{\frac{\tilde{K}}{K^*}}\,\breve{\xi}\,\breve{z}\right)\mathrm{d}\breve{\xi},\end{aligned} \tag{38}$$

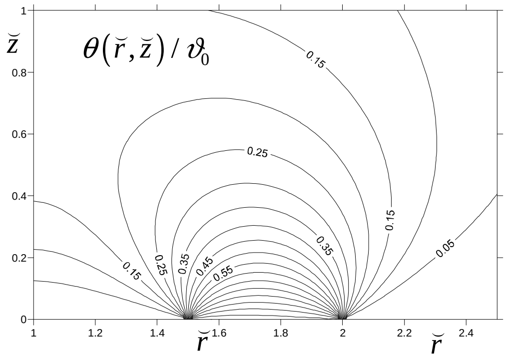
$$\begin{aligned}q_r^{(j)}(\breve{r},\breve{z})/\left(\vartheta_0 K^*\right) = \frac{K_j}{K^*}\int_0^\infty \frac{C_{11}(\breve{\xi}\breve{r},\breve{\xi})}{J_1^2(\breve{\xi})+Y_1^2(\xi)}\cdot\xi\Big[& Y_1(\breve{\xi})\left(-\breve{b}J_1(\breve{b}\breve{\xi})+\breve{c}J_1(\breve{c}\breve{\xi})\right)- \\ & J_1(\breve{\xi})\left(-\breve{b}Y_1(\breve{b}\breve{\xi})+\breve{c}Y_1(\breve{c}\breve{\xi})\right)\Big]\exp\left(-\sqrt{\frac{\tilde{K}}{K^*}}\,\breve{\xi}\,\breve{z}\right)\mathrm{d}\breve{\xi},\end{aligned} \tag{39}$$

The integrals In equations (37)-(39) have calculated numerically and the results are presented in the form of figures.

Figures 5a, b, c show the distribution of dimensionless temperature $\theta(\breve{r},\breve{z})/\vartheta_0$ (the isothermal curves) for $\breve{b}=1.5$, $\breve{c}=2.0$, $\eta=0.5$ and three cases of the ratio $K_1/K_2=1.0$; 4.0 ; 8.0. Figure 5a presents the solution for the homogeneous half-space.

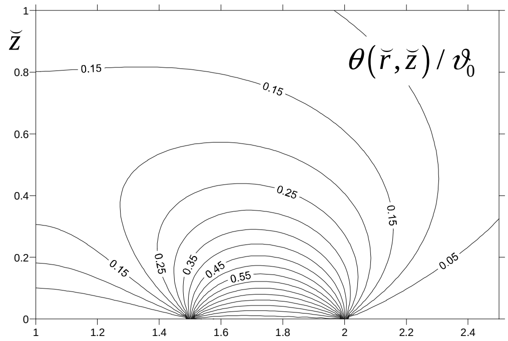
The dimensionless component of heat flux normal to the layering $q_z^{(j)}(\breve{r},\breve{z})/\left(\vartheta_0 K^*\right)$ as a function of $\breve{r}$ is presented in Figures 6a, b, c, on the four cases of depths $\breve{z}=\delta$; 2δ ; 5δ ; 10δ for three cases of the ratio $K_1/K_2=1.0$; 4.0 ; 8.0.

a)



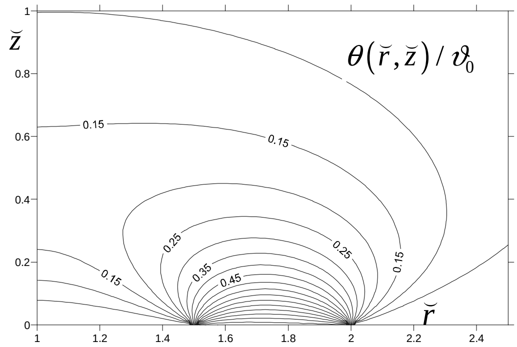
$\breve{b}=1.5, \breve{c}=2.0, K_1/K_2=1.0, \eta=0.5$

b)



$\breve{b}=1.5, \breve{c}=2.0, K_1/K_2=4.0, \eta=0.5$

c)



$\breve{b}=1.5, \breve{c}=2.0, K_1/K_2=8.0, \eta=0.5$

Fig. 5. The dimensionless temperature θ/ϑ_0 for Problem (2) and $\breve{b}=1.5$, $\breve{c}=2.0$, $\eta=0.5$; Fig. 5a for $K_1/K_2=1.0$, Fig. 5b for $K_1/K_2=4.0$, Fig. 5c for $K_1/K_2=8.0$

The dimensionless radial component of heat flux $q_r^{(j)}\left(\breve{r},\breve{z}\right)/\left(\vartheta_0 K^*\right)$ as a function of $\breve{z}$ is presented in Figures 7a, b, c, for $\breve{b}=1.5$, $\breve{c}=2.0$, $\eta=0.5$, $\breve{r}=1.75$ for three cases of the ratio $K_1/K_2=1.0;\ 4.0;\ 8.0$. The radial heat flux $q_r^{(j)}$ is discontinuous on the interfaces and the jumps take the greatest values near boundary $z=0$ for $K_1/K_2=8.0$.

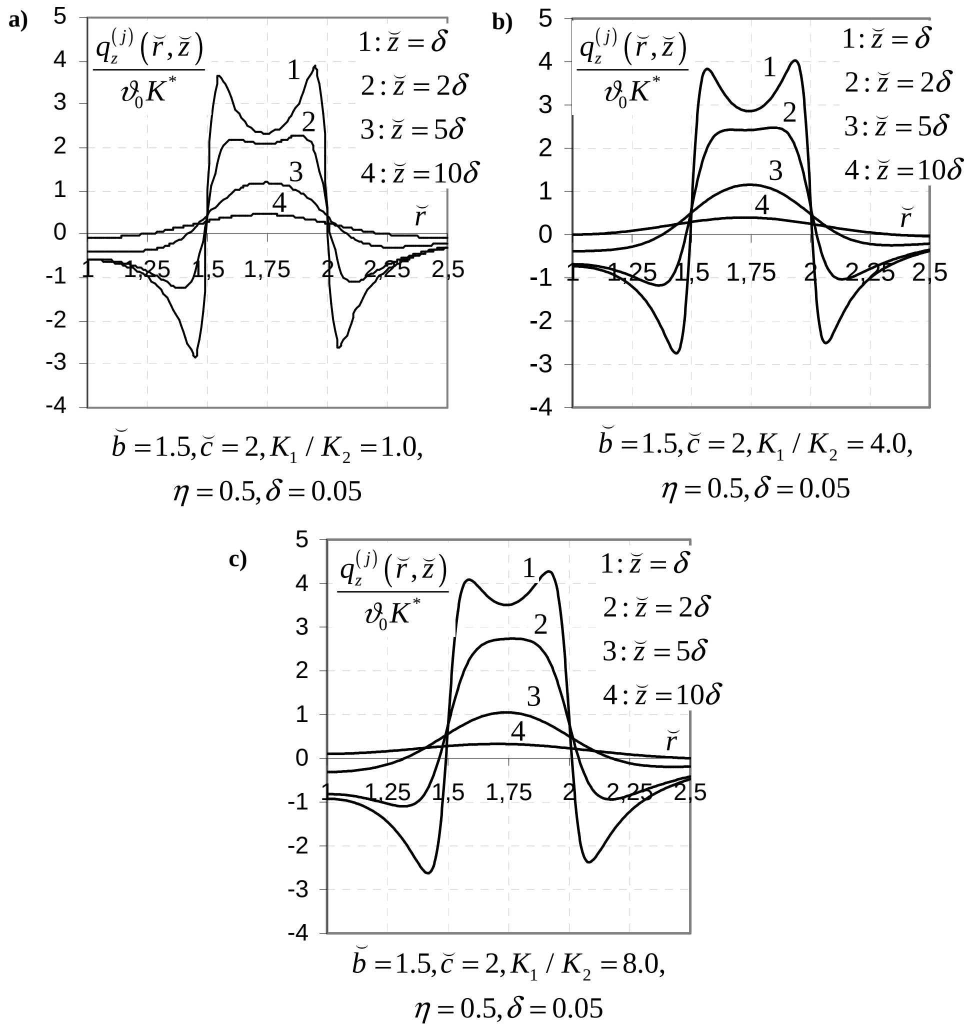


Fig. 6. The dimensionless heat flux $q_z^{(j)}/\left(\vartheta_0 K^*\right)$ θ/ϑ_0 for Problem (1) and $\breve{b}=1.5$, $\breve{c}=2.0$, $\eta=0.5$, $\delta=0.05$; Fig. 6a for $K_1/K_2=1.0$, Fig. 6b for $K_1/K_2=4.0$, Fig. 6c for $K_1/K_2=8.0$, and the dimensionless depths $\breve{z}=\delta;\ 2\delta;\ 5\delta;\ 10\delta$

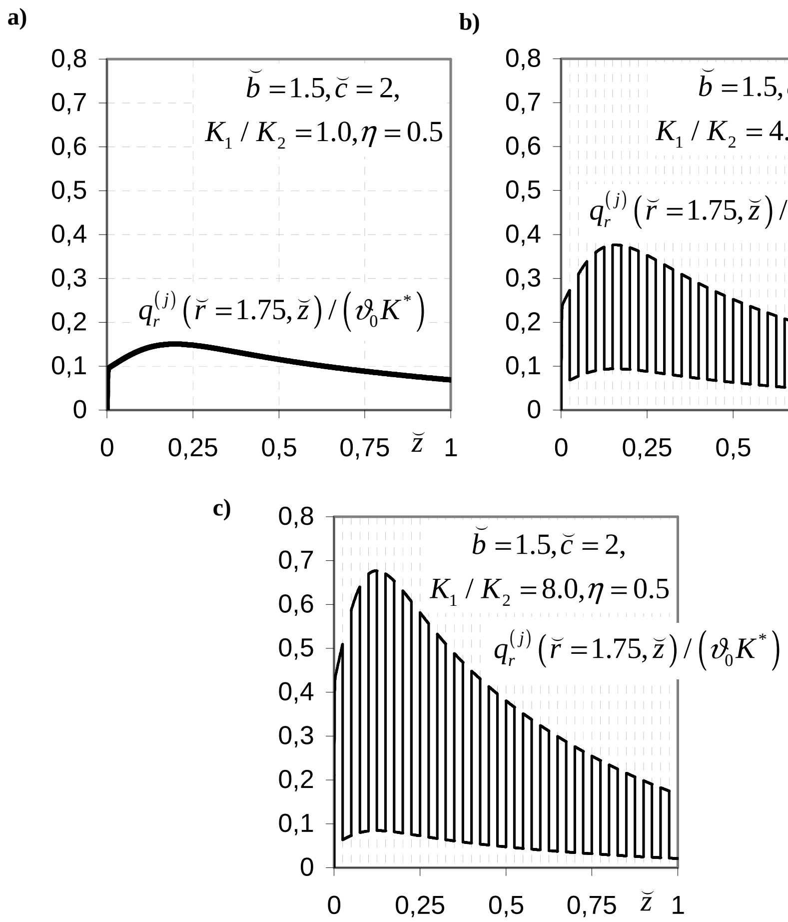


Fig. 7. The dimensionless radial heat flux $q_r^{(j)} / \left(\vartheta_0 K^*\right)$ for Problem (2) and $\breve{r} = 1.75$, $\breve{b} = 1.5$, $\breve{c} = 2.0$, $\eta = 0.5$, and Fig 7a - for $K_1 / K_2 = 1.0$, Fig 7b - for $K_1 / K_2 = 4.0$, Fig 7b - for $K_1 / K_2 = 8.0$

Final remarks

The obtained analytical solutions for temperature and heat flux distributions in the periodically two-layered half-space with cylindrical hole allow on the numerical analysis of influence of the composite structure on the thermal fields. The results presented in figures showed that the layered structure and thermal properties of the composite constituents have an essential influence on the radial heat fluxes for both considered problems.

References

[1] Matysiak S.J., Woźniak C., On the modelling of heat conduction problem in laminated bodies, Acta Mechanica 1986, 65, 223-238.

[2] Woźniak C., A nonstandard method of modelling of thermoelastic periodic composites, International Journal of Engineering Science 1987, 25, 483-499.

[3] Matysiak S.J., On the microlocal parameter method in modelling of periodically layered thermoelastic composites, Journal of Theoretical and Applied Mechanics 1995, 33, 481-487.

[4] Woźniak C., Woźniak M., Modelowanie w dynamice kompozytów, Teoria i zastosowanie, Prace IPPT PAN 1995, 25.

[5] Titchmarsh E.C., Weber's integral theorem, Proceedings of the London Mathematical Society 1923, 22, 15-28.

[6] Titchmarsh E.C., Eigenfunction expansion associated with the second order differential equations, Clarendon Press, Oxford 1946.

[7] Olesiak Z., A note on the extended Weber transforms, [in:] Topics in contemporary mechanics, CISM Courses and Lectures, No. 210 Springer, Udine 1974.

[8] Krajewski J., Olesiak Z., Associated Weber Integral Transforms of $W_{\nu-1,\nu}[;]$ and $W_{2-\nu,\nu}[;]$ Types, Bulletin De L'Academie Polonaise Des Sciences, Serie des sciences techniques 1982, 30, 31-37.

[9] Matysiak S.J., Konieczny S., Yevtushenko A., Distribution of friction heat during cold - rolling of metals by using composite rolls, Numerical Heat Transfer 1998, Part A, 34, 719-729.

[10] Matysiak S.J., Yevtushenko A.A., Ivanyk E., Temperature field in a microperiodic two - layered composite caused by a circular laser heat source, Heat and Mass Transfer 1998, 34127-133.

[11] Yevtushenko A.A., Matysiak S.J., Ivanyk E., Influence of periodically layered material structure on the frictional temperature during braking, International Journal of Heat and Mass Transfer 1997, 40, 2115-2122.

[12] Kulchytsky-Zhyhailo R., Matysiak S.J., On some heat conduction problem in a periodically two-layered body: Comparative results, International Communications in Heat and Mass Transfer 2005, 32 332-340.

[13] Gradshteyn I.S., Ryzhik I.M., Tables of integrals, series and products, Academic Press, New York 1965.