INVESTIGATION OF HM-NETWORK WITH UNRELIABLE QUEUEING SYSTEMS AND RANDOM INCOMES

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Abstract. Expressions for expected incomes and variances in systems of HM (Howard--Matalytski)-queueing network are obtained. Queueing systems are unreliable, service channels in them are exposed to random failure. It is supposed, that service rate of messages, rate of work of serviceable channels and restoration rate of faulty channels depends on messages number in these systems. The case when incomes from transitions between network's states are random variables with the given moments of first two orders is thus considered.

Introduction

In [1-4] expressions for expected incomes and variances of incomes in systems queueing networks of arbitrary structure with service disciplines FIFO have been received. The incomes of transitions between network's states are random variables (RV) with specified moments of the first and the second orders. In [5, 6] considered HM-queueing network (QN) with unreliable queueing system (QS) in a case when incomes of transitions between their states are time-dependent determined functions.

Let us examine open exponential QN with one type messages which consist of n queueing systems (QS) $S_1, S_2, \ldots, S_n$. The Poisson flow of one type messages with arrival rate λ comes into network. Let the system S_i will consist of m_i identical service channels, the service time in each of which has exponential distribution with parameter $\mu_i(k_i)$, where k_i - messages number in this system, $i = \overline{1,n}$.

Let's suppose, that service channels of system S_0 (outside) are absolutely reliable. At the other QS $S_1, S_2, \ldots, S_n$ service channels are exposed to random failure and serviceable work time of each channel of system S_i has exponential distribution with parameter $\beta_i(k_i)$. After failure the service channel immediately starts to be restored and restoration time also has exponential distribution with parameter $\gamma_i(k_i)$, $i = \overline{1,n}$. Let's consider, that service times of messages, durations of serviceable work of channels and restoration time of service channels are independent random variables. State of such network could be described via vector $Z(t)=(d(t),k(t))=(d_1,d_2,\ldots,d_n,k_1,k_2,\ldots,k_n,t)$, where $d_i(t)$ - number of serviceable channels in system S_i at the moment t, $0\le d_i(t)\le m_i$, k_i - messages number in system S_i at the moment t, $t\in[0,+\infty)$, $i=\overline{1,n}$. Let p_{0j} - probability of message enter from outside to the system S_j, $\sum_{j=1}^{n}p_{0j}=1$; p_{ij}- probability of message transition from system S_i to the system S_j, $\sum_{j=0}^{n}p_{ij}=1$, $i=\overline{1,n}$. Matrix $P=\|p_{ij}\|_{(n+1)\times(n+1)}$ is matrix of passage probabilities of irreducible Markovian chains. Service rate of message occurs according to discipline FIFO.

In the given article the approximating expressions for expected (mean) incomes of QS at the any moment t are discovered. Provided that values of these incomes in initial moment of time are known.

1. Expected incomes of network's systems

Let us consider dynamics of income changes of some network system S_i. Let at the initial moment income of this system equal v_{i0}. Denote income at the moment t as $V_i(t)$. Dividing time interval $[0,t]$ on m equal parts in length $\Delta t=\frac{t}{m}$, suppose that m is large. Let $\Delta V_{il}(\Delta t)$ represents of income change of system S_i an l-th time interval lenght Δt, $l=\overline{1,m}$. For determination income $V_i(t)$ of system S_i we will write probabilities of those events which can appear on time interval l. Following situations are possible.

1) With probability

$$1-\left\{\lambda+\sum_{j=1}^{n}\left[\mu_j(k_j(l))+\beta_j(k_j(l))+\gamma_j(k_j(l))\right]u(k_j(l))\right\}\Delta t+o(\Delta t)$$

state changes of system S_i will not appear, system S_i increases its income by value $r_i\Delta t$ at the expense of percents on the money r which are in it. Let also suppose, that r_i is RV with distribution function (d.f.) $F_i(x)$, $i=\overline{1,n}$; $k_i(l)$ - message number in i-th QS on l-th time iterval, $u(x)=\begin{cases}1, & x>0\\ 0, & x\le 0\end{cases}$ - Heaviside function.

2) With probability

$$\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{i0}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{j=1\\ j\neq i}}^{n}\left[\mu_j\left(k_j(l)\right)+\beta_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)+\right.\right.$$

$$\left.\left.+\left[\beta_i\left(k_i(l)\right)+\gamma_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

message becomes processed in system S_i and will pass to outside, income of system S_i will decrease by value R_{i0}, where R_{i0} - RV with d.f. $F_{i0}(x)$, $i=\overline{1,n}$.

3) With probability

$$\left[\lambda p_{0i}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda\sum_{\substack{j=1\\ j\neq i}}^{n}p_{0j}+\sum_{j=1}^{n}\left[\mu_j\left(k_j(l)\right)+\beta_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

message from outside will enter to system S_i and will bring to it income of size r_{0i}, where r_{0i} - RV with d.f. $F_{0i}(x)$, $i=\overline{1,n}$.

4) With probability

$$\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{ij}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{c=1\\ c\neq i}}^{n}\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)+\gamma_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\right.\right.$$

$$\left.\left.+\left[\beta_i\left(k_i(l)\right)+\gamma_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

message from the system S_i will pass to the system S_j, income of *i*-th QS change on $\Delta V_{il}(\Delta t)=-R_{ij}+r_i\Delta t$, where R_{ij} - RV with d.f. $F_{1ij}(x)$, $i,j=\overline{1,n}$, $i\neq j$.

5) With probability

$$\left[\mu_j\left(k_j(l)\right)u\left(k_j(l)\right)p_{ji}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{c=1\\c\neq j}}^{n}\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)+\gamma_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\right.\right.$$

$$\left.\left.+\left[\beta_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

message from the system S_j will pass to the system S_i, income of *i*-th system will change on $\Delta V_{il}(\Delta t)=r_{ji}+r_i\Delta t$, where r_{ji} - RV with d.f. $F_{2ji}(x)$, $i,j=\overline{1,n}$, $i\neq j$. Evidently that $r_{ji}=R_{ji}$ with probability 1, i.e.

$$F_{1ij}(x)=F_{2ij}(x),\ i,j=\overline{1,n},\ i\neq j \tag{1}$$

6) With probability

$$\left[\beta_j\left(k_j(l)\right)u\left(k_j(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{c=1\\c\neq j}}^{n}\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)+\gamma_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\right.\right.$$

$$\left.\left.+\left[\mu_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right]\Delta t+o(\Delta t)\right\}$$

number of serviceable channels in system S_j will decrease, income of *i*-th QS change on $\Delta V_{il}(\Delta t)=r_i\Delta t$, $i,j=\overline{1,n}$.

7) With probability

$$\left[\beta_j\left(k_j(l)\right)u\left(k_j(l)\right)\Delta t+o(\Delta t)\right]\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{i0}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{c=1\\c\neq i,j}}^{n}\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)+\gamma_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\right.\right.$$

$$+\left[\mu_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)+$$

$$\left.\left.+\left[\beta_i\left(k_i(l)\right)+\gamma_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

number of serviceable channels in system S_j will decrease, message becomes processed in system S_i and will pass to outside, then $\Delta V_{il}(\Delta t) = -R_{i0} + r_i \Delta t$, $i, j = \overline{1,n}$.

8) With probability

$$\left[\beta_j\left(k_j(l)\right)u\left(k_j(l)\right)\Delta t + o(\Delta t)\right]\left[\lambda p_{0i}\Delta t + o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda\sum_{\substack{s=1\\s\neq i}}^{n} p_{0s} + \sum_{\substack{s=1\\s\neq j}}^{n}\left[\mu_s\left(k_s(l)\right)+\beta_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)+\right.\right.$$

$$\left.\left.+\left[\mu_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right\}\Delta t + o(\Delta t)\right]$$

number of serviceable channels in system S_j will decrease, message from outside will enter to system S_i; $\Delta V_{il}(\Delta t) = r_{0i} + r_i \Delta t$, $i, j = \overline{1,n}$.

9) With probability

$$\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t + o(\Delta t)\right]\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{ij}\Delta t + o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq i,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)+$$

$$\left.\left.+\left[\beta_i\left(k_i(l)\right)+\gamma_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\right\}\Delta t + o(\Delta t)\right]$$

number of serviceable channels in system S_s will decrease, message from the system S_i will pass to the system S_j; $\Delta V_{il}(\Delta t) = -R_{ij} + r_i \Delta t$, $i, j = \overline{1,n}$, $i \neq j$.

10) With probability

$$\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t + o(\Delta t)\right]\left[\mu_j\left(k_j(l)\right)u\left(k_j(l)\right)p_{ji}\Delta t + o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq j,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)+$$

$$\left.\left.+\left[\beta_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

number of serviceable channels in system S_s will decrease, message from the system S_j will pass to the system S_i; $\Delta V_{il}(\Delta t)=r_{ji}+r_i\Delta t$, $i,j=\overline{1,n}$, $i\neq j$.

11) With probability

$$\left[\gamma_j\left(k_j(l)\right)u\left(k_j(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{c=1\\c\neq j}}^{n}\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)+\gamma_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\right.\right.$$

$$\left.\left.+\left[\mu_j\left(k_j(l)\right)+\beta_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

number of serviceable channels in system S_j will increase and $\Delta V_{il}(\Delta t)=r_i\Delta t$, $i,j=\overline{1,n}$, $i\neq j$.

12) With probability

$$\left[\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{c=1\\c\neq i}}^{n}\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)+\gamma_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\right.\right.$$

$$\left.\left.+\left[\mu_i\left(k_i(l)\right)+\beta_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored, income decrease on $\Delta V_{il}(\Delta t)=-g_i+r_i\Delta t$, where g_i - RV with d.f. $H_i(x)$, $i=\overline{1,n}$. In this case g_i it's payment for restoration of service channel in system S_i.

13) With probability

$$\left[\gamma_j\left(k_j(l)\right)u\left(k_j(l)\right)\Delta t+o(\Delta t)\right]\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{i0}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq i,j}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$+\left[\mu_j\left(k_j(l)\right)+\beta_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)+$$

$$\left.\left.+\left[\beta_i\left(k_i(l)\right)+\gamma_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_j will be restored, message becomes processed in system S_i and will pass to outside; $\Delta V_{il}(\Delta t)=-R_{i0}+r_i\Delta t$, $i,j=\overline{1,n}$, $i\neq j$.

14) With probability

$$\left[\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Delta t+o(\Delta t)\right]\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{i0}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq i}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$\left.\left.+\beta_i\left(k_i(l)\right)u\left(k_i(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored, message becomes processed in this system and will pass to outside; $\Delta V_{il}(\Delta t)=-g_i-R_{i0}+r_i\Delta t$, $i=\overline{1,n}$.

15) With probability

$$\left[\gamma_j\left(k_j(l)\right)u\left(k_j(l)\right)\Delta t+o(\Delta t)\right]\left[\lambda p_{0i}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda\sum_{\substack{s=1\\s\neq i}}^{n}p_{0s}+\sum_{\substack{s=1\\s\neq j}}^{n}\left[\mu_s\left(k_s(l)\right)+\beta_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)+\right.\right.$$

$$\left.\left.+\left[\mu_j\left(k_j(l)\right)+\beta_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_j will be restored, message from outside will enter to system S_i; $\Delta V_{il}(\Delta t)=r_{0i}+r_i\Delta t$, $i,j=\overline{1,n}$, $i\neq j$.

16) With probability

$$\left[\gamma_i(k_i(l))u(k_i(l))\Delta t+o(\Delta t)\right]\left[\lambda p_{0i}\Delta t+o(\Delta t)\right]\times$$
$$\times\left[1-\left\{\lambda\sum_{\substack{s=1\\ s\neq i}}^{n}p_{0s}+\sum_{\substack{s=1\\ s\neq i}}^{n}\left[\mu_s(k_s(l))+\beta_s(k_s(l))+\gamma_s(k_s(l))\right]u(k_s(l))+\right.\right.$$
$$\left.\left.+\left[\mu_i(k_i(l))+\beta_i(k_i(l))\right]u(k_i(l))\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored and message from outside will enter to this system; $\Delta V_{il}(\Delta t)=-g_i+r_{0i}+r_i\Delta t$, $i=\overline{1,n}$.

17) With probability

$$\left[\gamma_s(k_s(l))u(k_s(l))\Delta t+o(\Delta t)\right]\left[\mu_i(k_i(l))u(k_i(l))p_{ij}\Delta t+o(\Delta t)\right]\times$$
$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\ q\neq i,s}}^{n}\left[\mu_q(k_q(l))+\beta_q(k_q(l))+\gamma_q(k_q(l))\right]u(k_q(l))+\right.\right.$$
$$+\left[\mu_s(k_s(l))+\beta_s(k_s(l))\right]u(k_s(l))+$$
$$\left.\left.+\left[\beta_i(k_i(l))+\gamma_i(k_i(l))\right]u(k_i(l))\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_s will be restored, message from the system S_i will pass to the system S_j, income of i-th QS change on $\Delta V_{il}(\Delta t)=-R_{ij}+r_i\Delta t$, $i,j,s=\overline{1,n}$, $i\neq s$, $i\neq j$.

18) With probability

$$\left[\gamma_i(k_i(l))u(k_i(l))\Delta t+o(\Delta t)\right]\left[\mu_i(k_i(l))u(k_i(l))p_{ij}\Delta t+o(\Delta t)\right]\times$$
$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\ q\neq i}}^{n}\left[\mu_q(k_q(l))+\beta_q(k_q(l))+\gamma_q(k_q(l))\right]u(k_q(l))+\right.\right.$$
$$\left.\left.+\beta_i(k_i(l))u(k_i(l))\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored, message from the system S_i will pass to the system S_j, income of i-th system change on $\Delta V_{il}(\Delta t) = -g_i - R_{ij} + r_i \Delta t$, $i, j = \overline{1,n}$, $i \neq j$.

19) With probability

$$\left[\gamma_s(k_s(l))u(k_s(l))\Delta t + o(\Delta t)\right]\left[\mu_j(k_j(l))u(k_j(l))p_{ji}\Delta t + o(\Delta t)\right]\times$$
$$\times\left[1 - \left\{\lambda + \sum_{\substack{q=1 \\ q\neq j,s}}^{n}\left[\mu_q(k_q(l)) + \beta_q(k_q(l)) + \gamma_q(k_q(l))\right]u(k_q(l)) + \right.\right.$$
$$+\left[\mu_s(k_s(l)) + \beta_s(k_s(l))\right]u(k_s(l)) +$$
$$\left.\left. +\left[\beta_j(k_j(l)) + \gamma_j(k_j(l))\right]u(k_j(l))\right\}\Delta t + o(\Delta t)\right]$$

service channel in system S_s will be restored, message from the system S_j will pass to the system S_i, income of i-th system change on $\Delta V_{il}(\Delta t) = r_{ji} + r_i \Delta t$, $i, j, s = \overline{1,n}$, $i \neq s$, $i \neq j$.

20) With probability

$$\left[\gamma_i(k_i(l))u(k_i(l))\Delta t + o(\Delta t)\right]\left[\mu_j(k_j(l))u(k_j(l))p_{ji}\Delta t + o(\Delta t)\right]\times$$
$$\times\left[1 - \left\{\lambda + \sum_{\substack{q=1 \\ q\neq i,j}}^{n}\left[\mu_q(k_q(l)) + \beta_q(k_q(l)) + \gamma_q(k_q(l))\right]u(k_q(l)) + \right.\right.$$
$$+\left[\mu_i(k_i(l)) + \beta_i(k_i(l))\right]u(k_i(l)) +$$
$$\left.\left. +\left[\beta_j(k_j(l)) + \gamma_j(k_j(l))\right]u(k_j(l))\right\}\Delta t + o(\Delta t)\right]$$

service channel in system S_i will be restored, message from the system S_j will pass to the system S_i; $\Delta V_{il}(\Delta t) = -g_i + r_{ji} + r_i \Delta t$, $i, j = \overline{1,n}$, $i \neq j$.

21) With probability

$$\left[\gamma_c(k_c(l))u(k_c(l))\Delta t + o(\Delta t)\right]\left[\beta_s(k_s(l))u(k_s(l))\Delta t + o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq c,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$
$$+\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+$$
$$\left.\left.+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_c will be restored, number of serviceable channels in system S_s will decrease, state changes of system S_i will not appear. Income of *i*-th system S_i change on $\Delta V_{il}(\Delta t)=r_i\Delta t$, $i,c,s=\overline{1,n}$, $i\neq c$.

22) With probability

$$\left[\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$
$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq i,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$
$$+\left[\mu_i\left(k_i(l)\right)+\beta_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)+$$
$$\left.\left.+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored, number of serviceable channels in system S_s will decrease, state changes of system S_i will not appear; $\Delta V_{il}(\Delta t)=-g_i+r_i\Delta t$, $i,s=\overline{1,n}$.

23) With probability

$$\left[\gamma_c\left(k_c(l)\right)u\left(k_c(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$
$$\times\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{i0}\Delta t+o(\Delta t)\right]\times$$
$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq i,c,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$
$$+\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)+$$

$$+\left[\beta_i\left(k_i(l)\right)+\gamma_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\Bigg\}\Delta t+o(\Delta t)\Bigg]$$

service channel in system S_c will be restored, number of serviceable channels in system S_s will decrease, message becomes processed in system S_i and will pass to outside; $\Delta V_{il}(\Delta t)=-R_{i0}+r_i\Delta t$, $i,c,s=\overline{1,n}$, $i\neq c$.

24) With probability

$$\left[\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{i0}\Delta t+o(\Delta t)\right]\times$$

$$\times\Bigg[1-\Bigg\{\lambda+\sum_{\substack{q=1\\q\neq i,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+$$

$$+\beta_i\left(k_i(l)\right)u\left(k_i(l)\right)+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)\Bigg\}\Delta t+o(\Delta t)\Bigg]$$

service channel in system S_i will be restored, number of serviceable channels in system S_s will decrease, message becomes processed in system S_i and will pass to outside; $\Delta V_{il}(\Delta t)=-g_i-R_{i0}+r_i\Delta t$, $i,s=\overline{1,n}$.

25) With probability

$$\left[\gamma_c\left(k_c(l)\right)u\left(k_c(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[\lambda p_{0i}\Delta t+o(\Delta t)\right]\Bigg[1-\Bigg\{\lambda\sum_{\substack{q=1\\q\neq i}}^{n}p_{0q}+\sum_{\substack{q=1\\q\neq c,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\right.$$

$$\left.+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+$$

$$+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)\Bigg\}\Delta t+o(\Delta t)\Bigg]$$

service channel in system S_c will be restored, number of serviceable channels in system S_s will decrease, message from outside will enter to system S_i; $\Delta V_{il}(\Delta t)=r_{0i}+r_i\Delta t$, $i,c,s=\overline{1,n}$, $i\neq c$.

26) With probability

$$\left[\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[\lambda p_{0i}\Delta t+o(\Delta t)\right]\left[1-\left\{\lambda\sum_{\substack{q=1\\q\neq i}}^{n}p_{0q}+\sum_{\substack{q=1\\q\neq c,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\right.\right.\right.$$

$$\left.\left.\left.+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored, number of serviceable channels in system S_s will decrease, message from outside will enter to system S_i; $\Delta V_{il}(\Delta t)=-g_i+r_{0i}+r_i\Delta t$, $i,s=\overline{1,n}$.

27) With probability

$$\left[\gamma_c\left(k_c(l)\right)u\left(k_c(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{ij}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq i,c,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$+\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)+$$

$$\left.\left.+\left[\beta_i\left(k_i(l)\right)+\gamma_i\left(k_i(l)\right)\right]u\left(k_i(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_c will be restored, number of serviceable channels in system S_s will decrease, message from the system S_i will pass to system S_j, income of i-th QS change on $\Delta V_{il}(\Delta t)=-R_{ij}+r_i\Delta t$, $i,j,c,s=\overline{1,n}$, $i\neq j$, $i\neq c$.

28) With probability

$$\left[\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{ij}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq i,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$\left.\left.+\beta_i\left(k_i(l)\right)u\left(k_i(l)\right)+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored, number of serviceable channels in system S_s will decrease. Message from the system S_i will pass to system S_j; $\Delta V_{il}(\Delta t)=-g_i-R_{ij}+r_i\Delta t$, $i,j,s=\overline{1,n}$, $i\neq j$.

29) With probability

$$\left[\gamma_c\left(k_c(l)\right)u\left(k_c(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[\mu_j\left(k_j(l)\right)u\left(k_j(l)\right)p_{ji}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq j,c,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$+\left[\mu_c\left(k_c(l)\right)+\beta_c\left(k_c(l)\right)\right]u\left(k_c(l)\right)+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)+$$

$$\left.\left.+\left[\beta_j\left(k_j(l)\right)+\gamma_j\left(k_j(l)\right)\right]u\left(k_j(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_c will be restored, number of serviceable channels in system S_s will decrease. Message from the system S_j will pass to system S_i; $\Delta V_{il}(\Delta t)=-r_{ji}+r_i\Delta t$, $i,j,c,s=\overline{1,n}$, $i\neq j$, $i\neq c$.

30) With probability

$$\left[\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Delta t+o(\Delta t)\right]\left[\beta_s\left(k_s(l)\right)u\left(k_s(l)\right)\Delta t+o(\Delta t)\right]\times$$

$$\times\left[\mu_j\left(k_j(l)\right)u\left(k_j(l)\right)p_{ji}\Delta t+o(\Delta t)\right]\times$$

$$\times\left[1-\left\{\lambda+\sum_{\substack{q=1\\q\neq j,s}}^{n}\left[\mu_q\left(k_q(l)\right)+\beta_q\left(k_q(l)\right)+\gamma_q\left(k_q(l)\right)\right]u\left(k_q(l)\right)+\right.\right.$$

$$\left. \left. +\beta_j\left(k_j(l)\right)u\left(k_j(l)\right)+\left[\mu_s\left(k_s(l)\right)+\gamma_s\left(k_s(l)\right)\right]u\left(k_s(l)\right)\right\}\Delta t+o(\Delta t)\right]$$

service channel in system S_i will be restored, number of serviceable channels in system S_s will decrease. Message from the system S_j will pass to system S_i; $\Delta V_{il}(\Delta t)=-g_i+r_{ji}+r_i\Delta t$, $i,j,s=\overline{1,n}$, $i\neq j$.

Let also suppose that RV R_{i0}, r_{0i}, R_{ij}, r_{ji}, g_i are independent with respect to RV r_i, $i,j=\overline{1,n}$, $i\neq j$.

Income of queueing system S_i can be presented as

$$V_i(t)=v_{i0}+\sum_{l=1}^{m}\Delta V_{il}(\Delta t)$$

Let us introduce denotations for proper mathematical expectations (m.e.):

$$M\left\{R_{ij}\right\}=\int_0^\infty xdF_{1ij}(x)=b_{ij}\ ,\ M\left\{r_{ji}\right\}=\int_0^\infty xdF_{2ji}(x)=a_{ji}\ ,\ i,j=\overline{1,n}\ ,$$

$$M\left\{r_i\right\}=\int_0^\infty xdF_i(x)=c_i\ ,\ M\left\{R_{i0}\right\}=\int_0^\infty xdF_{i0}(x)=b_{i0}\ ,$$

$$M\left\{r_{0i}\right\}=\int_0^\infty xdF_{0i}(x)=a_{0i}\ \ M\left\{g_i\right\}=\int_0^\infty xdH_i(x)=h_i\ ,\ i=\overline{1,n} \tag{2}$$

according to equality (1)

$$a_{ji}=b_{ji}\ ,\ i,j=\overline{1,n} \tag{3}$$

Let's finding approximate expression for the expected income of QS S_i at the moment *t*. Under fixed realization of the process $k(t)$, subject to probabilities of events, states of incomes change 1)-30) in QS S_i and denominations (2) we can write:

$$M\left\{\Delta V_{il}(\Delta t)/k(l)\right\}=\left(c_i+\lambda a_{0i}p_{0i}+\sum_{\substack{j=1\\ j\neq i}}^{n}a_{ji}p_{ji}\mu_j\left(k_j(l)\right)u\left(k_j(l)\right)-\right.$$

$$\left.-\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)\sum_{\substack{j=0\\ j\neq i}}^{n}a_{ij}p_{ij}-h_i\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\right)\Delta t+o(\Delta t)\ ,\ i=\overline{1,n} \tag{4}$$

Allow that $m\Delta t = t$ and equality (3), we will have

$$M\left\{V_i(t)/k(t)\right\} = \sum_{l=1}^{m} M\left\{\Delta V_{il}(\Delta t)/k(l)\right\} = v_{i0} + (c_i + \lambda a_{0i} p_{0i})t +$$

$$+\sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ji} p_{ji} \sum_{l=1}^{n} \mu_j(k_j(l)) u(k_j(l))\Delta t - \sum_{\substack{j=0 \\ j\neq i}}^{n} b_{ij} p_{ij} \sum_{l=1}^{m} \mu_i(k_i(l)) u(k_i(l))\Delta t -$$

$$-h_i \sum_{l=1}^{m} \gamma_i(k_i(l)) u(k_i(l))\Delta t + o(\Delta t),\ i=\overline{1,n}$$

With $m \to \infty,\ \Delta t \to 0$

$$\sum_{l=1}^{m} \mu_j(k_j(l)) u(k_j(l))\Delta t \xrightarrow[\Delta t \to 0]{} \int_0^t \mu_j(k_j(x)) u(k_j(x))dx,$$

$$\sum_{l=1}^{m} \gamma_j(k_j(l)) u(k_j(l))\Delta t \xrightarrow[\Delta t \to 0]{} \int_0^t \gamma_j(k_j(x)) u(k_j(x))dx,\ j=\overline{1,n}$$

therefore

$$M\left\{V_i(t)/k(t)\right\} = v_{i0} + (c_i + \lambda a_{0i} p_{0i})t +$$

$$+\sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ji} p_{ji} \int_0^t \mu_j(k_j(x)) u(k_j(x))dx - \sum_{\substack{j=0 \\ j\neq i}}^{n} b_{ij} p_{ij} \int_0^t \mu_i(k_i(x)) u(k_i(x))dx -$$

$$-h_i \int_0^t \gamma_i(k_i(x)) u(k_i(x))dx,\ i=\overline{1,n}$$

Average the last relation by $k(t)$ with taking to account normalization condition $\sum_k P(k(t)=k)=1$, for expected income of system S_i we will obtain

$$v_i(t) = M\left\{V_i(t)\right\} = v_{i0} + \sum_k P(k(t)=k) M\left\{V_i(t)/k(t)\right\} =$$

$$= v_{i0} + (c_i + \lambda a_{0i} p_{0i})t + \sum_k P(k(t)=k) \times$$

$$\times\left\{\sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ji} p_{ji} \int_0^t \mu_j(k_j(x)) u(k_j(x))dx - \sum_{\substack{j=0 \\ j\neq i}}^{n} b_{ij} p_{ij} \int_0^t \mu_i(k_i(x)) u(k_i(x))dx\right\} -$$

$$-h_i \int_0^t \gamma_i(k_i(x)) u(k_i(x))dx,\ i=\overline{1,n}$$

Let at the moment t system S_i contains $d_i(t)$ serviceable channels, $0 \le d_i(t) \le m_i$, service rate of messages μ_i, restoration rate of faulty channels γ_i in it linearly depend on messages number, i.e.

$$\mu_i(k_i(x))u(k_i(x)) = \begin{cases} \mu_i k_i(x), \ k_i(x) \le d_i(x), \\ \mu_i d_i(x), \ k_i(x) > d_i(x), \end{cases} =$$

$$= \mu_i \min(k_i(x), d_i(x)), \ i = \overline{1,n}, \tag{5}$$

$$\gamma_i(k_i(x))u(k_i(x)) = \begin{cases} \gamma_i k_i(x), \ k_i(x) \le (m_i - d_i(x)), \\ \gamma_i(m_i - d_i(x)), \ k_i(x) > (m_i - d_i(x)), \end{cases} =$$

$$= \gamma_i \min(k_i(x), (m_i - d_i(x))) \ \ i = \overline{1,n} \tag{6}$$

Then from (5), (6) follows:

$$v_i(t) = M\{V_i(t)\} = v_{i0} + (c_i + \lambda a_{0i} p_{0i})t +$$

$$+ \sum_{\substack{j=1 \\ j \ne i}}^{n} a_{ji} p_{ji} \int_0^t \mu_j \min(k_j(x), d_j(x)) dx -$$

$$- \sum_{\substack{j=0 \\ j \ne i}}^{n} b_{ij} p_{ij} \int_0^t \mu_i \min(k_i(x), d_i(x)) dx -$$

$$- h_i \int_0^t \gamma_i \min(k_i(x), (m_i - d_i(x))) dx, \ i = \overline{1,n} \tag{7}$$

Let us suppose that averaging of expression $\min(k_i(x), d_i(x))$ brings $\min(N_i(x), \overline{d}_i(x))$, i.e.

$$M\{\min(k_i(x), d_i(x))\} = \min(N_i(x), \overline{d}_i(x)),$$

$$M\{\min(k_i(x), (m_i - d_i(x)))\} = \min(N_i(x), (m_i - \overline{d}_i(x))), \ i = \overline{1,n},$$

where $N_i(x) = M\{k_i(x)\}$ average number of messages (waiting and serving), $\overline{d}_i(x) = M\{d_i(x)\}$ average number of serviceable channels in QS S_i on the time interval $[0,x]$, $i=\overline{1,n}$. Then from (7) we will receive the following approximate expression:

$$v_i(t)=M\{V_i(t)\}=v_{i0}+(c_i+\lambda a_{0i}p_{0i})t+$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{n}\mu_j a_{ji}p_{ji}\int_0^t \min\left(N_j(x),\bar{d}_j(x)\right)dx-\sum_{\substack{j=0\\ j\neq i}}^{n}\mu_i b_{ij}p_{ij}\int_0^t \min\left(N_i(x),\bar{d}_i(x)\right)dx-$$
$$-\gamma_i h_i\int_0^t \min\left(N_i(x),\left(m_i-\bar{d}_i(x)\right)\right)dx,\ i=\overline{1,n} \quad (8)$$

From (3) and

$$\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\mu_j a_{ji}p_{ji}\int_0^t \min\left(N_j(x),\bar{d}_j(x)\right)dx=$$
$$=\sum_{i=1}^{n}\mu_i\int_0^t \min\left(N_i(x),\bar{d}_i(x)\right)dx\sum_{\substack{j=1\\ j\neq i}}^{n}b_{ij}p_j\ ,$$

expected income of queuing network can be written as

$$M\{W(t)\}=\sum_{i=1}^{n}\Bigg[v_{i0}+(c_i+\lambda a_{0i}p_{0i})t-$$
$$-\mu_i b_{i0}p_{i0}\int_0^t \min\left(N_i(x),\bar{d}_i(x)\right)dx-\gamma_i h_i\int_0^t \min\left(N_i(x),\left(m_i-\bar{d}_i(x)\right)\right)dx\Bigg]$$

2. Variances of incomes of network systems

From (3) expression (8) can be written

$$v_i(t)=M\{V_i(t)\}=v_{i0}+(c_i+\lambda a_{0i}p_{0i})t-$$
$$-\mu_i b_{i0}p_{i0}\int_0^t \min\left(N_i(x),\bar{d}_i(x)\right)dx+$$
$$+\sum_{\substack{j=1\\ j\neq i}}^{n}\left[\mu_j a_{ji}p_{ji}\int_0^t \min\left(N_j(x),\bar{d}_j(x)\right)dx-\mu_i a_{ij}p_{ij}\int_0^t \min\left(N_i(x),\bar{d}_i(x)\right)dx\right]-$$
$$-\gamma_i h_i\int_0^t \min\left(N_i(x),\left(m_i-\bar{d}_i(x)\right)\right)dx,\ i=\overline{1,n} \quad (9)$$

Let us introduce denotations

$$M\left\{R_{ij}^2\right\}=b_{2ij}\,,\ M\left\{r_{ji}^2\right\}=a_{2ji}\,,\ i,j=\overline{1,n}\,,$$

$$M\left\{r_i^2\right\}=c_{2i}\,,\ M\left\{R_{i0}^2\right\}=b_{2i0}\,,\ M\left\{r_{0i}^2\right\}=a_{20i}\,,$$

$$M\left\{g_i^2\right\}=h_{2i}\,,\ i=\overline{1,n}\,, \tag{10}$$

and will consider expression

$$M\left\{\left(V_i(t)-v_{i0}\right)^2/k(t)\right\}=M\left\{\left(v_{i0}+\sum_{l=1}^{m}\Delta V_{il}(\Delta t)-v_{i0}\right)^2/k(t)\right\}=$$

$$=M\left\{\left(\sum_{l=1}^{m}\Delta V_{il}(\Delta t)\right)^2/k(t)\right\}=\sum_{l=1}^{m}M\left\{\Delta V_{il}^2(\Delta t)/k(t)\right\}+$$

$$+\sum_{l=1}^{m}\sum_{\substack{q=1\\q\neq l}}^{m}M\left\{\Delta V_{il}(\Delta t)\Delta V_{iq}(\Delta t)/k(t)\right\},\ i=\overline{1,n}$$

Considering incomes and probabilities of state 1)-30), descriptions (10) it is had

$$M\left\{\Delta V_{il}^2(\Delta t)/k(t)\right\}=\Bigg[\lambda a_{20i}p_{0i}+b_{2i0}\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{i0}+$$

$$+\sum_{\substack{j=1\\j\neq i}}^{n}\Big[a_{2ji}\mu_j\left(k_j(l)\right)u\left(k_j(l)\right)p_{ji}+b_{2ij}\mu_i\left(k_i(l)\right)u\left(k_i(l)\right)p_{ij}\Big]+$$

$$+h_{2i}\gamma_i\left(k_i(l)\right)u\left(k_i(l)\right)\Bigg]\Delta t+o(\Delta t),\ i=\overline{1,n}\,. \tag{11}$$

Under fixed realization of the process $k(t)$ values $\Delta V_{il}(\Delta t)$, $\Delta V_{iq}(\Delta t)$ are independent at $l\neq q$. Then using (3), (4) we can write

$$M\left\{\Delta V_{il}(\Delta t)\Delta V_{iq}(\Delta t)/k(t)\right\}=o(\Delta t)^2 \tag{12}$$

With $\Delta t\to 0$ from (11), (12) and $m\Delta t=t$

$$M\left\{\left(V_i(t)-v_{i0}\right)^2/k(t)\right\}=\sum_{l=1}^{m}M\left\{\Delta V_{il}^2(\Delta t)/k(t)\right\}+$$

$$+\sum_{l=1}^{m}\sum_{\substack{q=1\\q\neq l}}^{m}M\left\{\Delta V_{il}(\Delta t)\Delta V_{iq}(\Delta t)/k(t)\right\}=$$

$$= \lambda a_{20i} p_{0i} t + \mu_i b_{2i0} p_{i0} \int_0^t \min\left(k_i(x), \bar{d}_i(x)\right) dx +$$

$$+\sum_{\substack{j=1 \\ j \neq i}}^{n} \left[\mu_j a_{2ji} p_{ji} \int_0^t \min\left(k_j(x), \bar{d}_j(x)\right) dx + \mu_i b_{2ij} p_{ij} \int_0^t \min\left(k_i(x), \bar{d}_i(x)\right) dx \right] +$$

$$+\gamma_i h_{2i} \int_0^t \min\left(k_i(x), \left(m_i - \bar{d}_i(x)\right)\right) dx, \ i = \overline{1,n}$$

Average the receive relation by $k(t)$ we will have

$$M\left\{\left(V_i(t) - v_{i0}\right)^2\right\} = \lambda a_{20i} p_{0i} t + \mu_i b_{2i0} p_{i0} \int_0^t \min\left(N_i(x), \bar{d}_i(x)\right) dx +$$

$$+\sum_{\substack{j=1 \\ j \neq i}}^{n} \left[\mu_j a_{2ji} p_{ji} \int_0^t \min\left(N_j(x), \bar{d}_j(x)\right) dx + \mu_i b_{2ij} p_{ij} \int_0^t \min\left(N_i(x), \bar{d}_i(x)\right) dx \right] +$$

$$+\gamma_i h_{2i} \int_0^t \min\left(N_i(x), \left(m_i - \bar{d}_i(x)\right)\right) dx, \ i = \overline{1,n} \qquad (13)$$

Expression for $M^2\left\{\left(V_i(t) - v_{i0}\right)\right\}$, using (9), looks like

$$M^2\left\{\left(V_i(t) - v_{i0}\right)\right\} = \left\{ \left(c_i + \lambda a_{0i} p_{0i}\right) t - \mu_i b_{i0} p_{i0} \int_0^t \min\left(N_i(x), \bar{d}_i(x)\right) dx + \right.$$

$$+\sum_{\substack{j=1 \\ j \neq i}}^{n} \left[\mu_j a_{ji} p_{ji} \int_0^t \min\left(N_j(x), \bar{d}_j(x)\right) dx - \mu_i b_{ij} p_{ij} \int_0^t \min\left(N_i(x), \bar{d}_i(x)\right) dx \right] -$$

$$\left. -\gamma_i h_i \int_0^t \min\left(N_i(x), \left(m_i - \bar{d}_i(x)\right)\right) dx \right\}^2, \ i = \overline{1,n} \qquad (14)$$

Expression for variance of income of QS S_i can be evaluated, using the formula $DV_i(t) = D\left(V_i(t) - v_{i0}\right) = M\left\{\left(V_i(t) - v_{i0}\right)^2\right\} - M^2\left\{\left(V_i(t) - v_{i0}\right)\right\}$ and (13), (14), $i = \overline{1,n}$.

We will note, for finding of values $N_i(x)$, $\bar{d}_i(x)$, $i = \overline{1,n}$ the method of diffusive approximation can be used. It wil be made in next article.

Conclusions

Thus, in current article the approximate expressions for expected incomes and variances of incomes of exponential HM-network in a case when incomes from transitions between network's states are random variables with the given moments of first two orders are received.

The further investigations in this area are associated with investigation of precision of the received expressions and distribution of the received results on a networks with other specialties.

References

[1] Koluzaeva E.V., Matalytski M.A., Analysis of incomes in open HM-networks of arbitrary architecture, Westnik GrSU 2008, 1, 22-29 (in Russian).

[2] Matalytski M., Koluzaeva K., Analysis and optimization Markov HM-networks with stochastic incomes from transitions between their states, Scientific Research of the Institute of Mathematics and Computer Science, Czestochowa University of Technology 2008, 1(7), 51-62.

[3] Matalytski M.A., Koluzaeva E.V., Investigation of Marcov networks with incomes and different-type messages of many classes by convergence method, Papers of the National Academy of Science of Belarus, Minsk 2008, 4, 113-118 (in Russian).

[4] Matalytski M.A., Koluzaeva E.V., Marcov queueing networks of arbitrary structure with incomes, Papers of the National Academy of Science of Belarus, Minsk 2009, 53(3), 10-17 (in Russian).

[5] Matalytski M.A., Statkevich S.E., Systems of the difference-differential equations for expected incomes of HM-network with unreliable queueing systems, Probability Theory, Mathematical Statistic And Their Application, Minsk 2010, 3, 233-238 (in Russian).

[6] Statkevich S.E., Investigation of HM-network with unreliable queueing systems, Westnik GrSU 2010, 2, 31-40 (in Russian).