MATHEMATICAL MODELING OF THE FILLING PROCESS OF A SLENDER MOULD CAVITY

Leszek Sowa

Institute of Mechanics and Machine Design, Czestochowa University of Technology, Poland
sowa@imipkm.pcz.czest.pl

Abstract. In this paper, three mathematical models of the solidification of a thin-walled casting, which take into account the filling process of the mould cavity with molten metal, have been proposed. In the general model, velocity and pressure fields were obtained by solving the momentum equations and the continuity equation, while the thermal fields were obtained by solving the heat conduction equation containing the convection term. In the simplified models, making assumptions relating to both the material and the geometry of the region, the general equations for momentum and continuity have been reduced to single an equation for pressure. This approach leads to the essential acceleration of fluid flow computations. In this model, coupling of the thermal and fluid flow phenomena by changes in the fluidity function and thermophysical parameters of alloy with respect to temperature has been taken into consideration. The problem has been solved by the finite element method.

Introduction

Casting processes are widely used to produce metal components. Much research has been devoted toward process development for the production of high quality casting goods at low costs. From a macroscopic point of view, casting processes involve the coupling of solidification, heat transfer and fluid flow [1]. The most widely studied area of casting is phase-change heat transfer [2]; natural convection effects and macro/micro segregation [3, 4] have only recently been studied in some detail. Fluid flow analysis during the mould filling process has been studied vigorously in recent decades due to the advent of computer hardware systems. The filling of simple mould geometry has been previously modelled and studies for an effective filling algorithm have also been reported [5].

Liquid metal cools as it flows into the mould. In the case of a thin cast part or a casting of molten metal with low superheat, the molten metal is cooled during filling and it is fully solidified before the mould is completely filled. Hence, it is necessary to simultaneously analyze the mould filling and solidification processes. Most of the previous works have considered only the solidification process after filling [2]. Therefore, at the time of analysis, the numerical simulation of casting solidification, taking into account the molten metal motion and the mould cavity filling process, is very often carried out. Velocity fields are usually obtained by solving the Navier-Stokes equations and the continuity equation, whereas the thermal fields are calculated by solving the Fourier-Kirchhoff equation with the convection term. This is a complex and difficult problem to solve numerically (general model) [1, 6-8]. The analysis of these phenomena is often limited only to their proceedings during the filling process of the cylindrical mould cavity [9] or to the thin plane cavity of the casting mould [6]. Just as for the case of the cavities, geometric considerations allow further simplification of the governing equations. Making assumptions relating to both a material and the geometry of a region, the general equations for continuity and momentum have been reduced to a single equation for pressure (simplified model). This approach leads to the essential acceleration of the numerical calculations [6, 9].

In this study, the simultaneous analysis models of the metal solidification process in the cavity of the casting mould during its filling have been proposed. Mathematical models take into consideration the interdependence of thermal and dynamical phenomena. Coupling of the thermal and fluid flow phenomena by changes in the fluidity function (in simplified models only) and the thermophysical parameters of alloy with respect to temperature has been taken into consideration. The whole task has been solved using the finite element method [2, 5-9].

1. A general mathematical model of metal molten flow in the cavity of the mould

The general mathematical model of metal alloy solidification in the filling cavity of the casting mould has been proposed. It is based on the solution of the following system of differential equations [1, 2, 5-8]:
- the heat conduction equation containing the convection term

$$\rho C_{ef}\left(\frac{\partial T(\mathbf{x},t)}{\partial t}+\nabla T\cdot\mathbf{v}\right)=\nabla\cdot(\lambda\nabla T)+\mu\Phi^2 \tag{1}$$

- the momentum equations

$$\rho\frac{d\mathbf{v}}{dt}=\rho\mathbf{g}-\nabla p+\mu\nabla^2\mathbf{v} \tag{2}$$

- the continuity equation

$$\nabla\cdot\mathbf{v}=0 \tag{3}$$

where T [K] is the temperature, t [s] is the time, $\rho=\rho(T)$ [kg/m^3] is the density, λ [W/(mK)] is the thermal conductivity coefficient, $\mathbf{v}$ [m/s] is the velocity vector of metal molten flow, $\mathbf{x}$ [m] is the position vector, $\mu(T)$ [Ns/m^2] is the dynamical viscosity coefficient, $C_{ef} = c + \dfrac{L}{(T_L - T_S)}$ [J/(kgK)] is the effective specific heat of a mushy zone, L [J/kg] is the latent heat of solidification, c [J/(kgK)] is the specific heat, p [N/m^2] is the pressure, Φ [1/s] is the shear rate, $\mathbf{g}$ [m/s^2] is the vector of the acceleration of gravity.

In the applied model of solid phase growth, the internal heat sources do not become evident in the equation of heat conductivity, because they are in the effective specific heat of the mushy zone [1, 2, 5]. The system of equations (1)-(3) is completed with a boundary and initial conditions for temperature and velocity or pressure fields [1, 2, 5, 6, 9, 10].

The initial conditions for temperature and velocity or pressure fields are given as:

$$T(\mathbf{x},t_0) = T_0(\mathbf{x}), \qquad \mathbf{v}(\mathbf{x},t_0) = \mathbf{v}_0(\mathbf{x}), \qquad p(\mathbf{x},t_0) = p_0(\mathbf{x}) \tag{4}$$

The boundary conditions specified in the considered problem were as follows:
- at the inlet gate

$$T = T_{in}, \qquad v_n = v_{in}, \qquad v_t = 0, \quad \text{or} \quad p = p_{in} \tag{5}$$

- at the mould wall

$$v_n = v_t = 0, \qquad \frac{\partial p}{\partial n} = 0 \tag{6}$$

$$\lambda_M \frac{\partial T_M}{\partial n} = -\alpha_M \left(T_M - T_a\right) \tag{7}$$

- at the flow front

$$T = T_{in}^*, \qquad v_n = v_{in}^*, \quad \text{or} \quad p = 0 \tag{8}$$

- at the cavity centre line

$$\frac{\partial T}{\partial n} = \frac{\partial v_t}{\partial n} = 0, \qquad v_n = 0, \qquad \frac{\partial p}{\partial n} = 0 \tag{9}$$

where T_a [K] is the ambient temperature, T_M [K] is the mould temperature, α_M [W/(m^{2}K)] is the heat-transfer coefficient between a mould and ambient, $\boldsymbol{n}$ is the outward unit normal surface vector.

2. A simplified mathematical model of metal molten flow in the narrow cavity of a mould

In certain coincidences, e.g. concerning the metal molten flow in the thin plane cavity of the casting mould [6], it is quite possible to simplify the mathematical description of the coupled thermal and fluid flow phenomena. This can be done by leaving the lower-order elements of the equation system (1)-(3) which are at least the order of magnitude less than the remaining. To simplify the above equations, we employ a technique called dimensional analysis. Basically the idea is to obtain estimates of the order of magnitudes of each term in the governing equations. Terms of sufficiently low order have little influence on the numerical simulation results and so are neglected. If liquid metal is Newtonian and the flow is laminar, the governing equations for energy, momentum and mass conservation in the Cartesian coordinate system are as follows [2, 6]:

$$C_{ef}\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)+\mu\Phi^2 \qquad (10)$$

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial z}\left(\mu\frac{\partial v_x}{\partial z}\right),\qquad \frac{\partial p}{\partial y}=\frac{\partial}{\partial z}\left(\mu\frac{\partial v_y}{\partial z}\right),\qquad \frac{\partial p}{\partial z}=0 \qquad (11)$$

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0 \qquad (12)$$

where $C_{ef}(T)=\rho_{LS}c_{LS}+\frac{\rho_S L}{T_L-T_S}$ [J/(m^{3}K)] is the effective heat capacity of a mushy zone, c_{LS} [J/(kgK)] is the specific heat of a mushy zone, $\rho_S, \rho_L, \rho_{LS}$ [kg/m^3] are the density of solid phase, liquid phase, and mushy zone, respectively, $\Phi^2=\left(\frac{\partial v_x}{\partial z}\right)^2+\left(\frac{\partial v_y}{\partial z}\right)^2$ [1/s^2] is the shear rate.

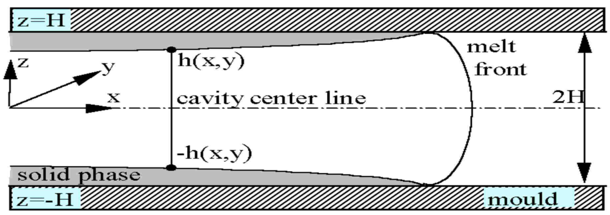

Fig. 1. Schematic diagram of the mould cavity filling

Further simplification is available by integrating the momentum and continuity equations. From the momentum equations (11) we see that the pressure is a function of coordinates only. For this reason it is convenient to integrate the momentum equation across the thin cavity with the aim of obtaining expression for the *x*, *y*-component of velocity (v_x, v_y) as follows

$$v_x(z) = \frac{\partial p}{\partial x}\left[\int_{-h}^{z}\frac{z'}{\mu}dz' - A(x,y)\int_{-h}^{z}\frac{1}{\mu}dz'\right]$$
$$v_y(z) = \frac{\partial p}{\partial y}\left[\int_{-h}^{z}\frac{z'}{\mu}dz' - A(x,y)\int_{-h}^{z}\frac{1}{\mu}dz'\right] \qquad (13)$$

where we have defined the constant

$$A(x,y) = \int_{-h}^{h}\frac{z'}{\mu}dz' \Bigg/ \int_{-h}^{h}\frac{1}{\mu}dz' \qquad (14)$$

where h [m] denotes the half-gap thickness of the cavity.

By integrating the continuity equation (12) over the area of the melt cavity (with respect to *z*) and using the definition of the average velocity (*x*, *y*-component) we obtain the following equation [6]

$$\frac{\partial}{\partial x}\left(S\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(S\frac{\partial p}{\partial y}\right) = 0 \qquad (15)$$

where we have defined

$$S = \int_{0}^{h}\frac{z^2}{\mu}dz \qquad (16)$$

which is often called the fluidity function.

This equation (15) is a single equation for pressure that combines the momentum and continuity equation. A system of equations (10), (15) is completed by the appropriate initial conditions and the boundary conditions (4)-(9).

3. A simplified mathematical model of metal molten flow in the cylindric cavity of a mould

The proposed, the simplified mathematical model of the metals alloy solidification, gives consideration to the motion of the metal liquid phase during the filling process of the cylindric mould and can be limited only to solving the following system of differential equations in a cylindrical axisymmetry coordinate system [2, 6, 9]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)-C_{ef}\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}\right)=-\mu\Phi^2 \tag{17}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v_z}{\partial r}\right)=\frac{\partial p}{\partial z} \tag{18}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_z}{\partial z}=0 \tag{19}$$

where r [m] is the internal radius of the channel, $\mathbf{x}(r, z)$[m] are the coordinates of the vector of the considered node's position, $C_{ef}(T)$ [J/(m^{3}K)] is the effective heat capacity of the mushy zone, $\mathbf{v}(v_r, v_z)$ [m/s] is the velocity vector of molten metal flow, $\Phi=\frac{\partial v_z}{\partial r}$ [1/s] is the shear rate.

Further simplification is available by integrating the momentum and continuity equations. From the momentum equation (18) we find the expression for the z-component of velocity (v_z) as follows

$$v_z(r)=\frac{1}{2}\frac{\partial p}{\partial z}\left[\int_0^r\frac{r'}{\mu}dr'-\int_0^{r_S}\frac{r'}{\mu}dr'\right] \tag{20}$$

where r_S [m] is the radius indicating melt-solid interface.

Integrating of the continuity equation (19) over the area of the melt cavity (with respect to r) and using the definition of the average velocity (z-component) we obtain the following equation [6, 9]

$$\frac{\partial}{\partial z}\left(r_S S_1\frac{\partial p}{\partial z}\right)=0 \tag{21}$$

where we have defined

$$S_1=\frac{1}{2r_S}\int_0^{r_S}\frac{r'^3}{\mu}dr' \tag{22}$$

that is called the fluidity function for one dimensional flow.

This equation (21) is a single equation for pressure that combines the momentum and continuity equations. After making an assumption regarding the material and using the effect of geometry, equations governing the flow of the molten metal in the mould of circular cross-sections may take the form of the system of equations (17, 21) that is completed by the appropriate initial conditions and the boundary conditions (4)-(9). The above problem was solved by the finite element method in the weighted residuals formulation [2, 5-10].

4. Example of numerical calculations

Calculations were made for the system casting-mould-ambient. The given dimensions of the essential elements of that system were as follows: d = 0.017, d_M = 0.067, h = 0.135 m, δ = 0.15 mm [9]. The numerical calculations were made for Al-4.5% Cu alloy which poured into a cast iron mould. The thermophysical properties were taken from works [2, 10-12]. The linear change of density (ρ) and thermal conductivity (λ) were assumed in $T_L \div T_S$ temperature interval. The variability of the dynamical viscosity coefficient (μ) with respect to temperature was determined according to the relationship in [12].

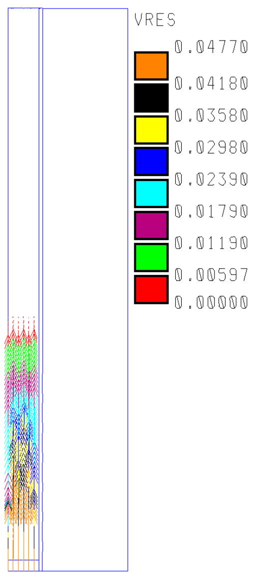

Fig. 2. Velocity vectors during the mould cavity filling (general model)

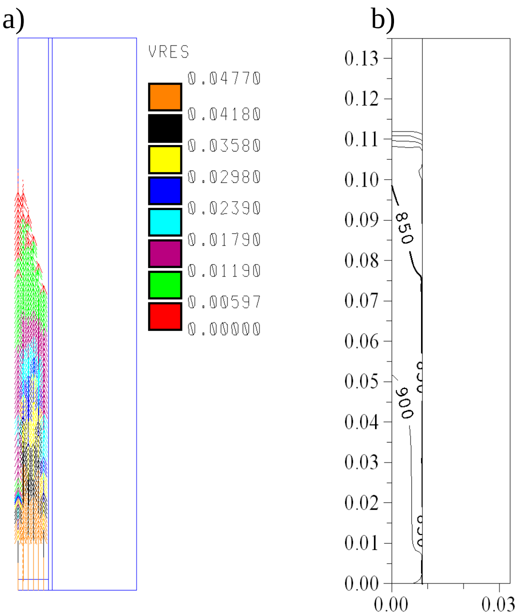

Fig. 3. Velocity vectors (a) and temperature field (b) after the stopping of the metal molten flow (general model)

The overheated metal with temperature T_{in} = 1003 K was poured with velocity v_{in} = 0.048 m/s or pressure p_{in} = 1 N/m^2 into the mould with the initial temperature T_M = 423 K. The remaining characteristic temperatures were equal to: T_L = 913 K, T_S = 850 K and T_a = 300 K. Thermal and flow phenomena, which proceeded in the mould cavity during filling until total solidification of the casting, were analysed by the general model (Figs. 2 and 3) and the simplified model (Fig. 4). The influences of liquid metal movement inside the mould on the solidification kinetics of the casting were determined. Examples of calculation results are shown in the form of the temperature and velocity fields.

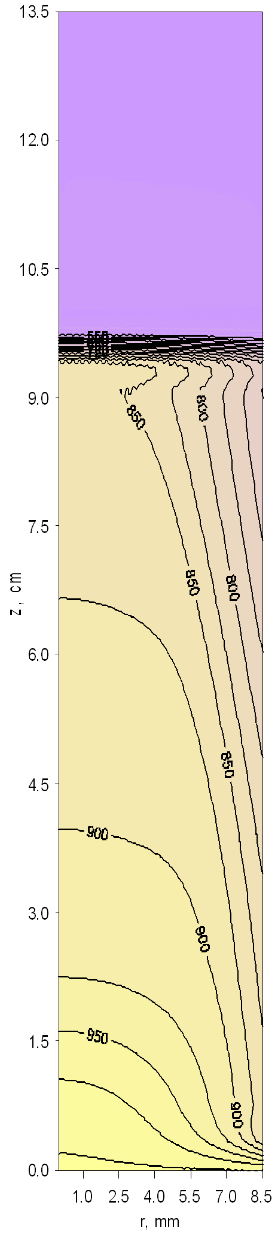

Fig. 4. Temperature field after the stopping of the metal molten flow (simplified model)

Conclusions

This paper presents the coupled model of solidification for the transient evaluation of fluid flow and heat transfer during casting solidification processes. The changes in the thermophysical parameters, with respect to temperature, were taken into consideration. Numerical analysis included the filling process of the mould cavity with molten metal, fluid flow, convectional motions of molten metal and the solidification process. The influence of velocity or pressure and the temperature of metal pouring on the solid phase growth kinetics, as well as the moment of stopping the fluid flow, were estimated. It was noticed that the distance of alloy flowing in the slender mould cavity, depends more on the speed or the pressure of pouring than on the temperature of pouring.

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