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INFLUENCE OF CRACK LOCATION ON DIVERGENCE AND FLUTTER INSTABILITY OF A COLUMN SUBJECTED TO GENERALIZED LOAD

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Abstract. In this paper, the results of numerical studies on the divergence and flutter instability and vibration of a geometrically nonlinear column subjected to generalized load are presented. The system is loaded by axially applied external force P. The direction of action of the force is dependent on follower factor η. The Hamilton principle was used to formulate the boundary problem. Due to the geometric nonlinearity, the solution to the problem was performed by means of the perturbation method. The main purpose of this paper is to investigate the influence of the location of the crack on divergence and flutter loading as well as natural vibration frequency. The presented results of numerical calculations also concern the influence of rotational spring stiffness and follower factor η on the investigated parameters.

Introduction

The study on natural vibration, divergence and flutter instability of geometrically nonlinear slender systems subjected to generalized loading have been the subject of numerous scientific investigations. The first papers in this field already appeared in the 1960s. Among others, the influence of the follower factor, asymmetry of the bending rigidity coefficient and stiffness of the supporting springs in the examined systems on the type of instability, bifurcation (divergence) and critical (flutter) loading were investigated.

In this study, the problem of the natural vibration of a geometrically nonlinear column consisting of three rods with divergence and flutter instability is taken into account. In the investigated system, the first element is a continuous rod and rods two and three are connected by a pin, strengthened by a rotational spring with stiffness C. In the physical system, the pin and the spring can represent the internal crack or connection of two rods made of two different materials. The scientific research of columns with cracks were performed by Kukla [1] and Wang [2].The investigated column is loaded by external force P. The direction of action of the force is dependant on follower factor η. The numerical calculations of divergence and flutter instability were performed by Przybylski [3], and Tomski [4]. The problem of instability and natural vibration has been formulated by means of Hamilton's principle [5]. Due to geometric nonlinearities, the solution to the problem has been performed by use of the small parameter method [6]. The main purpose of this study is to investigate the influence of the location of a crack on the divergence and flutter loading and natural vibration frequency. The presented results of the numerical calculations also concern the influence of the rotational spring stiffness and follower factor η on the investigated parameters.

1. Formulation of the problem

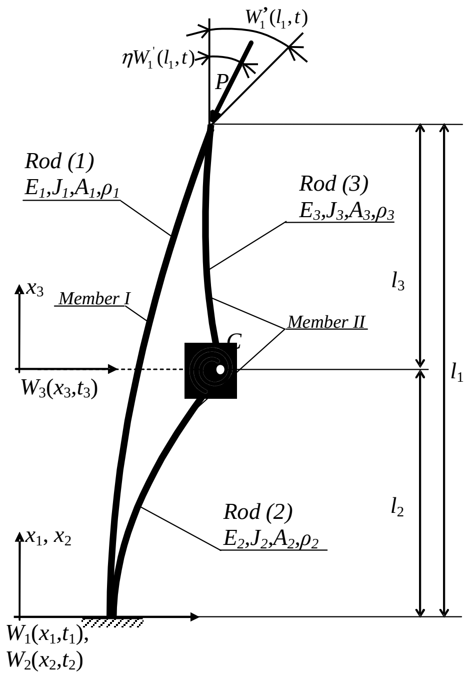


Fig. 1. Nonlinear system under consideration subjected to generalized load

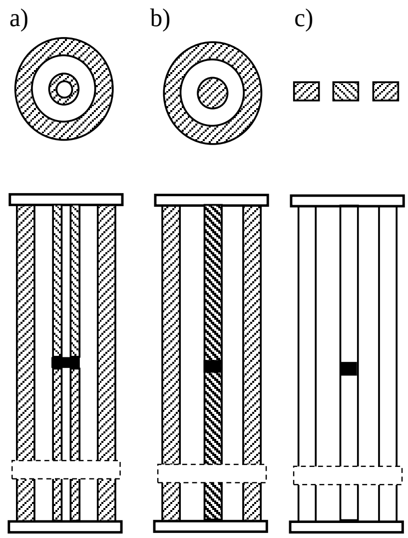


Fig. 2. Exemplary models of real life structures

In Figure 1 the nonlinear cantilever column under investigation is presented. Member *I* consists of rod (1), and member *II* elements are rods (2) and (3) connected by a pin strengthened by a rotational spring of stiffness *C*. The smaller value of *C*, the greater the crack. The investigated system is loaded by a concentrated axially applied force *P* at the point of connection of rods (1) and (3). The deflection angles of these rods are identical. The direction of action of the force is dependant on follower factor η. The rods have length $l_i = 1, 2, 3$ respectively. The physical structures of the considered system are shown in Figure 2: a) two coaxial tubes, b) tube and rod, c) flat frame.

The problem presented in this paper has been formulated by means of the Hamilton principle:

$$\delta\int_{t_1}^{t_2}(T - V - L_n)dt = 0 \tag{1}$$

where kinetic T and potential V energy and work L_n of non-conservative forces are expressed by the following formulas:

$$T = \frac{1}{2}\sum_{i=1}^{3}\int_0^{l_i}\rho_i A_i\left(\frac{\partial W_i(x_i,t)}{\partial t}\right)^2 dx_i \tag{2}$$

$$V = \frac{1}{2}\left\{\sum_{i=1}^{3}\int_0^{l_i}E_i J_i\left[\frac{\partial^2 W_i(x_i,t)}{\partial x_i^{\;2}}\right]^2 dx_i + \int_0^{l_i}E_i A_i\left[\frac{\partial U_i(x_i,t)}{\partial x_i} + \frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]^2 dx_i\right\} + \\ + \frac{1}{2}C\left(\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|_{x_3=0} - \left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=l_2}\right)^2 + PU_1(l_1,t) \tag{3}$$

$$L_n = \eta P\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=l_1} \tag{4}$$

Introducing kinetic (2) and potential (3) energy and work (4) into (1) and performing the variational and integration operations, and assuming that virtual longitudinal and transversal displacements for $i = 1, 2, 3$ are arbitrary and independent for $0 < x_i < l_i$, the following equations of motion in a transversal direction were obtained:

$$E_i J_i\frac{\partial^4 W_i(x_i,t)}{\partial x_i^{\;4}} - E_i A_i\frac{\partial}{\partial x_i}\left[\left[\frac{\partial U_i(x_i,t)}{\partial x_i} + \frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]\frac{\partial W_i(x_i,t)}{\partial x_i}\right] + \rho_i A_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2} = 0$$

$$i = 1, 2, 3 \tag{5}$$

The compressive axial force is defined as follows:

$$S_i(t) = -E_i A_i\left(\frac{\partial U_i(x_i,t)}{\partial x_i} + \frac{1}{2}\left[\frac{\partial W_i(x_i,t)}{\partial x_i}\right]^2\right), \; i = 1, 2, 3 \tag{6}$$

Introducing the definition of axial force (6) into the equation of motion, equation (5) has the form:

$$E_i J_i \frac{\partial^4 W_i(x_i,t)}{\partial x_i^4} + S_i(t)\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2} + \rho_i A_i \frac{\partial^2 W_i(x_i,t)}{\partial t^2} = 0 \quad i = 1, 2, 3 \tag{7}$$

The axial displacement in each rod is expressed by the formula:

$$U_i(x_i,t) = -\frac{S_i(t)x_i}{E_i A_i} - \frac{1}{2}\int_0^{x_i}\left[\frac{\partial W_i(x_i,t)}{\partial x_i}\right]^2 dx_i \quad i = 1, 2, 3 \tag{8}$$

Introducing geometrical boundary conditions into the variational equation:

$$W_1(0,t) = W_1^I(x_1,t)\Big|_{x_1=0} = W_2(0,t) = W_2^I(x_2,t)\Big|_{x_2=0} = 0, \qquad W_1^I(x_1,t)\Big|_{x_1=l_1} = W_3^I(x_3,t)\Big|_{x_3=l_3},$$
$$W_2(l_2,t) = W_3(0,t), \qquad W_1(l_1,t) = W_3(l_3,t), \qquad U_1(0,t) = U_2(0,t) = 0,$$
$$U_2(l_2,t) = U_3(0,t), \qquad U_1(l_1,t) = U_3(l_3,t). \tag{9 a-k}$$

the following set of natural boundary conditions were obtained:

$$E_1J_1W_1^{II}(x_1,t)\Big|_{x_1=l_1} + E_3J_3W_3^{II}(x_3,t)\Big|_{x_3=l_3} = 0,$$
$$E_1J_1W_1^{III}(x_1,t)\Big|_{x_1=l_1} + PW_1^I(x_1,t)\Big|_{x_1=l_1} + E_3J_3W_3^{III}(x_3,t)\Big|_{x_3=l_3} = 0,$$
$$E_2J_2W_2^{III}(x_2,t)\Big|_{x_2=l_2} + S_2W_2^I(x_2,t)\Big|_{x_2=l_2} - E_3J_3W_3^{III}(x_3,t)\Big|_{x_3=0} - S_3W_3^I(x_3,t)\Big|_{x_3=0} = 0,$$
$$-E_3J_3W_3^{II}(x_3,t)\Big|_{x_3=0} + C\left[W_3^I(x_3,t)\Big|_{x_3=0} - W_2^I(x_2,t)\Big|_{x_2=l_2}\right] = 0,$$
$$E_2J_2W_2^{II}(x_2,t)\Big|_{x_2=l_2} - C\left[W_3^I(x_3,t)\Big|_{x_3=0} - W_2^I(x_2,t)\Big|_{x_2=l_2}\right] = 0,$$
$$S_2 = S_3, \qquad S_1 + S_2 = P \tag{10 a-g}$$

The small parameter ε method [6] has been used to solve the boundary problem. According to this method, the longitudinal and transversal displacements, axial force and vibration frequency of each rod are written in a power series:

$$w_i(\xi,\tau) = \sum_{n=1}^{N}\varepsilon^{2n-1}w_{i2n-1}(\xi,\tau) + O(\varepsilon^{2N+1}) \quad u_i(\xi,\tau) = u_{i0}(\xi) + \sum_{n=1}^{N}\varepsilon^{2n}u_{i2n}(\xi,\tau) + O(\varepsilon^{2N+1})$$

$$k_i(\tau) = k_{io} + \sum_{n=1}^{N}\varepsilon^{2n}k_{i2n}(\tau) + O(\varepsilon^{2N+1}) \quad \omega_i^2 = \omega_{0i}{}^2 + \sum_{n=1}^{N}\varepsilon^{2n}\omega_{i2n}{}^2 + O(\varepsilon^{2N+1}) \tag{11a-d}$$

The magnitudes obtained from equations (11a-d) are introduced into the equation of motion, axial force and boundary conditions. Then, the terms are grouped at the same power of small parameter ε, which leads to an infinite sequence of equations. The solution presented in this paper was obtained on the basis of a system of equations with the small parameter in the first power.

2. Results of numerical calculations

At the beginning, the relation curves force *p* versus natural vibration frequency ω for different a follower factor and crack location has been presented (Figs 3 and 4). The continuous curves stand for the divergence instability and the dotted curves stand for the flutter instability. With the increasing value of follower factor η, the increase of the maximum magnitude of the external load has been achieved irrespective of the location of the crack. The crack location changes the natural vibration frequency of the system. It has been concluded that if the crack is located near the free end of the column, the point at which the system loses instability through flutter occurs for a smaller follower factor value. For example, when the stiffness of the rotational spring is equal to one and the pin is located in the middle of the column, the flutter instability occurs for an η greater than 0.5, while for location d_2 = 0.7, the flutter begins with an η greater than 0.4. The point at which the natural vibration frequency curves cross each other has also been found. At this point the force and vibration frequency does not depend on the follower factor.

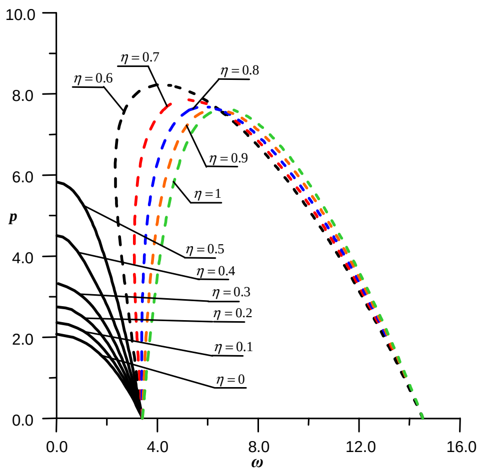


Fig. 3. Influence of follower factor η on natural vibrations, c = 1, d_2 = 0.5, r_m = 0.76

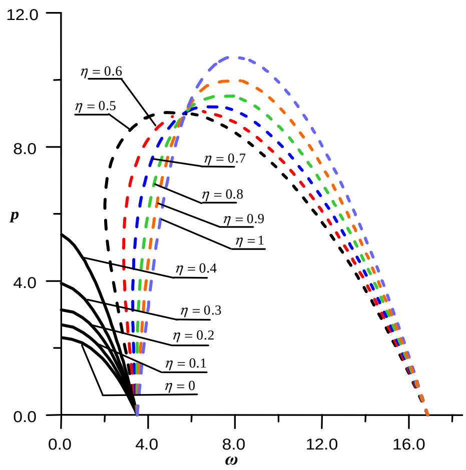
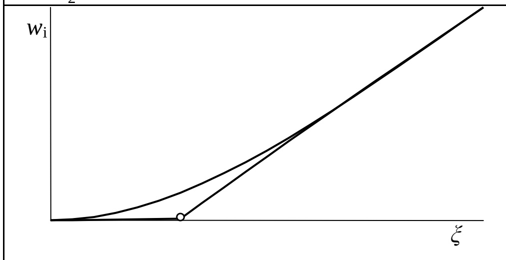
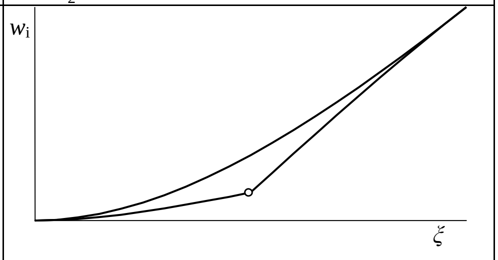
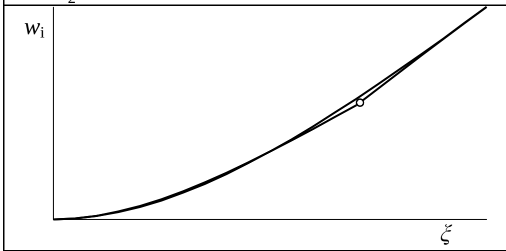
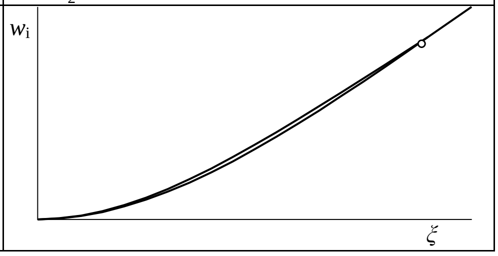


Fig. 4. Influence of follower factor η on natural vibrations, c = 1, d_2 = 0.7, r_m = 0.76

The next step in the numerical calculations project was to investigate the location of the crack on the shape modes. The sample results of this study are presented in Table 1.

Table 1

Influence of location of crack on shape mode, *c = 1*

$d_2 = 0.3$	$d_2 = 0.5$
	
$d_2 = 0.7$	$d_2 = 0.9$
	

It has been concluded that for $c > 5$, the location of the pin has no influence on the shape modes, while for smaller c values, the shape modes vary on each other. As shown in Table 1, when $c = 1$, translation of the crack along the columns length changes the shape modes. When the crack is located near the free end of the system, the shape mode is close to the linear system, that is why the external load value in this location is the greatest.

The relation force-crack location for a different spring stiffness and follower factor is presented in Figures 5 and 6.

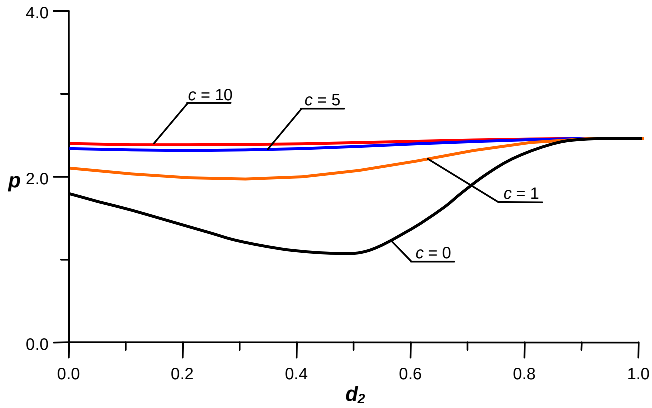


Fig. 5. Influence of crack location on maximum load for different spring stiffness, $\eta = 0$, $r_w = 1$, $r_m = 0.76$

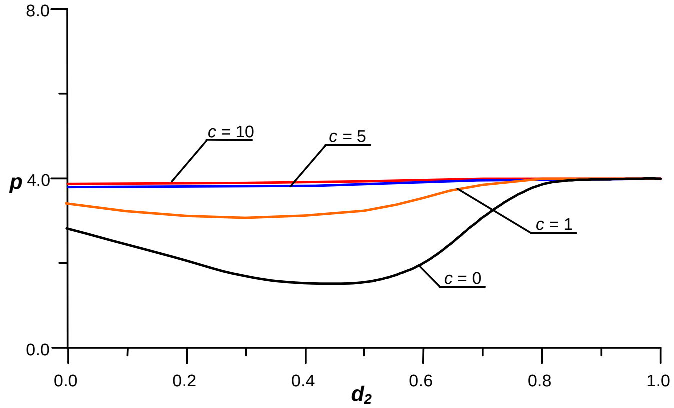


Fig. 6. Influence of crack location on maximum load for different spring stiffness, $\eta = 0.3$, $r_w = 1$, $r_m = 0.76$

When the rotational spring stiffness tends towards infinity, the change in location of the pin has no influence on the bearing capacity of the system. Reduction of the stiffness of the connection between rods 2 and 3 up to 5, with translation of the pin along the columns length from the fixed end up to free one causes the capacity of the system to stabilize. Further reduction of connection stiffness c allows one to obtain greater load changes under the influence of the pin localization. With $c = 0$, the bearing capacity of the column is the smallest. Despite changes of the rotational spring stiffness which connects rods 2 and 3, the maximum external load value stabilizes when the pin is located close to the free end of the column.

Conclusions

In this paper the influence of the crack location along the column length on the divergence and flutter load and natural vibration of a geometrically nonlinear column subjected to generalized load P is presented. After analysis of the results of numerical calculations it was found that:

- For $c < 5$ the location of the pin has a great influence on the critical loading and natural vibration frequency. By changing the location of the pin, instability regions can be controlled.
- When the crack is located near the free end of the column, the influence of the spring stiffness on the maximum loading and natural vibration frequency is negligible.
- There is a value of rotational spring stiffness above which the location of the pin has no influence on the investigated parameters. This effect occurs for $c > 5$.

- Points in which, irrespective of the follower factor, the natural vibration frequency is constant were found.
- It has been concluded that the influence on the type of instability (divergence or flutter) also has the direction of action of the external load, which is dependent on follower factor η.

Nomenclature

A_i	Cross section area	r_m	Bending stiffness ratio E_1J_1/E_2J_2
E_i	Young's modulus	r_w	Bending stiffness ratio E_2J_2/E_3J_3
J_i	Area moment of inertia	k_i	Non-dimensional axial force S_il^2/E_iJ_i
P	External force	w_i	Non-dimensional transversal displacement W_i/l
C	Rotational spring stiffness	u_i	Non-dimensional axial displacement U_i/l
U_i	Axial displacement	d_i	Non-dimensional length of a rod l_i/l
W_i	Transversal displacement	ξ_i, τ	Non-dimensional space and time variable, respectively
η	Follower factor	c	Non-dimensional spring stiffness $Cl/(E_1J_1+E_2J_2)$
ρ_i	Density of a material	ω_i^2	Non-dimensional natural frequency $\Omega^2(\rho_iA_il^4/E_iJ_i)$
Ω_i	Natural vibration frequency	p	Non-dimensional external load $Pl^2/(E_1J_1+E_2J_2)$

References

[1] Kukla S., Free vibration and stability of stepped columns with cracks, Journal of Sound and Vibration 2009, 319, 3-5, 1301-1311.

[2] Wang C.Y., Buckling of an internally hinged column with an elastic support, Engineering Structures 2002, 24, 1357-1360.

[3] Przybylski J., Drgania i stateczność dwuczłonowych układów prętowych wstępnie sprężonych przy obciążeniach niezachowawczych, Częstochowa 2002.

[4] Tomski L., Szmidla J., Uzny S., The local and global instability and vibration of the system subjected to non-conservative loading, Thin Walled Structures 2007, 45, 945-949.

[5] Skalmierski B., Mechanika, PWN, Warszawa 1982, 283-285.

[6] Osiński Z., Teoria drgań, PWN, Warszawa 1978, 52-54.