

CRITICAL SEISMIC COEFFICIENTS OF HOMOGENEOUS EARTH DAMS PREDICTION BY A FELA-ANN APPROACH

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Received: 12 September 2018; Accepted: 1 October 2019

Abstract. In this research study, a combination of lower and upper bound finite element limit analysis (FELA) and artificial neural network (ANN) has been adopted in order to forecast critical seismic coefficients (kc) of homogeneous earth dams (HED) subjected to pseudo-static seismic loading. To achieve this, the results of kc obtained by OptumG2 software were used in the development of the ANN and MR models. The ANN models have shown higher prediction performance than the MR models based on the performance indices. The most appropriate architecture was found 8-14-1, as this gave the best kc predict with the minimum statistical measures of error and the high determination coefficient (> 99%). Consequently, the ANN model can be used to easily and accurately predict kc value of the HED as the best substitute for the conventional methods.

MSC 2010: 62-07, 65-04

Keywords: critical seismic coefficient, pseudo-static analysis, finite element limit analysis, artificial neural network, homogeneous earth dams

1. Introduction

The critical seismic coefficient of a slope is an important parameter in deciding the seismic safety of an earth dam. In practice, the stability of small earth dams under seismic loading is usually evaluated by applying the conventional pseudo-static method (PSM). The PSM is convenient when critical seismic coefficients (kc) can be estimated. In addition, with these coefficients, permanent deformations of earth dam slopes can be determined using the Newmark simplified procedure [1]. In fact, the kc is calculated as the quotient of the horizontal seismic acceleration, a_h , to the acceleration of gravity ($kc = a_h/g$), the critical state corresponding to a factor of safety (FS) equals one [2]. However, this procedure requires much effort and time during iteration [3].

In slope stability analysis, the evaluation of FS and k_c is often performed using either conventional approaches (limit equilibrium and limit analysis) or numeric ones (the strength reduction technique). Therefore, different techniques have been proposed to determine the critical seismic coefficient k_c (upper bound formulation [4], variational formulation of limiting equilibrium and a strength reduction technique [5], limit equilibrium method and stress acceptability criteria [6],...etc.). Particularly, there is no exacting formulation for selecting the pseudo-static coefficient for designing earth dams with special conditions as anisotropy seepage.

Recently, the finite element limit analysis (FELA), an alternative approach that offers higher-quality results than the conventional methods [7]. So, FELA is a powerful combination of a mathematical and numerical tool with great finite element discretization capabilities for manipulating complex soil, load, and boundary conditions and bound plastic theorems to frame the exact limit load with upper and lower limit solutions [8].

As an alternative practice to the analytical and numerical methods, artificial neural networks (ANNs) have been developed as powerful mathematical and numerical modeling manners that can interconnect complex nonlinear relationships. The ANNs learning capability have used in multivariate complex modeling and have enlarged uses in various fields of geotechnical engineering [9].

The aim of this study is to develop an ANN model in order to achieve quick forecasting of the critical seismic coefficient of homogeneous earth dams (k_c) under full reservoir long-term stability condition and anisotropy seepage ($K_h \neq K_v$). The k_c assessed by the developed ANN models corresponds well with those obtained by the conventional trial and error approach and, thus, can be applied efficiently for assessing the stability and performance of HED during earthquake shaking.

2. Numerical model of analysis

In the present study, the numerical finite element limit analysis software, OptumG2 [10], was used to accurately determine the critical seismic coefficients of homogeneous earth dams (HED). OptumG2 software has been efficiently applied to solve various problems in geotechnical engineering [11].

In the following modeling steps, an HED on an impervious foundation with the geometric parameters shown in Figure 1a is considered. The numerical model of a HED in OptumG2 is shown in Figure 1b. An automatically adaptive mesh refinement with 2,000 elements and 3 adaptivity steps was used in both the upper and lower bound analyses. The mesh was composed of six noded, second-order, plane-strain triangular elements. The boundary condition of this problem is defined as the following: the left boundaries of the problem are allowed to move only in a vertical direction, while the bottom boundary of the model is fixed in both directions. Whereas, an elastoplastic constitutive model following the Mohr-Coulomb failure criterion was assigned to the soil.

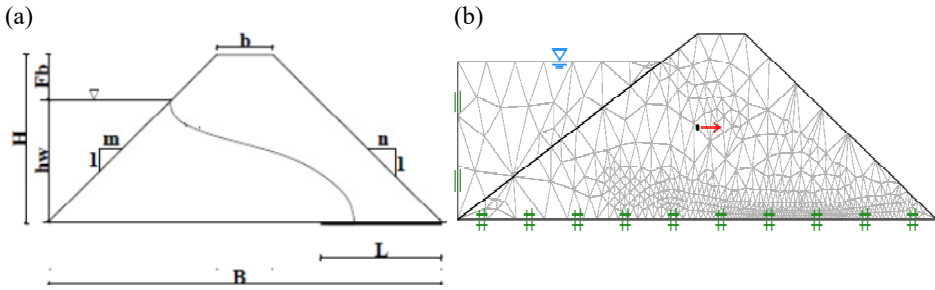


Fig. 1. (a) Geometric parameters of HED with horizontal drain on impervious foundation (m/n - Upstream/downstream slope of dam; Fb - Free broad; b - top width of the dam; hw - Water depth in upstream of the dam); (b) adaptive mesh using OptumG2 software

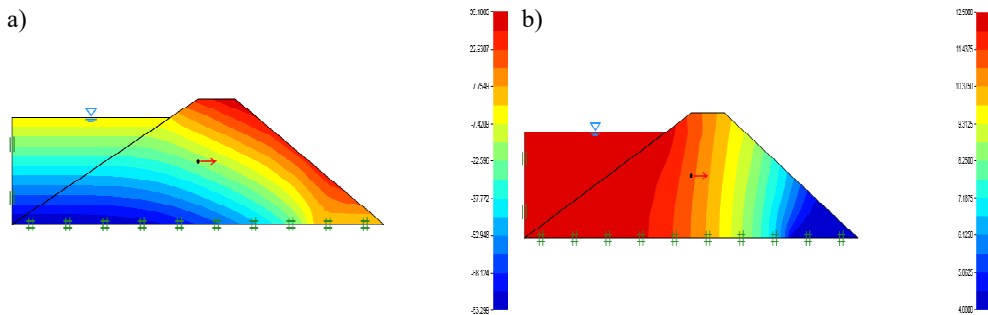


Fig. 2. Sample of results of pseudostatic analysis using OptumG2 software: a) pore pressure [kPa], b) head water [m]

3. Research methodology

The methodology proposed in this study to develop the ANN models contains the following steps:

1. Construction tow representative databases D1 and D2 using the OptumG2 (2017) software. The input variables of the first database contains the different parameters influencing the k_c of the HED values: Base width of the dam (B), Height of the dam (H), length of horizontal toe drain (L), unit weight of dam soil (γ), effective cohesion (c'), effective friction angle (ϕ'), horizontal permeability (K_h), and vertical permeability (K_v), while the second database (compacted) present the input data in a dimensionless manner (γ/H_c , K_v/K_h , $\tan\phi$, L/B).
2. Developing the direct relationships between the selected sets of input variables and the obtained output variables using either Multiple Regression (MR and MR2) and feed forward back-propagation neural network method (BPNN1, BPNN2).
3. Exploitation of the developed ANN models: sensitivity analysis (BPNN1), mathematical equation (BPNN2).
4. Accuracy of the developed ANN models is compared with the existing methods for estimation of k_c for some new cases with input values that do not exist in the database constructed in step (1) above.

4. Models development

The detail of development of MR and ANN models for prediction of k_c of HED has been discussed hereunder.

4.1. Artificial neural network models

ANN can be defined as structures consisting of mathematical processing element famed as neurons, which performs parallel calculations for data processing [12]. The feed forward back-propagation (FB) is often the most common model of neural networks used in geotechnical applications [13, 14]. In this model, the neuron of the network receives input signals, trains them and sends an output signal. The connection between neurons is represented by a number called weights (w).

4.1.1. Database and normalization

The k_c evaluation of HED with horizontal drain by FELA in the case of the full reservoir under pseudostatic loading needs to identify a number of parameters. These parameters can be classified into three groups: first, geometric parameters (B, H, L), secondly, geotechnical parameters (γ, c', ϕ') and third one seepage parameters (K_h, K_v).

A dataset of 306 different cases of above eight independent variables as inputs and one dependent variable k_c as an output was built through OptumG2 FELA software and was then used to produce artificial neural network (ANN). Hence, k_c is the mean value of the upper bound and lower bound critical seismic coefficient.

Table 1 presents the summary statistics for each variable of the critical seismic coefficient dataset.

Table 1. Summary statistics of the critical seismic coefficient database

Parameters	Max	Min.	Mean	St.dv.
B [m]	142	50	86.23	27.77
L [m]	40	0	17.57	11.82
H [m]	20	10	14,98	4.14
γ [kN/m ³]	21	15	17.23	1.99
c [kPa]	50	5	22.66	16.21
ϕ [°]	50	10	29.90	10.55
K_h [m/day]	0.1	0.001	0.06	0.05
K_v [m/day]	0.1	E-5	0.04	0.04
γ/H_c	0.38	0.02	0.12	0.12
K_v/K_h	1	0.01	0.40	0.40
$\tan \phi$	1.19	0.18	0.60	0.25
L/B	0.41	0	0.21	0.13
k_c	0.701	0	0.25	0.16

For FBNN1, before applying the training, the input and output values should be normalized, since most training algorithms are sensitive to the scale of the data. In the present study, the dataset has been scaled to make them bounded into the range $[-1,1]$ using the min-max normalization formula as follows:

$$X_{normalized} = \frac{2(X_i - X_{i,min})}{(X_{i,max} - X_{i,min})} - 1 \quad (i = 1, 2, \dots, N) \quad (1)$$

where $X_{normalized}$ and X_i represent normalised and actual value respectively. $X_{i,min}$ and $X_{i,max}$ represent the minimum and maximum values of the attribute X_i .

In this study, ANN's modeling has been developed by the use of Matlab software. So, for a better form of the elements that frame the ANN architecture with its developments, our selection of modeling sets were made as follows:

- A random dataset has been distributed into three sets, which are: training set, testing set, and validation set. So, in order to study the training process, a 70% (214/306) of data was used, while the remaining (30%) was used equally for the testing and validation steps (46/311; 46/311).
- The linear purelin transfer function for the output neuron was selected since it is fitting for continuous-valued targets [15].
- The trial-and-error technique based on the statistical measures was used to determine the optimum neurons in the hidden layer to optimize the ANN architecture.

The principal settings of ANN used in this study are mentioned in Table 2.

Table 2. Settings of ANN used in this study

Setting	ANN1	ANN2
Topology of the network	Feed forward	
Learning paradigm	Back propagation	
Training algorithm	Levenberg-Marquardt	
Number of hidden layers	1	
Normalization	$[-1, 1]$	/
Activation function	hyperbolic tangent sigmoid (tansig)	Log sigmoid (logsig)
Performance evaluation	R^2 , RMSE, MAE	
Network typology	$8-j-1/j = 4, \dots, 18.$	$3-j-1/j = 2, \dots, 9.$

4.1.2. Performance evaluation

In order to quantify the agreement between computed and predicted values of the critical seismic coefficient of HED, we used three statistical measures: root mean-square error (RMSE), mean absolute error (MAE), coefficient of determination (R^2). These statistical performance evaluation criteria are identified as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^N (P_i - C_i)^2}{\sum_{i=1}^N (C_i - \bar{C})^2} \quad (2)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (P_i - C_i)^2}{N}} \quad (3)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |P_i - C_i| \quad (4)$$

where: P_i - predicted value; C_i - computed value; N - number of observations; C_{max} - maximum computed value; C_{min} - minimum computed value; and \bar{C} - averaged computed value.

The methodology practiced to estimate the performance of ANN architecture includes obtaining the minimum statistical measures of error between computed and predicted values. Therefore, the best ANN model was selected on the basis of the lowest (RMSE, MAE) and highest R^2 values.

4.2. Multiple regression (MR) models

The multiple regression (MR) is a statistical technique that examines cause-effect relations between dependent and independent variables [16]. The data used while developing the ANN models were used in the development of the MR models. The two MR models revealed the following correlations:

$$kc = -0.293 + 0.002B + 0.004L - 0.014H + 0.008c + 0.008\varphi + 0.007\gamma - 0.306K_h + 0.169K_v \quad (5)$$

$$kc = 0.083 - 0.931 \frac{\gamma}{H_c} - 0.026 \frac{K_v}{K_h} + 0.365 \tan\varphi + 0.329 \frac{L}{B} \quad (6)$$

5. Results and discussion

In this study, the overall performance of the developed models was assessed using the statistical analyses of the predicted and calculated kc values. The results of the statistical performance of the optimal ANN models with best geometries are shown in Table 3. As it can be seen, for ANN1, the R^2 values are very close to one while MSE and MAE values are close to zero, indicating excellent agreement between the assessment results and forecasted results of kc . Also, it can be noticed that the predicted kc values from ANN2 and MR1 models are in good agreement with kc observed, as their $R^2 > 0.8$ [17].

Table 3. Statistics and performance parameters of the optimal models

Optimal model	Set	R ²	RMSE	MAE
ANN1 (8-14-1)	Training	0.999	0.0412	0.0025
	Validation	0.987	0.0591	0.0048
	Test	0.993	0.0707	0.0084
MR1	All data	0.817	0.0697	0.0548
ANN2 (4-4-1)	Training	0.836	0.0648	0.0364
	Validation	0.873	0.0656	0.0283
	Test	0.810	0.0728	0.0554
MR2	All data	0.650	0.0964	0.0736

Figure 3 shows that the values forecast by the developed ANN models fit perfectly with the computed values. High performances of the training step, testing step, and validation step indicate that the learning process of the optimal predictive model is successful, hence, the R-value is over 0.899 for the whole response for both models. Obviously, it can be noted that the ANN1 model developed exhibits higher prediction performance than the ANN2 model based on the performance criteria, similarly for MR models. Also, the ANN models developed demonstrate a higher prediction performance than MR models, which demonstrates the effectiveness of the ANN approach.

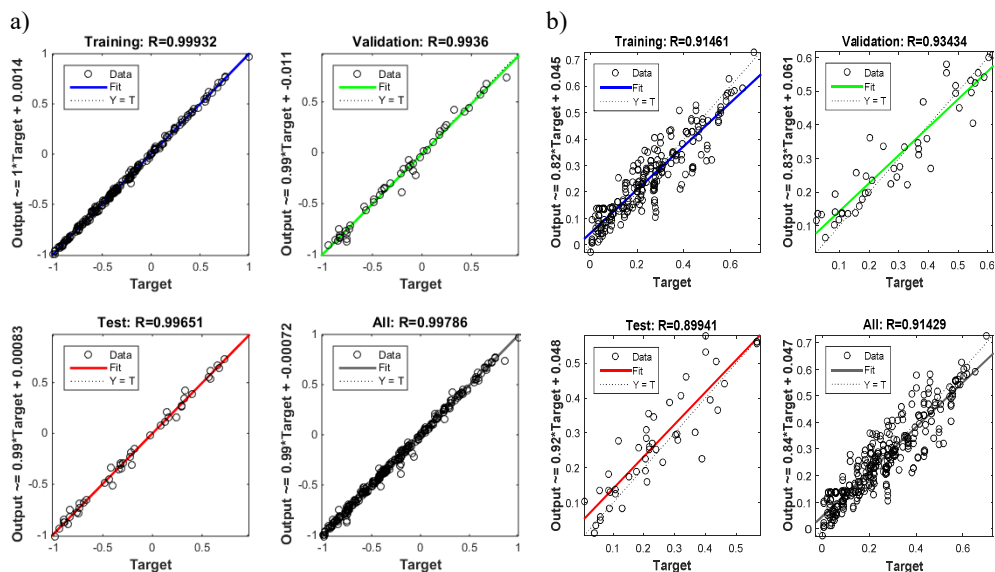


Fig. 3. R values of training, validation, test and all data for optimal models: a) FBNN1, b) FBNN2

The plots of network performance (MSE) at different epochs for both models (FBNN1 and FBNN2) are shown in Figure 4. According to these plots, the devel-

oped models are processed because the MSE of the network starting at a high value and decreasing to a lower value. This figure also shows that the test set error and the validation set errors have similar characteristics and that there does not seem to have been a significant overfitting.

On the other hand, to look for a statistically significant difference between calculated and predicted kc values, a paired t-test was performed by using the GraphPad Software. The p value was found as (0.345, 0.427) and (0.651, 0.473) for the ANN and MR models, respectively, indicating that no significant difference in kc values was observed between computed and forecasted values for all models ($p > 0.05$).

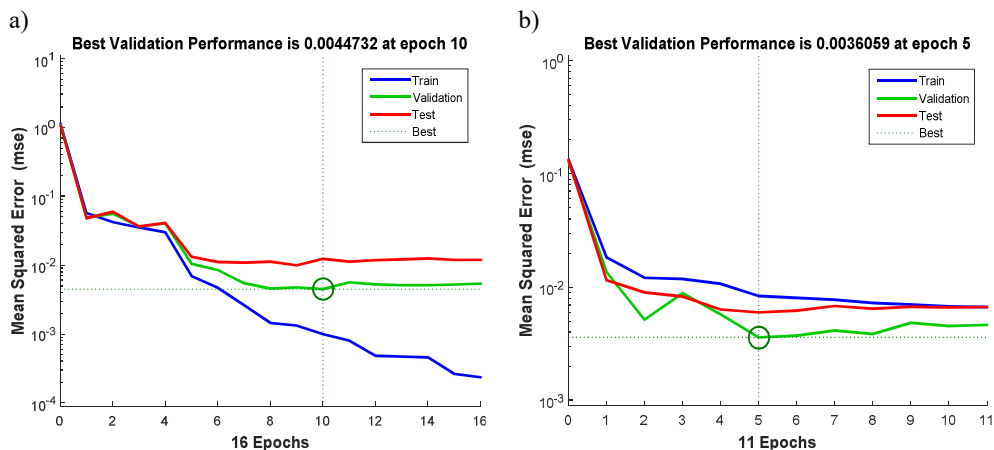


Fig. 4. Best validation performance: a) FBNN1, b) FBNN2

5.1. Sensitivity analysis

The relative importance of the various input issues can be assessed by examining the connection weights. Garson [18] proposed an equation that included partitioning the hidden-output connection weights of each hidden neuron into components associated with each input neuron:

$$(RI)_j = \frac{\sum_{m=1}^{m=Nh} \left(\left(\frac{|w_{jm}^{ih}|}{\sum_{k=1}^{Ni} |w_{km}^{ih}|} \right) \times |w_{mn}^{ho}| \right)}{\sum_{k=1}^{Ni} \left\{ \sum_{m=1}^{Nh} \left(\frac{|w_{km}^{ih}|}{\sum_{k=1}^{Ni} |w_{km}^{ih}|} \right) \times |w_{mn}^{ho}| \right\}} \quad (7)$$

where RI_j is the relative importance of the j -th input variable on the output variable, w is connection weight, Ni and Nh are the numbers of input and hidden neurons respectively, the subscripts "k", "m" and "n" refer to input, hidden and output neurons respectively, and the superscripts "i", "h" and "o" refer to input, hidden and output layers respectively.

Table 4 shows the weights of the input-hidden layer connections, the hidden-output layer connections, and biases for model (4-4-1). The weights and biases can be used for sensitivity analysis and framing an ANN model in equation form.

Table 4. Connection weights and biases of the optimal ANN2 model

Neuron	<i>w1</i>				<i>w2</i>	Biases	
	Input neurons				Output		
	γ/H_c	K_v/K_h	$\tan\phi$	L/B	kc	b_{hk}	b_0
1	2.3123	-1.6702	0.43082	0.39334	-0.47731	-2.4001	-0.32977
2	-1.8787	0.25268	-1.3399	-0.60174	-0.50553	1.716	
3	1.0478	1.8986	-2.7067	-0.36643	-2.7263	0.28588	
4	4.2262	-0.61591	0.67505	4.0793	3.1077	-0.51371	

The results of relative importance of inputs are shown in Table 5. The results indicate that the ϕ , H, K_v , K_h , and c are the most important parameters affecting kc. The results also show that the B, L and γ have less effect on kc.

Table 5. Relative importance of different input variables

Input variable	B	L	H	γ	c	ϕ	K_h	K_v
Relative Importance	3.188	4.395	17.435	6.110	9.360	29.398	14.603	15.510

5.2. Explicit formulation of ANN2 model

The model equation for the output can be formulated based on the trained weights of the ANN model [19]. In this study, the basic mathematical equation relating the independent variables and the critical seismic coefficient value based on trained neural network can be written as:

$$kc = f\{b_0 + \sum_{k=1}^h [w_k * f(b_{hk} + \sum_{i=1}^m w_{ik} X_i)]\} \tag{8}$$

where w_{ik} is the connection weight between i -th input variable and k -th neuron of hidden layer; X_i is the input variable and f is the sigmoid transfer function. h is the number of neurons in the hidden layer.

So, such a model equation for critical seismic coefficient (kc) was established using the values of the weights and biases shown in Table 4 as per the following expressions:

$$\begin{cases} A1 = 2.3123 \left(\frac{\gamma}{H_c}\right) + -1.6702 \left(\frac{K_v}{K_h}\right) + 0.43082(\tan \phi) + 0.39334 \left(\frac{L}{B}\right) - 2.4001 \\ A2 = -1.8787 \left(\frac{\gamma}{H_c}\right) + 0.25268 \left(\frac{K_v}{K_h}\right) - 1.3399(\tan \phi) - 0.60174 \left(\frac{L}{B}\right) + 1.716 \\ A3 = 1.0478 \left(\frac{\gamma}{H_c}\right) + 1.8986 \left(\frac{K_v}{K_h}\right) - 2.7067(\tan \phi) - 0.36643 \left(\frac{L}{B}\right) + 0.28588 \\ A4 = 4.2262 \left(\frac{\gamma}{H_c}\right) - 0.61591 \left(\frac{K_v}{K_h}\right) + 0.67505(\tan \phi) + 4.0793 \left(\frac{L}{B}\right) - 0.51371 \end{cases} \tag{9}$$

$$\begin{cases} B_1 = -0.47731 \left(\frac{1}{1 + e^{-A1}} \right) \\ B_2 = -0.50553 \left(\frac{1}{1 + e^{-A2}} \right) \\ B_3 = -2.7263 \left(\frac{1}{1 + e^{-A3}} \right) \\ B_4 = -2.7263 \left(\frac{1}{1 + e^{-A4}} \right) \end{cases} \quad (10)$$

$$kc = \frac{1}{1 + e^{-(-0.32977+B1+B2+B3+B4)}} \quad (11)$$

5.3. Comparison with other methods

For this purpose, we have arbitrarily chosen three new cases with input values that are not similar to values adopted in the database. Table 6 shows the selected input variables for each of the three cases used. Table 7 shows the results obtained from FELA (OptumG2), FEM (GEO5) and the developed models (MR1, ANN1) for new cases.

Table 6. Selected input data for new cases

Cases	Input variable							
	B	L	H	c	ϕ	γ	K_h	K_v
1	98	30	20	5	25	17	0,1	0,1
2	102	0	20	25	40	19	0,1	0,02
3	142	40	20	10	20	15	0,001	0,0002

Table 7. Comparison of results obtained from different methods

Cases	kc (OptumG2)	kc (GEO5FEM)	Relative error [%]	kc (ANN1)	Relative error [%]	kc (MR1)	Relative error (%)
1	0.0925	0.00	89.19	0.1026	-10.92	0.0883	4.45
2	0.355	0.35	0.17	0.3057	13.89	0.25678	27.66
3	0.146	0.1	31.51	0.1731	-18.56	0.21573	-47.76

6. Conclusions

In this study, the critical seismic coefficient of homogenous earth dams has been predicted by Artificial Neural Network models. The prediction of these critical

seismic coefficients requires the use of OptumG2 FELA software with identified ranges of variables to build a representative database. For a specified set of independent variables, the (8-14-1) and (4-4-1) ANN models were found to be capable of estimating the k_c accurately. The error estimated is very low which indicates that the approach is applicable accurately and rapidly as compared to other techniques (conventional and numeric methods). Moreover, the results of sensitivity analysis showed that angle of friction (ϕ) is the most important parameter affecting k_c with relative importance of 29.40%.

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