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NUMERICAL SOLUTION OF MHD CHANNEL FLOW IN A POROUS MEDIUM WITH UNIFORM SUCTION AND INJECTION IN THE PRESENCE OF AN INCLINED MAGNETIC FIELD

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Abstract. In this paper, the steady fully developed MHD flow of a viscous incompressible electrically conducting fluid through a channel filled with a porous medium and bounded by two infinite walls is investigated numerically for the cases (i) Poiseuille flow and (ii) Couette-Poiseuille flow; with uniform suction and injection at the walls in the presence of an inclined magnetic field. The Brinkman equation is used for the flow in the porous channel and solved numerically using the finite difference method. Numerical results are obtained for velocity. The effects of various dimensionless parameters such as Hartmann number (M), suction/injection parameter (S), permeability parameter (α) and angle of inclination (θ) on the flow are discussed and presented graphically.

MSC 2010: *76D05, 34B60, 65L12*
Keywords: *MHD flow, Brinkman equation, permeability parameter, Hartmann number, uniform suction and injection, finite difference method*

Nomenclature

a	Distance between two plates
B_0	Applied magnetic field
h	Increment along y-axis
i	Index refers to y
k	Permeability of the medium
M	Hartmann number
n	Number of grid points along y-direction
p	Pressure gradient
S	Suction/injection parameter
U	Characteristic velocity
u'	Velocity along x'-axis
u	Non-dimensional velocity along x-axis
V_0	Constant suction/injection velocity pressure gradient
x', y'	Cartesian coordinate
x, y	Non-dimensional cartesian coordinate
Greek letters	
θ	Angle between x'-axis and direction of the applied magnetic field
ρ	Density of liquid
σ	Electrical conductivity
μ	Coefficient of viscosity
α	Permeability parameter

1. Introduction

The study of magnetohydrodynamic (MHD) flow through channels filled with a porous medium having uniform suction/injection at the plates have received great interest for many researchers due to its wide applications such as in MHD generators, air filters, water filters, water coolers, artery blood flow and in petroleum engineering and chemical engineering.

Verma and Gupta [1] studied steady state fully developed MHD flow of a viscous incompressible conducting fluid within a channel filled with a porous medium and bounded by two infinite walls. Singh and Sapna [2] investigated the effect of injection/suction on the flow of electrically conducting fluid through a porous channel filled with porous material considering effect of heat radiation, heat source and chemical reaction into consideration. Hamza [3] studied the effects of suction and injection on steady MHD flow of a viscous and electrically conducting fluid in an annular porous region between two concentric cylinders. Srivastava and Deo [4] considered the Poiseuille and Couette flow of an electrically conducting fluid through a porous medium of variable permeability under the transverse magnetic field. They used the Brinkman equation for flow through the porous medium and obtained a numerical solution for velocity and the volumetric flow rate using the Galerkin method. Pantokratoras and Fang [5] investigated the fully developed flow in a fluid-saturated porous medium channel with an electrically conducting fluid under the action of a parallel Lorentz force. Verma and Singh [6] studied the steady flow of an electrically conducting viscous incompressible fluid in a circular channel filled with a saturated porous medium in the presence of a transverse magnetic field. Chamkha [7] developed unsteady flow and heat transfer of an electrically conducting fluid through a porous medium channel in the presence of transverse magnetic field. Manglesh and Gorla [8] investigated unsteady free convective flow through porous media of viscous, incompressible, electrically conducting fluid through a vertical porous channel with thermal radiation. Manyange et al. [9] analytically examined the motion of a two dimensional steady flow of a viscous, electrically conducting, incompressible fluid flowing between two infinite parallel porous plates under the influence of inclined magnetic field and constant pressure gradient. Ramakrishnan [10] studied the effects of a slip parameter and an inclined magnetic field on MHD flow through two infinite parallel plates filled by a porous medium. Kuiry and Bahadur [11] analyzed the effect of an inclined magnetic field on the magnetohydrodynamic behaviour of two dimensional Poiseuille flow between two parallel porous plates of which the lower plate is taken to be porous. Sandeep [12] analysed the laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of an inclined magnetic field with volume fraction, heat source and considering porous parameter. Attia et al. [13] investigated the transient flow with heat transfer through a porous medium of an incompressible viscous fluid between two infinite horizontal porous plates. Falade et al. [14] investigated the effect of suction/injection on the unsteady oscillatory flow through a vertical channel subjected to a transverse magnetic field. Dwivedi et al. [15] studied

MHD flow problem through a porous channel containing a porous medium placed under an inclined magnetic field. Chandrasekar et al. [16] studied the problem of steady laminar flow of an electrically conducting viscous incompressible fluid flow between two parallel porous plates of a channel in the presence of a transverse magnetic field with bottom injection and top suction.

In this work, the steady flow of a viscous, incompressible, electrically conducting fluid through a horizontal channel filled with a porous material and bounded by two infinite parallel walls with constant suction and injection at the walls in the presence of inclined magnetic field has been investigated. The flow within the channel is driven by a constant pressure gradient. Two cases of interest are considered; (i) Poiseuille flow; i.e., when both the walls are stationary and (ii) Couette-Poiseuille flow; i.e., when the upper wall is moving and the lower is stationary. The dimensionless governing equation is solved numerically by the finite difference method using Scilab programming. Numerical results are obtained for velocity and the effects of various parameters on the fluid velocity are analyzed and presented graphically.

2. Mathematical formulation

We consider the steady fully developed flow of a viscous incompressible electrically conducting fluid in a parallel channel filled with a porous medium of permeability k. The flow in the channel is driven by a constant applied pressure gradient $\frac{\partial p}{\partial x'}$ in the x'-direction and the uniform magnetic field of strength B_0 is applied in the direction which makes an angle θ with the positive direction of the x'-axis. We assume that the magnetic Reynolds number is too small so that the induced magnetic field can be neglected. The fluid particles are being injected into the channel through the lower wall with constant velocity V_0 along the Y'-axis and fluid particle are being sucked out of the channel at the same velocity V_0 through the upper wall as shown in Figure 1. The governing Brinkman equation for the present flow in the porous medium (Verma and Gupta [1]) in the presence of inclined magnetic field is given by

$$\frac{d^2u'}{dy'} - \frac{\rho V_0}{\mu}\frac{du'}{dy'} - \frac{u'}{k} - \frac{\sigma B_0^2 sin^2\theta}{\mu}u' = \frac{1}{\mu}\frac{\partial p'}{\partial x'} \tag{1}$$

where u', μ, ρ, σ are velocity, viscosity, density and conductivity of the fluid respectively; k is the permeability of the porous medium and V_0 is the constant suction/ /injection velocity at the walls.

We introduce dimensionless variables follows

$$u = \frac{u'}{U},\ y = \frac{y'}{a},\ P = \frac{a^2}{\mu U}\frac{\partial p}{\partial x'} \tag{2}$$

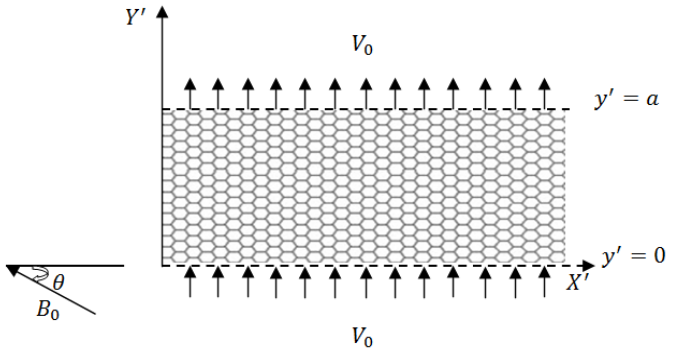


Fig. 1. Geometrical sketch of the channel flow problem

Here, U is the characteristic velocity and a is the distance between two walls of the channel. Using the dimensionless variables in Eq. (2), we get

$$\frac{d^2u}{dy^2} - S\frac{du}{dy} - (\alpha^2 + M^2 sin^2\theta)u = -P \tag{3}$$

Where, $S = \frac{\rho a V_0}{\mu}$ is the suction or injection parameter, $\alpha = \frac{a}{\sqrt{k}}$ is the Permeability parameter, and $M = aB_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number.

3. Numerical solution

The dimensional differential Eq. (3) subjected to the boundary conditions given in Eq. (6) and Eq. (8) is solved using the finite difference technique. In this method, the derivative terms, occurring in the differential equations, have been replaced by their finite difference approximations. Central difference approximations of second order accuracy have been used because they are more accurate than forward and backward differences. Then, an iterative scheme is used to solve the linearized system of difference equations. The linearized system of equations based on what representd in the present paper the step size by *h,* the finite difference equations corresponding to Eq. (3) is as follows:

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} - S\left(\frac{u_{i+1} - u_{i-1}}{2h}\right) - (\alpha^2 + M^2 sin^2\theta)u_i = -P \tag{4}$$

where index i represents y. Then, simplifying Eq. (4), we get

$$u_i = C_1(u_{i+1} + u_{i-1}) - C_2(u_{i+1} - u_{i-1}) + C_3 \tag{5}$$

where, $C_1 = \frac{1}{2+h^2(\alpha^2+M^2sin^2\theta)}$, $C_2 = \frac{hS}{2\{2+h^2(\alpha^2+M^2sin^2\theta)\}}$ and $C_3 = \frac{h^2P}{2+h^2(\alpha^2+M^2sin^2\theta)}$ are constants.

4.1. Poiseuille flow

In Poiseuille flow, both upper and lower walls of the channel are kept stationary, and the flow in the channel is driven by the constant applied pressure gradient. Boundary conditions in dimensionless form at the walls with a no-slip condition yields

$$\left.\begin{array}{l} u = 0,\ \text{at}\ y = 0 \\ u = 0,\ \text{at}\ y = 1 \end{array}\right\} \tag{6}$$

The discretized boundary conditions (6) take the form as follows:

$$\left.\begin{array}{l} u_i = 0, \quad \text{at}\ i = 1 \\ u_i = 0,\ \text{at}\ i = n + 1 \end{array}\right\} \tag{7}$$

4.2. Couette-Poiseuille flow

In Couette-Poiseuille flow, one wall is stationary and the other moves with a constant velocity. We consider the lower wall as fixed and the upper wall as moving with constant velocity U in the X-direction. The boundary condition at the walls of the channel in dimensionless form with a no-slip condition at the walls yields

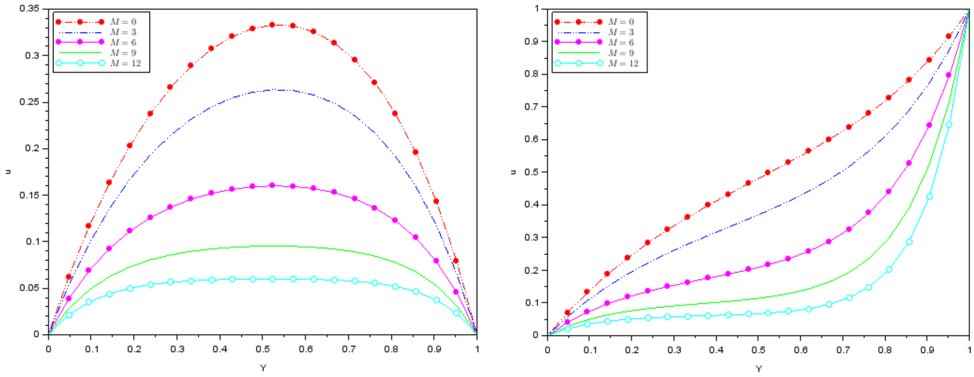
$$\left.\begin{array}{l} u = 0,\ \text{at}\ y = 0 \\ u = 1,\ \text{at}\ y = 1 \end{array}\right\} \tag{8}$$

The discretized boundary conditions (6) take the form as follows:

$$\left.\begin{array}{l} u_i = 0, \quad \text{at}\ i = 1 \\ u_i = 1,\ \text{at}\ i = n + 1 \end{array}\right\} \tag{9}$$

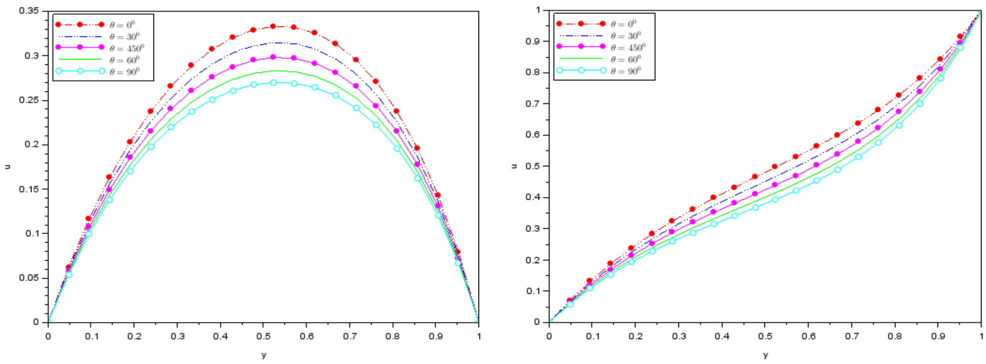
5. Results and discussion

Steady fully developed MHD flow through a channel filled with a porous medium and bounded by two infinite walls is investigated numerically for the cases (i) Poiseuille flow and (ii) Couette-Poiseuille flow; with uniform and injection at the walls in the presence of an inclined magnetic field have been investigated numerically by the finite difference method. In this technique, we have divided the region $0 \le y \le 1$ into 101 grid points. Numerical values of fluid velocity (u_i) have been iterated by the Gauss Seidal iteration method in Scilab programming [17] to a suitable number. Thus, the convergent solution of u_i is considered to be achieved when the maximum differences between two successive iterations are less than a tolerance, 10^{-7} [18]. The computed results of velocity are presented graphically for relevant parameters such as the Hartmann number (M), permeability parameter (α) suction/ /injection parameter (S) and inclination angle θ in Figures 2-6.



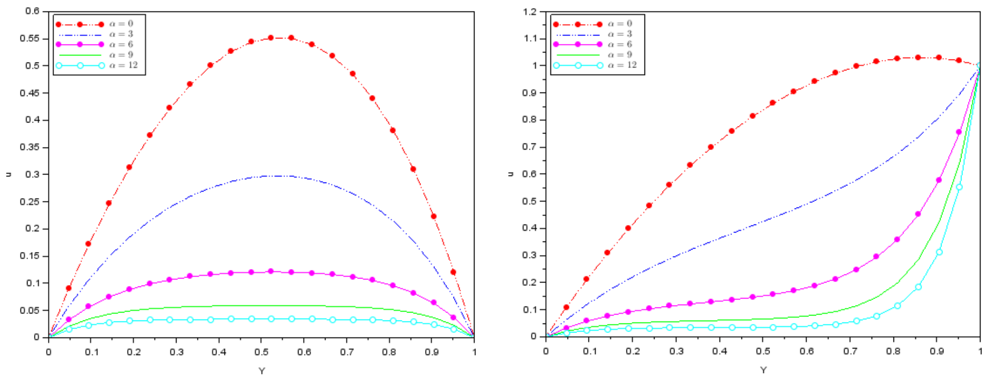
(a) (b)

Fig. 2. Velocity distributions for Poiseuille flow (a) and Couette-Poiseuille flow (b) for different values of Hartmann number M



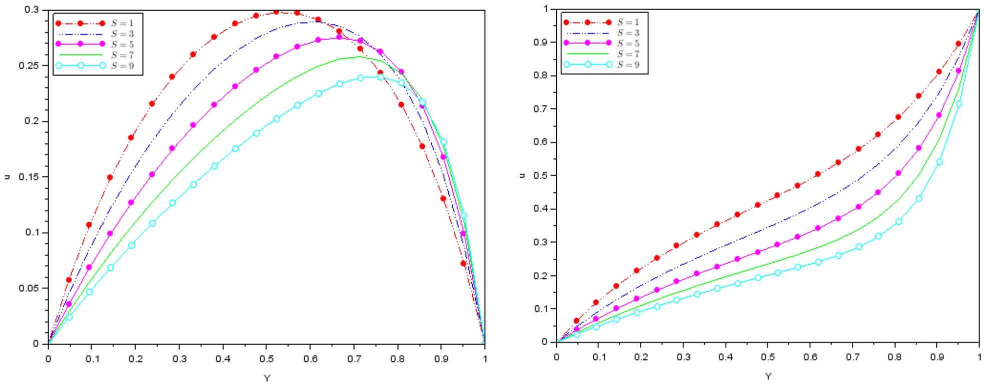
(a) (b)

Fig. 3. Velocity distributions for Poiseuille flow (a) and Couette-Poiseuille flow (b) for different values of inclination angle θ



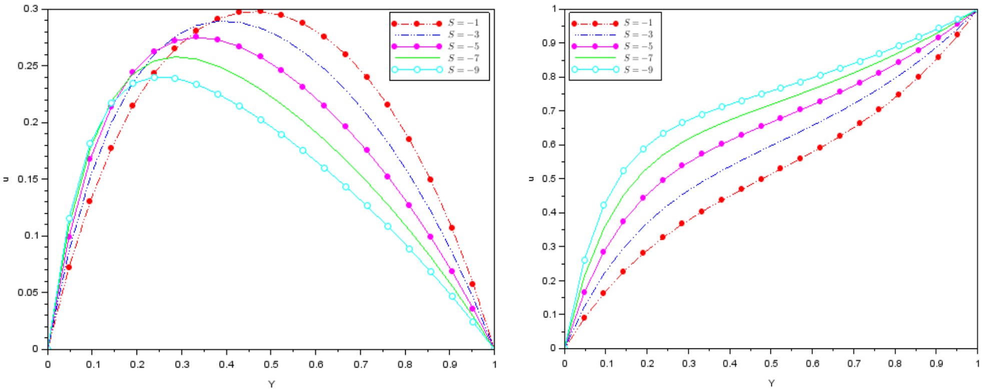
(a) (b)

Fig. 4. Velocity distributions for Poiseuille flow (a) and Couette-Poiseuille flow (b) for different values of permeability parameter α



(a) (b)

Fig. 5. Velocity distributions for Poiseuille flow (a) and Couette-Poiseuille flow (b) for different values of suction parameter S ($S > 0$)



(a) (b)

Fig. 6. Velocity distributions for Poiseuille flow (a) and Couette-Poiseuille flow (b) for different values of injection parameter S ($S < 0$)

Figures 2a and 2b present the velocity distributions within the channel for different values of Hartmann number M when $\alpha = 3$, $P = 5$, $\theta = 45°$ and $S = 1$ respectively for Poiseuille flow and Couette-Poiseuille flow. It is observed in both cases that the fluid velocity decreases as the Hartmann number M increases. This is due to fact that the Lorentz force increases as M increases, and creates a body force which acts opposite to the flow direction and thereby reducing fluid velocity.

Figures 3a and 3b depict the velocity distributions respectively for Poiseuille flow and Couette-Poiseuille flow within the channel for different values of inclination angle θ when $\alpha = 3$, $P = 5$, $M = 2$ and $S = 1$. It shows that fluid velocity decreases in both cases as the angle of inclination θ increases. This is because the intensity of the magnetic field increases as the angle of inclination increases thereby enhancing the Lorentz force.

Figures 4a and 4b depict the velocity distributions respectively for Poiseuille flow and Couette-Poiseuille flow within the channel for different values of permeability parameter α when $\theta = 45°$, $P = 5$, $M = 2$ and $S = 1$. Figures reveal that fluid velocity decreases as permeability parameter α decreases. Since $\alpha = \frac{a}{\sqrt{k}}$, so an increase in α means a decrease in permeability of the porous medium and reduces fluid velocity throughout the channel.

Figures 5 and 6 depict the effects of suction/injection parameter S on velocity distributions for Poiseuille flow and Couette-Poiseuille flow within the channel when $\theta = 45^0$, $P = 5$, $M = 2$ and $\alpha = 3$. It is observed in Figure 5a that fluid velocity decreases and swells toward the upper wall (i.e., sucked wall) as the suction parameter increases in the case of Poiseuille flow. Where in the case of Couette--Poiseuille flow, it is seen in Figure 5b that fluid velocity decreases as S increases. Reverse effects are observed for both cases of Poiseuille flow and Couette-Poiseuille flow respectively in Figures 6a and 6b as the injection parameter S $(S < 0)$ increases.

6. Conclusions

In this numerical investigation, we have considered steady fully developed MHD flow through a channel filled with a porous medium and bounded by two infinite walls is investigated for the cases (i) Poiseuille flow and (ii) Couette-Poiseuille flow; with uniform suction and injection at the walls in the presence of an inclined magnetic field have been investigated numerically by the finite difference method. It is noted that when inclination angle $\theta = 90°$, then this problem reduces to the problem considered by Verma and Gupta [1]. Computed results presented through graphs are summarized below:

1. For both Poiseuille and Couette-Poiseuille flow, fluid velocity decreases as the value of Hartmann number M increases.
2. For both Poiseuille and Couette-Poiseuille flow, fluid velocity decreases as the value of inclination angle θ increases.
3. For both Poiseuille and Couette-Poiseuille flow, fluid velocity decreases as the value of permeability parameter α increases.
4. For Poiseuille flow, fluid velocity decreases and swells toward the sucked wall as the value of suction parameter S $(S > 0)$ increases. Reverse effects occur when the value of injection parameter S $(S < 0)$ increases.

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