Journal of Applied Mathematics and Computational Mechanics 2024, 23(3), 74-85
www.amcm.pcz.pl p-ISSN 2299-9965
DOI: 10.17512/jamcm.2024.3.07 e-ISSN 2353-0588

DYNAMIC ANALYSIS OF THE WHEEL SUSPENSION ARM

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Received: 12 July 2024; Accepted: 20 September 2024

Abstract. This article is focused on the creation of a model and its dynamic analysis. The model analyzed is a part of the car wheel suspension mechanism. Input load effects are entered as nonlinear waveforms of individual components of forces and moments determined experimentally. Load curves in the pin are defined as a function of time in a three-component force. The load in the bushings is implemented in a six-component force bond (three force components and three moments) as a function of deformation. The output of the simulation process are defor-mations as well as reaction effects in bushings. In order to verify the results, the task was modelled in two simulation programs, namely in Adams and Matlab R2020b Simscape multibody. The monitored parameters of force effects and displacements differed minimally.

MSC 2010: *37N30, 68U20*
Keywords: *bushing, dynamic analysis, wheel suspension, Matlab, Adams*

1. Introduction

The automotive industry is one of the most dynamic and fastest growing industries globally. This has resulted in the continuous improvement and development of technologies used in this industry. This paper deals with the dynamic simulation of a part of the wheel suspension assembly, i.e., the multibody system (MBS), which has an essential impact on the comfort and drivability of the vehicle. Specifically, it concerns the lower triangular arm of a trapezoidal wheel suspension [1-3].

The paper includes an analysis of the attachment of the arms to the vehicle frame, for which bushings are generally used. The bushing is a flexible element of the joint that must fulfil the requirements related to passenger comfort, vibration reduction and the prolongation of the life of the suspension parts [4-6].

In the practical part of the thesis, there is a detailed description of the creation of the simulation of the arm using tools that simulate the bushings located at two points of this arm. The simulation is created in two programs, namely: MSC Adams and Matlab – Simscape multibody. The results of these two programs are compared with each other at different input loads.

The aim of this work is to describe the different design solutions for wheel suspension and their bushing connections [7, 8]. The simulation results in the practical part can be used to compare the states of the arm and bushings at different input loads using the nonlinear stiffness characteristics of the bushings [9].

2. Principles of analysis in ADAMS

The equations of motion are a system of differential and algebraic equations. In the ADAMS environment, the equations describing the behaviour of the system are generated during the model building process, and their system is composed of:

- 6 dynamic equations for each term (relating accelerations and forces),
- 6 kinematic equations for each term (relating position and velocity of each member),
- one algebraic equation for each prescribed motion,
- one algebraic equation for each scalar force component,
- any number of user-prescribed algebraic and differential equations I. order.

The solver compiles these equations into a single system of DAE equations, which the algorithm solves simultaneously for time-dependent values of the state vector.

The state vector of the resulting DAE system consists of:

- 3 Cartesian positions of the center of gravity of each member,
- 3 angles of orientation of the body with respect to the reference system,
- 3 sliding velocities for each member,
- 3 angular velocities for each member,
- applied forces,
- reaction forces (Lagrange multipliers).

Lagrange's equations of motion of the 2nd kind are used to describe the dynamical system of bound bodies:

$$f_j = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \sum_{i=1}^{m_v} \frac{\partial \Phi_i}{\partial q_j}\lambda_i - Q_j = 0, \text{ for } j = 1,2 \ldots n, \quad (1)$$

where:

L – Lagrangian ($L = E_K - E_p$),
E_K – kinetic energy of the system,
E_p – the potential energy of the system,
q_j – the generalised coordinates of the system,
Q_j – the genealised external forces acting on the system,
Φ – a system of constraint equations (algebraic equations of geometric and kinematic constraints),
n – the number of generalized coordinates,

m_v – number of constraints,
λ_i – Lagrange multiplier.

Thus, it is necessary to solve a system of algebraic and differential equations (1) of the form:

$$\boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{\lambda}, t) = \boldsymbol{0}, \tag{2}$$

under known initial conditions

$$\boldsymbol{q}(t = 0) = \boldsymbol{q}(0) \text{ and } \dot{\boldsymbol{q}}(t = 0) = \dot{\boldsymbol{q}}(0) \tag{3}$$

By introducing substitution $\boldsymbol{u}_v = \dot{\boldsymbol{q}}$, enter the following equation for the time step $n + 1$:

$$\boldsymbol{f}\left(\boldsymbol{q}^{n+1}, {\boldsymbol{u}_v}^{n+1}, {\dot{\boldsymbol{u}}_v}^{n+1}, \boldsymbol{\lambda}^{n+1}, t\right) = \boldsymbol{0} \tag{4}$$

Where $\boldsymbol{u}_v$ is an algebraic vector of generalized velocities.

In general, complex systems of bound bodies represented by DAEs cannot be solved in closed form. Therefore, different iterative procedures are used to solve with specific parameters. However, most of the approaches do not allow us to track the effect of changes in the individual parameters of the system. However, by using parameterization, we can eliminate this drawback to some extent.

The implicit Geare integration algorithm with a modified Newton-Raphson iterative method is used to solve strongly nonlinear problems (shocks, rapidly varying forces or stiffnesses depending on system parameters).

The general form of Geare's k-th order multistep backward BDF equation is:

$$\boldsymbol{q}^{n+1} = h\beta_0\dot{\boldsymbol{q}}^{n+1} + \sum_{j=1}^{k}\left(\alpha_j\boldsymbol{q}^{n-f+1} + h\beta_j\dot{\boldsymbol{q}}^{n-f+1}\right) \tag{5}$$

where:
$\boldsymbol{q}, \dot{\boldsymbol{q}}$ – an algebraic vector of generalised coordinates, or their velocities,
h – the time step from the n-th step to $n + 1$,
β_j, α_j – the integration coefficients,
k – the order of integration (degree of polynomial).

This equation relates displacements and velocities in the time step $n + 1$, and thus reduces the number of unknowns from $2n$ on n. This step does increase the number of equations, but it also yields a system of equations with fewer unknowns whose Jacobi matrix has a sparse structure. Using the substitution used in equation (4), we also rewrite the Geare equation:

$$\begin{gathered}\boldsymbol{g}(\boldsymbol{q}^{n+1}, {\boldsymbol{u}_v}^{n+1}, t) = \boldsymbol{0},\\ \boldsymbol{g} \equiv {\boldsymbol{u}_v}^{n+1} + \frac{E}{h\beta_0}\boldsymbol{q}^{n+1} + \frac{E}{h\beta_0}\left[\sum_{j=1}^{k}\left(\alpha_j\boldsymbol{q}^{n-j+1} + h\beta_j{\boldsymbol{u}_v}^{n-j+1}\right)\right]\end{gathered} \tag{6}$$

where E is the unit matrix.

3. Input parameters for the computational model

The specified model for the dynamic analysis is part of a trapezoidal axle, which is classified as an independent wheel suspension. The trapezoidal axle consists of a pair of superimposed arms, usually triangular in shape. Two peaks are located on the vehicle frame, the third on the wheel bearing house [10-12]. A spring with a shock absorber is most often mounted between the lower arm and the vehicle frame. This is one of the most common types of suspension.

The lower arm model of this type of suspension was chosen for the analysis. This is the most robust part of the suspension assembly, as it transmits the largest proportion of the forces acting on the wheel. For the analysis of the behaviour of the arm, which is loaded with a time-varying force, the positions of the three points of the body located at the tie points were sufficient. At point C, there is a spherical pin that is attached to the wheel bearing house, points A and B are attached to the vehicle frame by means of a flexible element – the bushing (Fig. 1). The coordinates of points A, B, C are given in Table 1.

Table 1. Coordinates of points A, B, C of the arm [mm]

Point		X	Y	Z
A	FAX PT3	1634.5	–400.6	341.8
B	FAX PT4	1952.5	–392.3	358.1
C	spherical pin	1606.7	–792.3	322.0

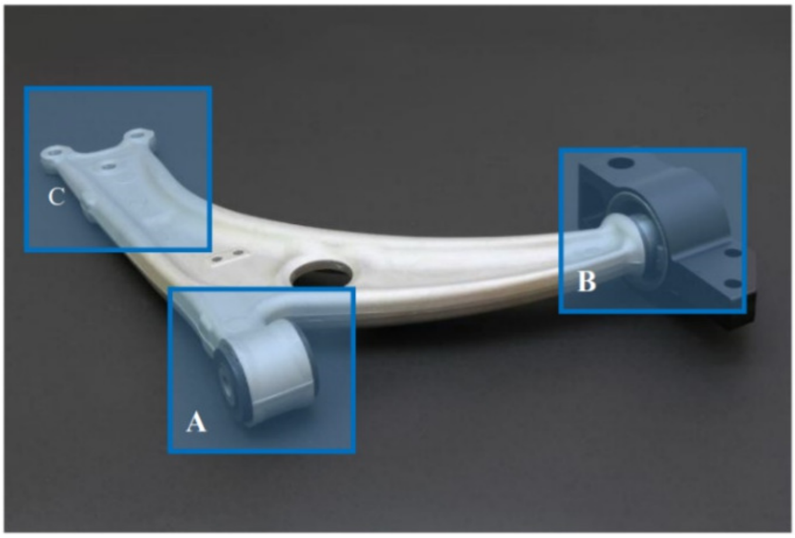

Fig. 1. Lower arm of the trapezoidal wheel suspension

Other input parameters were the force effects of the bushings (forces, moments) which were determined experimentally for each bushing. In the input data, the values of loading forces versus deformation (Fig. 2) or moments versus torsion (Fig. 3) were tabulated. This is all relative to the axes of the chosen coordinate system. It should be said that these dependencies did not show a linear progression, that is, we entered the stiffnesses into the constraints in the form of spline curves as a function of load and displacement/strain (Fig. 4). It is then possible to plot the waveforms of the force and moment components in the respective constraints (A, B). Figure 5 presents the waveform of the moment in constraint A acting in the Z-axis as a function of time. The waveforms can be plotted for all loads in each constraint.

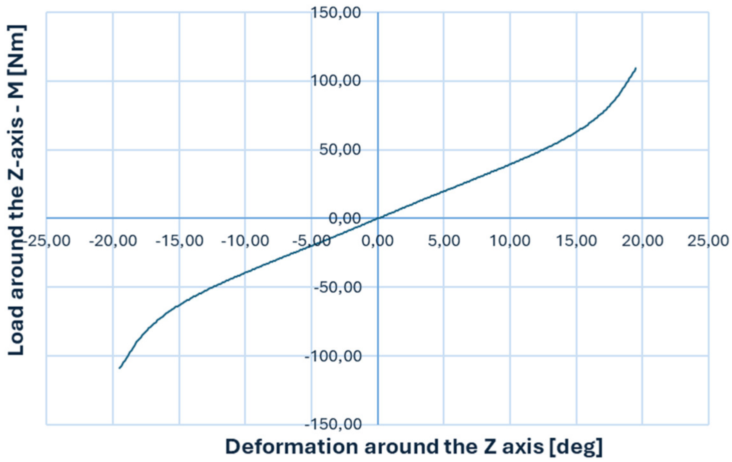

Fig. 2. Dependence of force and deformation in the Z-axis direction

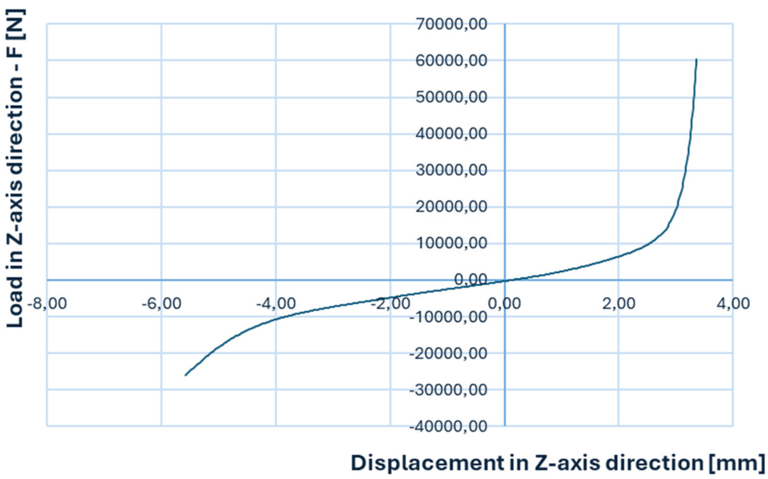

Fig. 3. Moment dependence relative to deformation/torsion about the Z-axis at point A

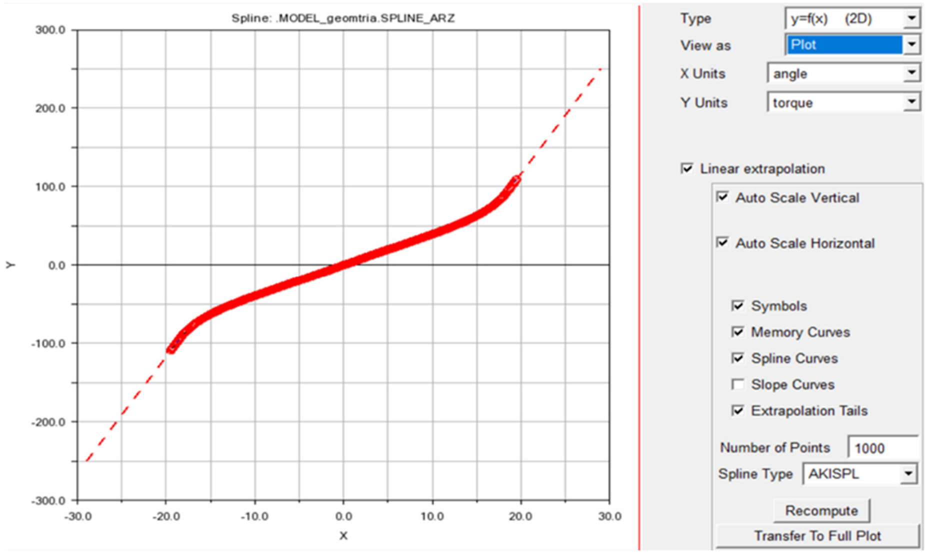

Fig. 4. Spline curve for constraint at point A as a function of the moment in Z-axis and the deformation (rotation) about Z

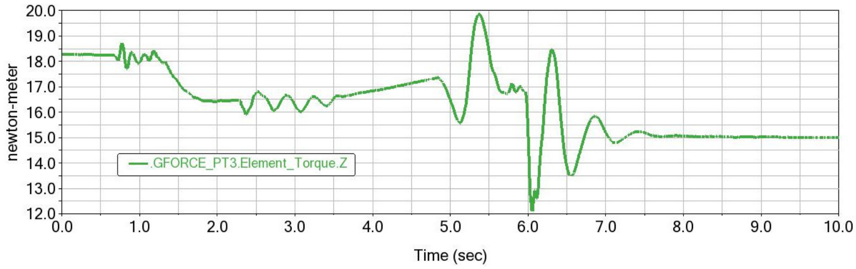

Fig. 5. The waveform of the moment in constraint A acting in the Z-axis as a function of time

4. Arm simulation in the Adams environment

The first step in creating the arm model is to create its geometry. The coordinates of points A, B, and C were adjusted so that the spherical pin is located at the origin of the coordinate system (0,0,0). The individual points were connected using construction lines called "polylines".

The disadvantage of using a simplified model lies in the inaccuracy of the results of a given simulation, but the inaccuracies can be eliminated if we know the individual inertia tensors, the gravity and the coordinates of the centre of gravity of this body. In Adams, we insert these characteristics using the "Modify body" window. In the case of an accurate solid model, we only enter the density of the material, and these parameters are automatically calculated by the software. Since these parameters were part of the assignment, a simplified model into which the input parameters are loaded is sufficient for the calculations.

General force (Gforce) between two bodies transfers translational and rotational loads. Here we will use a specific property of this 6 component force, namely: Bushing Like – which allows you to specify stiffness and damping coefficients and also allows the Adams View application to create a function expression for damping and stiffness based on the values of the coefficients. The individual components of this force represent the bushing properties. The results of the experimental measurements of the bushings are input into the spline curves.

The splines (curves) are entered into GForce as the characteristics of the bushings in each axis and around each axis using the AKISPL function.

The AKISPL function uses the Akima interpolation method to create a spline function in a set of data points. The data points to be interpolated are defined by the SPLINE command in the ADAMS/Solver dataset. SPLINE can represent a curve (x-y points) or a surface (x-y-z points) in the dataset. Interpolation in the y-axis direction is cubic, and interpolation in the z-direction is linear. AKISPL is very fast and gives good results for the value of the approximated function.

After all the input parameters were entered, the simulation was performed. The simulation was set according to the input forces that were part of the task, simulation time of 11.76 seconds and step length of 0.001 seconds. The Adams environment allows a graphical representation of the given simulation (Fig. 6).

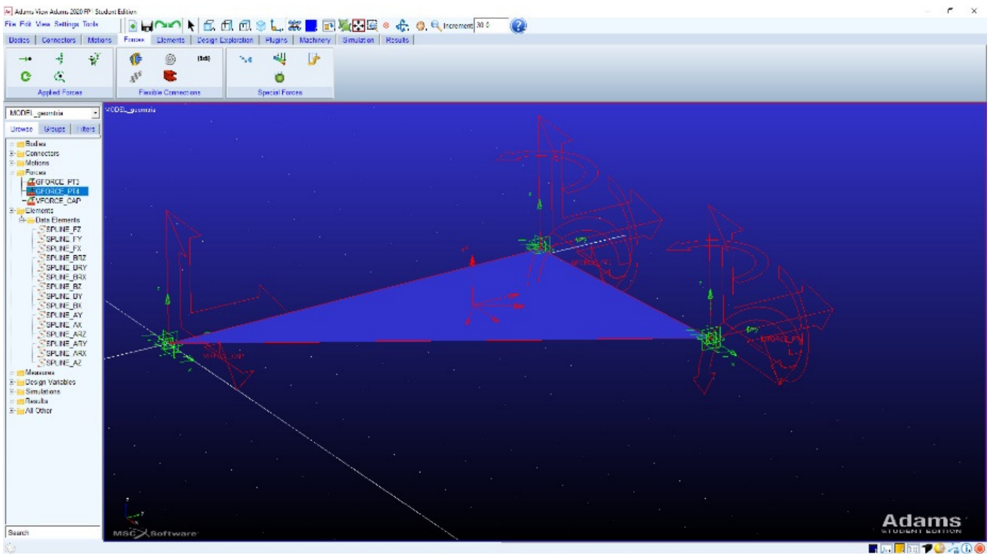

Fig. 6. Arm model in Adams

5. Arm simulation in the Matlab environment – Simscape multibody

Using the "Model" block, we have inserted a simplified model of the arm into the simulation, it is a triangular profile with a thickness of 50 mm, in the vertices of which points representing the bushings and the stud on which the specified force is applied. In order to be as close as possible to the real simulation, we have determined the moments of inertia of the body, its weight and the location of the centre of gravity according to the input to the "Model" block.

The properties of the bushings are entered using the "Bushing" block, and since it is only possible to enter the linear dependence of the bushing stiffness into the parameters, we also enter the stiffness via a spline curve. The change in stiffness is evident in the change in the resistance force (Force – Provided by input), the magnitude of which varies depending on the deformation (Sensing – Position) in the direction of the given axis.

We saved the stiffness dependencies on strain as a table in workspace and these values are then inserted as input data in the "Akima spline" block. This block uses interpolation to look for pairs of values, the deformation of a point in or around the axis enters the block. The output value of the block is the resistive force attributed to the bushing at a given deformation.

The so-called "Signal builder" was used to generate the input forces. This function allows you to generate a signal with arbitrary values in time. The values used in the simulation were loaded directly from the Excel file according to the input, the block processed them and created a time waveform and signal values. When testing this simulation in different conditions, it is easy to modify the signal or upload a new signal. The signal values come out of the block as a dimensionless number, which has to pass through the "Simulink-PS converter" block in order for the signal values to be entered into the next block as a load value.

A signal representing the magnitude of the load over time is applied to the body by the "External Force and Torque" block. Since only forces were applied to the pin in the specification, the moments remain zero in this case. If the input force values were constant, the "Constant" block can be used where the user simply sets a constant value and also enters the value into the model using the "Simulink PS-converter".

After connecting all the blocks, we get a complete model that is able to simulate the lower suspension arm under different conditions. From this model shown in Figure 7, it is possible to read out all the values needed for further analysis: displacements of points, rotational forces, etc.

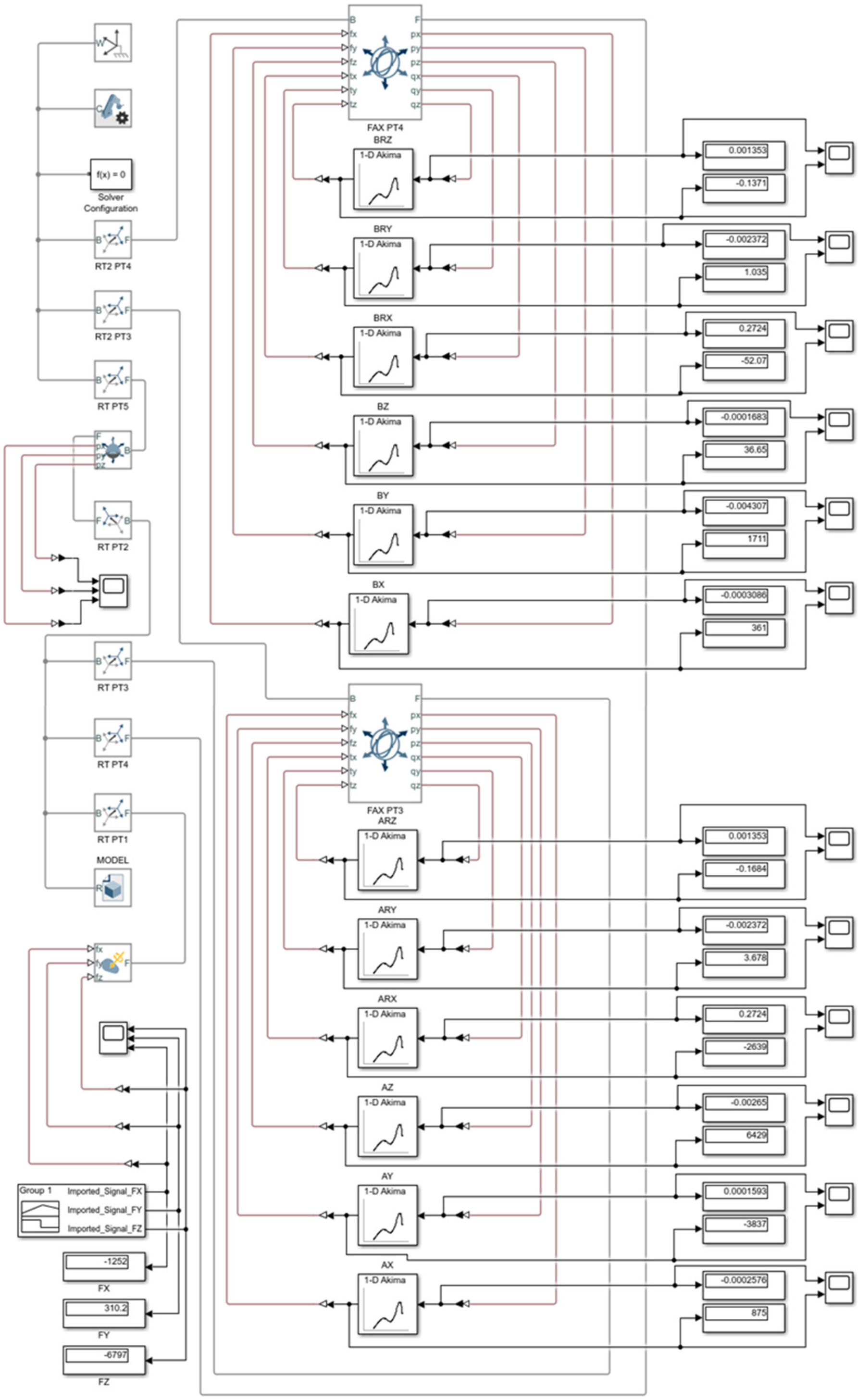

Fig. 7. Illustration of the complete computational model

6. Simulation processing results

Since the dynamic model of the suspension (arm) part of the car wheels was carried out in two environments, namely Adams and Matlab, in this chapter the loading, reaction effects and deformation at selected specific simulation times are presented and compared. The comparisons are made at a maximum load time of 4 seconds. It can be concluded that the observed parameters coincide with negligible deviations.

Table 2. Resulting values of loads and deformations at time t = 4 s

Input parameters						
Load values in time t = 4 s	Point	Axis	Input force [N]			
	C	X	2692.972			
		Y	–3672.658			
		Z	–5107.972			
Simulation results						
Program used for simulation	Point	Axis	Reaction in the bushing [N]	Moment applied to the bushing [Nm]	Deformation [mm]	Deviation of deformation [mm]
Adams View 2020	A	X	–3005.5	–1592.1	–0.6	–
		Y	7024.5	–44.3	–0.3	–
		Z	6346.3	–9.9	–1.6	–
	B	X	312.7	–22.3	–0.9	–
		Y	–3351.9	1.7	6.0	–
		Z	–1238.1	–5.5	–2.7	–
	C	X	–	–	11.4	–
		Y	–	–	11.4	–
		Z	–	–	–81.4	–
Matlab R2020b Simscape multibody	A	X	–3015.2	–1592.9	–0.8	–0.2
		Y	7018.7	–44.7	–0.5	–0.2
		Z	6337.8	–10.5	–1.8	–0.2
	B	X	306.9	–22.8	–1.7	–0.8
		Y	–3356.5	0.7	5.4	–0.6
		Z	–1243.6	–5.7	–3.1	–0.4
	C	X	–	–	11.4	–0.1
		Y	–	–	11.3	–0.1
		Z	–	–	–82.0	–0.6

7. Conclusions

The solution of the paper was the dynamic simulation and analysis of a part of the suspension assembly of a car wheel. The lower triangular arm of the trapezoidal wheel suspension was investigated.

The first part of the article presents the creation of the model with a detailed description of the procedure.

In the Adams environment, the arm was modeled as a triangle and fixed in three links. One of them was a spherical pin, modeled by a 3-component force (VForce), in which the components Fx, Fy and Fz exerted a variable loading force. This load was experimentally measured by the manufacturer, and its progression was modelled in this work in the form of a spline as a function of time.

In the other two constraints, the bushings were modelled with a six-component load into which measured functions were defined that did not exhibit a linear progression, i.e. we entered the stiffnesses into the constraints in the form of spline curves as a function of load and displacement/rotation. After running the simulation, the force effects at the locations defined by Gforce (bushings) were plotted.

In the Simscape multibody environment, a model was created consisting of preprogrammed "blocks" that were interconnected to form the final simulation algorithm. The simplified solid model enters the simulation as a "Solid" block in which a model prepared using an external modelling program is embedded. The magnitude of the input forces over time are represented by the "Signal builder" block, and enter the solid using the "External force" block.

The individual stiffnesses were entered using the "Bushing" block. The output value from this block was the deformation in the individual axes of the coordinate system; this value enters the "Akima spline" block. By interpolating the spline curves of the entered stiffnesses, the result is a load that presents us with the actual stiffness value at a given deformation. The load value enters as a reaction force into the bushing thus achieving the balance of forces.

After running the simulation, it is possible to read the results from the displays that show the actual values of loads and deformations. After the simulation, we can see the individual waveforms in the "Scope" blocks as a dependence on time t.

In conclusion, the observed parameters of force effects and displacements differed minimally, which confirms a correctly developed model with nonlinear waveforms of the modeled functions.

Acknowledgements

This work has been supported by Projects VEGA 1/0753/24 and KEGA 011ŽU-4/2022.

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