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TWO APPROACHES TO THE FORMATION OF A TOLERANCE AVERAGED EQUATIONS FOR ELASTODYNAMICS OF PERIODIC SOLIDS

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Abstract. The aim of this contribution is to formulate two different approaches to the computation of averaged moduli in the equations of the tolerance averaged model of microperiodic linear-elastic composites. In the first approach, which will be called basis function approach, we shall use some of the *FEM* concepts in order to establish the basis functions in the Galerkin approximation to the non-stationary periodic cell problem of elastodynamics. In the second approach, which will be called mode shape function approach, we shall also use *FEM* concepts in order to determine functions describing the character of free vibrations of the periodicity cell. This approach makes it possible to reduce a large number of unknowns occuring in the first approach.

1. Formulation of the problem

The subject of considerations is a linear-elastic microperiodic composite with a periodicity cell Δ. The exact dynamic behaviour of this composite is governed by the well-known equations of motion of the linear elasticity theory

$$\nabla \cdot (\mathsf{C} : \nabla \mathbf{u}) - \rho \ddot{\mathbf{u}} + \mathbf{f} = \mathbf{0} \tag{1}$$

where $\mathbf{u}(\cdot, t)$ is the displacement field at time t, $\mathsf{C}(\cdot), \rho(\cdot)$ are Δ-periodic piecewise-constant functions representing the elasticity tensor field and the mass density field, respectively, and $\mathbf{f}(\cdot, t)$ is the external force vector field at time t. Equations (1) are assumed to be satisfied in a region Ω occupied by the composite in its reference configuration. Moreover, they have non-continuous functional coefficients and that is why we look for various simplified models of such composites, which are governed by PDE's with constant coefficients.

In this contribution we restrict ourselves to the models obtained by an application of the tolerance averaging technique to equations (1) [1]. In contrast to the known homogenized model of a linear-elastic periodic medium, cf. [2], in the framework of the aforementioned model the dispersion phenomena can be analyzed. Using the tolerance averaging technique and denoting by $\mathbf{x}$ a position vector in E^3 we shall apply the averaging operator

$$\langle f\rangle(\mathbf{x},t)=\frac{1}{|\Delta|}\int\limits_{\Delta(\mathbf{x})} f(\mathbf{y},t)d\mathbf{y},\qquad \Delta\,(\mathbf{x}):=\mathbf{x}+\Delta$$

for every integrable function $f(\cdot,t)$. Setting $\mathbf{v}(\mathbf{x},t)=\langle\mathbf{u}\rangle(\mathbf{x},t)$, we shall decompose the displacement field into the sum $\mathbf{u}=\mathbf{v}+\mathbf{v}^*$, where $\langle\rho\mathbf{v}^*\rangle=\mathbf{0}$. Fields $\mathbf{v}(\cdot,t)$ and $\mathbf{v}^*(\cdot,t)$ will be called the macroscopic and the oscillating parts of the displacement field, respectively. The fundamental idea of the tolerance averaging is to supply the pair ($\mathbf{v}$, $\mathbf{v}^*$) with the suitable physical background to make it possible to treat these fields as an acceptable description of the kinematics of the composite. To this end we introduce concepts of slowly varying and periodic like functional spaces, denoted by $SV_\Delta(T)$ and $PL_\Delta(T)$, respectively, related to a certain tolerance system T [1]. We shall formulate the fundamental assumptions of the tolerance averaging modelling as certain restrictions imposed on the fields $\mathbf{v}$ and $\mathbf{v}^*$. Namely, away from the boundary of Ω, the macroscopic part $\mathbf{v}(\cdot,t)$ of the displacement field must be a slowly varying field, $\mathbf{v}(\cdot,t)\in SV_\Delta(T)$, and the oscillating part $\mathbf{v}^*(\cdot,t)$ of the displacement field must be a periodic like field, $\mathbf{v}^*(\cdot,t)\in PL_\Delta(T)$. The direct consequence of the second from these assumptions is that in a certain neighborhood of every periodicity cell $\Delta(\mathbf{x})$, situated away from the boundary of Ω, the oscillating part $\mathbf{v}^*(\cdot,t)$ of a displacement field can be approximated by a certain Δ-periodic field $\mathbf{v}^*_{\mathbf{x}}(\cdot,t)$, i.e. $\mathbf{v}^*(\mathbf{y},t)\cong\mathbf{v}^*_{\mathbf{x}}(\mathbf{y},t)$, $\mathbf{y}\in\Delta(\mathbf{x})$. Let us introduce a functional space

$$\widetilde{H}^1_{per}(\Delta):=\{\widetilde{\mathbf{v}}(\cdot)\in H^1_{per}(\Delta):\langle\widetilde{\mathbf{v}}\rangle=\mathbf{0}\}$$

It can be shown [1] that equations (1), under a decomposition $\mathbf{u}=\mathbf{v}+\mathbf{v}^*$ and assumptions $\mathbf{v}(\cdot,t)\in SV_\Delta(T)$, $\mathbf{v}^*(\cdot,t)\in PL_\Delta(T)$, imply the following periodic cell problem for the oscillating part $\mathbf{v}^*_{\mathbf{x}}(\cdot,t)$ of a displacement field:

For given $\mathbf{v}(\cdot)$ *and* $\Delta(\mathbf{x})\subset\Delta$ *find* Δ*-periodic field* $\mathbf{v}^*_{\mathbf{x}}(\cdot,t)\in\widetilde{H}^1_{per}(\Delta)$ *such that the variational equation*

$$\begin{aligned}\langle\nabla\widetilde{\mathbf{v}}:\mathsf{C}:\nabla\mathbf{v}^*_{\mathbf{x}}\rangle(\mathbf{x},t)+\langle\rho\widetilde{\mathbf{v}}\cdot\ddot{\mathbf{v}}^*_{\mathbf{x}}\rangle(\mathbf{x},t)=\qquad\qquad (CP)\\ =-\langle\nabla\widetilde{\mathbf{v}}:\mathsf{C}\rangle\cdot\nabla\mathbf{v}(\mathbf{x},t)-\langle\rho\widetilde{\mathbf{v}}\rangle\cdot\ddot{\mathbf{v}}(\mathbf{x},t)+\langle\widetilde{\mathbf{v}}\cdot\mathbf{f}\rangle(\mathbf{x},t)\end{aligned}$$

holds for every Δ*-periodic test field* $\widetilde{\mathbf{v}}(\cdot)\in\widetilde{H}^1_{per}(\Delta)$.

For details the reader is referred to [1]. As a rule, it is impossible to find the exact solution to the above problem. That is why we shall look for an approximate solution to (*CP*) in the form (here and hereafter summation over $A = 1,..., N$ holds)

$$\mathbf{v}^{*}_{\mathbf{x}}(\mathbf{y},t) = h^{A}(\mathbf{y})\mathbf{q}^{A}(\mathbf{x},t), \qquad \mathbf{y} \in \Delta(\mathbf{x}), \qquad A = 1,...,N$$

where $\{h^{A}\}_{A=1}^{N}$ is a basis which determines a certain N-dimensional subspace $W^{h}(\Delta)$ in $\widetilde{H}^{1}_{per}(\Delta)$ and $\mathbf{q}^{A}(\mathbf{x},t)$, $A = 1,...,N$, are new kinematical unknowns. Hence, $h^{A}(\cdot)$ are continuous Δ-periodic linear independent functions satisfying condition $\langle h^{A} \rangle = 0$. Moreover, it can be shown [1] that $\mathbf{q}^{A}(\cdot,t)$ are slowly varying fields, $\mathbf{q}(\cdot,t) \in SV_{\Delta}(T)$, $\mathbf{q} = (\mathbf{q}^{1},...,\mathbf{q}^{N})$. Under aforementioned assumptions and taking into account the above approximation to (*CP*), from the procedure given in [1] we obtain the averaged model of elastodynamics for periodic solids represented by the following system of equations

$$\begin{aligned} &\nabla \cdot (\langle \mathsf{C} \rangle : \nabla \mathbf{v} + \langle \nabla h^{B} \cdot \mathsf{C} \rangle \cdot \mathbf{q}^{B}) - \langle \rho \rangle \ddot{\mathbf{v}} - \langle \rho h^{B} \rangle \ddot{\mathbf{q}}^{B} = \langle \mathbf{f} \rangle \\ &\langle \rho h^{A} h^{B} \rangle \ddot{\mathbf{q}}^{B} + \langle \rho h^{A} \rangle \ddot{\mathbf{v}} + \langle \nabla h^{A} \cdot \mathsf{C} \cdot \nabla h^{B} \rangle \cdot \mathbf{q}^{B} + \langle \nabla h^{A} \cdot \mathsf{C} \rangle : \nabla \mathbf{v} = \mathbf{0} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad A = 1,...,N \end{aligned} \tag{2}$$

for slowly varying fields $\mathbf{v}(\cdot,t)$ and $\mathbf{q}^{A}(\cdot,t)$ where A, B run over 1,...,N (summation convention holds) and where for the sake of simplicity we have assumed that $\mathbf{f}(\cdot,t)$ is a slowly varying function. Equations (2) have constant coefficients which will be called the averaged moduli.

The aim of this contribution is to propose two different approaches in the method of computations of the averaged moduli in Eq. (2). In both cases we have to determine the basis functions $\{h^{A}\}_{A=1}^{N}$. This problem was discussed in [3] for the second order scalar elliptic equation. Similarly as in [3] we shall adapt some of the *FEM* concepts in order to establish the basis functions h^{A}, $A = 1,...,N$. For the sake of simplicity we shall restrict considerations to the plane problems by assuming that all fields are independent of x^{3}. Hence by $\mathbf{x}$, $\mathbf{y}$ we shall denote position vectors on the $Ox^{1}x^{2}$-plane.

2. Computation of the averaged moduli

2.1. Basis function approach

Let Λ be a Bravais lattice in E^{2} with the vector basis $\{\mathbf{d}^{1}, \mathbf{d}^{2}\}$ and let Δ be a two-dimensional periodicity cell related to Λ in the sense given in [4]. For example,

Δ can be a parallelogram in E^2 with a center at $\mathbf{x} = \mathbf{0}$. The proposed approach will be realized in eight steps. The *FEM* concepts and the terminology are based on those given in [5].

***Step* 1**. *Periodic partition of* Δ *into symplexes* Δ^e, $e = 1,...,E$:

$$\bar{\Delta} = \bigcup_e \overline{\Delta^e}, \qquad \Delta^e \cap \Delta^f = \varnothing \quad \textit{for every} \quad e \neq f$$

The above partition determines in Δ a certain set N of what are called the global nodes, cf. [5], p. 198.

***Step* 2**. *Numbering of global nodes in* $\bar{\Delta}$

$$\mathsf{N} = \{\mathbf{y}^1,..., \mathbf{y}^n\}$$

We define

$$\mathsf{N}^{\mathrm{o}} := \mathsf{N} \cap \Delta = \{\mathbf{x}^a \in \mathsf{N}: \mathbf{x}^a \in \Delta\}$$

as a set of what will be called the internal nodes; hence $\mathsf{N} \setminus \mathsf{N}^{\mathrm{o}}$ will be referred to as a set of boundary nodes.

Definition. Nodes $\mathbf{y}^a, \mathbf{y}^b \in \mathsf{N}$ will be called equivallent, $\mathbf{y}^a \sim \mathbf{y}^b$, if and only if

$$\mathbf{y}^a - \mathbf{y}^b \in \Lambda$$

Conclusion. If $\mathbf{y}^a \in \mathsf{N}^{\mathrm{o}}$ then $\mathbf{y}^a$ is equivalent only to itself, i.e. the equivalent classes of nodes belonging to N^{o} are singletons.

The vertexes of every $\overline{\Delta^e}$ will be called the local nodes in $\overline{\Delta^e}$, cf. [5], p. 198.

***Step* 3**. *Numbering of local nodes in every* $\overline{\Delta^e}$, $e = 1,...,E$

$$\mathsf{N}^e = \{\mathbf{y}^1_e, \mathbf{y}^2_e, \mathbf{y}^3_e\}$$

Hence

$$\mathbf{y}^i_e = \sum_{a=1}^{n} {}^{(e)}B^i_a \mathbf{y}^a, \quad i = 1,2,3$$

where $\{{}^{(e)}\mathrm{B}^i_a\}$ is for every e a Boolean $3 \times n$ matrix, cf. [5], p. 202.

***Step* 4**. *Numbering of sets of equivalent nodes.*

$$\mathsf{N}/\sim = \{\mathsf{N}^1, \mathsf{N}^2, ..., \mathsf{N}^{N+1}\}$$

Hence N^K, $K = 1,...,N+1$, are disjoined sets of equivalent nodes, $N+1 < n$. Setting $I_K := \{a \in \{1,...,n\}: \mathbf{x}^a \in \mathsf{N}^K\}$ we have $\mathsf{N}^K := \{\mathbf{y}^a \in \mathsf{N}: a \in I_K\}$.

Step 5. *Formation of the local interpolation function*, cf. [5], p. 206.

The local interpolation functions will be denoted by ψ_i^e, $e = 1,...,E$, $i = 1,2,3$, and defined by conditions:

1) $\psi_i^e(\mathbf{y}_e^j) = \delta_i^j$,

2) $\psi_i^e = 0$ *in every* $\overline{\Delta} \setminus \overline{\Delta}^e$,

3) ψ_i^e *are linear in every* $\overline{\Delta}^e$.

Step 6. *Formation of the global interpolation functions*, cf. [5], p. 212.

The global interpolation functions are denoted by $\phi_a(\mathbf{y})$, $\mathbf{y} \in \overline{\Delta}$, and will be defined for every $\mathbf{y} \in \overline{\Delta}$ by

$$\phi_a(\mathbf{y}) = \sup_e \{\sum_{i=1}^{3} B_a^{i(e)} \psi_i^e(\mathbf{y})\} \tag{3}$$

Hence ϕ_a satisfy conditions

(*i*) $\phi_a \in C^0(\overline{\Delta})$,

(*ii*) $\phi_a(\mathbf{y}^b) = \delta_a^b$ *for every* $a,b \in \{1,...,n\}$,

(*iii*) ϕ_a *are linear in every* $\overline{\Delta}^e$, $e = 1,...,E$.

Step 7. *Formulation of the periodic interpolation functions*.

Periodic interpolation functions will be denoted by $\widetilde{\phi}_K(\mathbf{y})$, $\mathbf{y} \in \overline{\Delta}$, $K = 1,...,N+1$, and defined by

$$\widetilde{\phi}_K(\mathbf{y}) = \sum_{a=1}^{n} B_{Ka} \phi_a(\mathbf{y}), \quad \mathbf{y} \in \overline{\Delta} \qquad K = 1,...,N+1 \tag{4}$$

where $\{B_{Ka}\}$ is a Boolean $(N+1) \times n$ matrix given by

$$B_{Ka} = \begin{cases} 1 & \text{if} \quad a \in I_K \\ 0 & \text{if} \quad \text{otherwise} \end{cases}$$

Conclusion. The global interpolation functions $\widetilde{\phi}_K(\cdot)$ satisfy condition

$$\sum_{K=1}^{N+1} \widetilde{\phi}_K(\mathbf{y}) = 1$$

and hence are linear dependent.

***Step* 8**. *Formulation of basis functions h^A in equations* (2).
The above functions will be defined by

$$h^A(\mathbf{y}) = \alpha_A(\mathbf{y})\widetilde{\phi}_A(\mathbf{y}) + \alpha_{A+1}(\mathbf{y})\widetilde{\phi}_{A+1}(\mathbf{y}), \qquad \mathbf{y} \in \overline{\Delta}, \qquad A = 1,...,N$$

where α_A, $A = 1,...,N$, are constants and $\alpha_A = \alpha_A(l) \in O(l)$, $l = diam\Delta$. Moreover

$$\alpha_A\langle\widetilde{\phi}_A\rangle + \alpha_{A+1}\langle\widetilde{\phi}_{A+1}\rangle = 0, \qquad A = 1,...,N$$

Hence $\langle h^A \rangle = 0$ for $A = 1,...,N$.

Conclusion. Functions $\{h^1, h^2,...,h^N\}$ constitute the basis in the N-dimensional subspace $W^h(\Delta)$ in $\widetilde{H}^1_{per}(\Delta)$.

Remark. Functions h^A are determined up to a multiplicative constant and, for example, can be assumed in the form

$$h^A(\mathbf{y}) = l\widetilde{\phi}_A(\mathbf{y})/\langle\widetilde{\phi}_A\rangle - l\widetilde{\phi}_{A+1}(\mathbf{y})/\langle\widetilde{\phi}_{A+1}\rangle, \qquad A = 1,...,N \tag{5}$$

After determining the basis functions h^A we can calculate, for the known Δ-periodic $\mathsf{C}(\cdot)$ and $\rho(\cdot)$, the averaged moduli $\langle\nabla h^B \cdot \mathsf{C}\rangle$, $\langle\nabla h^B \cdot \mathsf{C} \cdot \nabla h^B\rangle$, $\langle\rho h^A h^B\rangle$ and $\langle\rho h^A\rangle$, which are constant coefficients in equations (2).

It can be shown that the averaged gradients of the shape functions h^A vanish, i.e. $\langle\nabla h^A\rangle = 0$ for $A = 1,...,N$.

2.2. Mode shape function approach

In most cases the basis function approach to the computation of the averaged moduli leads to a large number N of functions $\mathbf{q}^A$, $A = 1,...,N$, and hence the model obtained may not constitute a proper tool for the analysis of special elastodynamic problems. To eliminate this drawback we shall propose an alternative approach, which leads to the concept of what are called mode shape functions. Roughly speaking, they are vector functions which are Δ-periodic, have the mean values over Δ equal to zero and describe free periodic vibrations of the periodicity cell. The above functions, restricted to the first m free vibrations modes, will be denoted

by $\mathbf{g}^A(\cdot)$, $A = 1,...,M$. To determine the above functions we shall assign to the periodic cell problem (*CP*) the eigenvalue problem given by

Find an eigenvalue λ *and a corresponding eigenfunction* $\mathbf{g}(\cdot) \in \widetilde{H}^1_{per}(\Delta)$ *for the variational equation*

$$\langle \nabla \widetilde{\mathbf{v}} : \mathsf{C} : \nabla \mathbf{g} \rangle - \lambda \langle \rho \widetilde{\mathbf{v}} \cdot \mathbf{g} \rangle = 0 \qquad (ECP)$$

which has to hold for every Δ*-periodic test field* $\widetilde{\mathbf{v}}(\cdot) \in \widetilde{H}^1_{per}(\Delta)$

As a rule, it is impossible to find the exact solution to the above problem. That is why we shall look for an approximate solution to (*ECP*) by the *FEM* approach. To this end we introduce the M dimensional subspace $H^h(\Delta)$ in $\widetilde{H}^1_{per}(\Delta)$ with the basis ϕ_a, $a = 1,...,n$, which has the same form as that introduced in the subsection 2.1. It follows that we shall look for an approximate solution to the eigenvalue problem (*ECP*) in the form

$$\mathbf{g}(\mathbf{x}) = \sum_{a=1}^{n} \mathbf{a}_a \phi_a(\mathbf{x}) \qquad (6)$$

for unknown vector coefficients $\mathbf{a}_a$. Assuming test functions related to (*ECP*) in the form

$$\widetilde{\mathbf{v}}(\mathbf{x}) = \sum_{b=1}^{n} \widetilde{\mathbf{a}}_b \phi_b(\mathbf{x}) \qquad (7)$$

for arbitrary vector coefficients $\widetilde{\mathbf{a}}_b$, after substituting (6), (7) to (*ECP*), we obtain the eigenvalue problem

$$\sum_{b=1}^{n} \langle \nabla \phi_a \cdot \mathsf{C} \cdot \nabla \phi_b \rangle \cdot \mathbf{a}_b = \lambda \sum_{b=1}^{n} \langle \rho \phi_a \phi_b \rangle \mathbf{a}_b \,, \qquad a = 1,...,n \qquad (8)$$

for $\mathbf{a} = (\mathbf{a}_1,..., \mathbf{a}_n) \in R^{2n}$ and $\lambda \in R$. Let $\mathbf{a} = (\mathbf{a}^1,..., \mathbf{a}^m)$ and $(\lambda^1,..., \lambda^m)$, where $\mathbf{a}^1 = (\mathbf{a}^1_1,..., \mathbf{a}^1_n)$,...,$\mathbf{a}^{\mathrm{m}} = (\mathbf{a}^m_1,..., \mathbf{a}^m_n)$, be the m-tuples of the first m eigenvectors and corresponding m eigenvalues, $m < n$, of the above problem. It follows that, for every $s = 1,...,m$, functions

$$\mathbf{g}^s(\mathbf{x}) = \sum_{a=1}^{m} \mathbf{a}^s_a \phi_a(\mathbf{x}) \,, \qquad s = 1,...,m \qquad (9)$$

describes periodic free vibrations of the periodic cell of the composite under consideration. The approximate solution to (*CP*) will be now assume in the form (here and hereafter summation over $s = 1,...,m$ holds)

$$\mathbf{v}^{*}_{\mathbf{x}}(\mathbf{y},t) = \mathbf{g}^{s}(\mathbf{y})w^{s}(\mathbf{x},t), \quad \mathbf{y} \in \Delta(\mathbf{x}), \qquad s = 1,...,m \tag{10}$$

where, as a rule, $m << n$. Fields $w^{s}(\cdot,t)$, $s = 1,...,m$, are new kinematical unknowns and $\mathbf{g}^{s}(\cdot)$ are the known continuous Δ-periodic linear independent functions satisfying condition $\langle \mathbf{g}^{s} \rangle = 0$. Moreover, it can be shown [1] that $w^{s}(\cdot,t)$ are slowly varying fields, $w^{s}(\cdot,t) \in SV_{\Delta}(T)$.

Substituting the *RHS* of (10) into (*CP*), after some manipulations, we obtain the system of equations for w^{s}. Using the procedure described in [1] we arrive finally at the averaged model for periodic solids represented by the system of equations

$$\begin{aligned} &\nabla \cdot (\langle \mathsf{C} \rangle : \nabla \mathbf{v} + \langle \nabla \mathbf{g}^{s} \cdot \mathsf{C} \rangle w^{s}) - \langle \rho \rangle \ddot{\mathbf{v}} - \langle \rho \mathbf{g}^{s} \rangle \ddot{w}^{s} = \langle \mathbf{f} \rangle \\ &\langle \rho \mathbf{g}^{s} \cdot \mathbf{g}^{r} \rangle \ddot{w}^{r} + \langle \rho \mathbf{g}^{s} \rangle \cdot \ddot{\mathbf{v}} + \langle \nabla \mathbf{g}^{s} : \mathsf{C} : \nabla \mathbf{g}^{r} \rangle w^{r} + \langle \nabla \mathbf{g}^{s} : \mathsf{C} \rangle : \nabla \mathbf{v} = \mathbf{0} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad s = 1,...,m \end{aligned} \tag{11}$$

After determining the system of vectors $\mathbf{a}^{s}_{a}$, $s = 1,...,m$, $a = 1,...,n$, and applying the formula (9), we can calculate, for the known Δ-periodic $\mathsf{C}(\cdot)$ and $\rho(\cdot)$, the averaged moduli $\langle \nabla \mathbf{g}^{s} \cdot \mathsf{C} \rangle$, $\langle \rho \mathbf{g}^{s} \rangle$, $\langle \rho \mathbf{g}^{s} \cdot \mathbf{g}^{r} \rangle$, $\langle \nabla \mathbf{g}^{s} : \mathsf{C} : \nabla \mathbf{g}^{r} \rangle$, which are constant coefficients in equations (10).

Conclusions

The new elements and informations related to the modelling of micro-periodic solids, which have been presented in this contribution, can be summarized as follows:

1° By enclosing the concept of the periodic interpolation functions to the known formulation of the *FEM* method, two approaches to the calculation of the averaged moduli in the tolerance averaging equations for microperiodic solids have been proposed.

2° In the first approach, named the *basis function approach*, the averaged moduli are calculated on the basis of functions h^{A} as certain periodic interpolation functions. This approach leads to the large number of equations in the periodic cell problem (*CP*).

3° In the second approach, named the *mode shape function approach*, the calculation of the averaged moduli is realized by the representing a solution to the periodic cell problem (*CP*) in the form of the eigenfunctions related to a certain

eigenvalue problem. Functional coefficients $\mathbf{g}^s(\cdot)$ in this form are certain approximation to free vibrations of the periodic cell. The mode shape function approach makes it possible to reduce a large number of equations the periodic cell problem (*CP*).

The examples of application of the proposed procedures will be published elsewhere.

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