# NUMERICAL SIMULATION OF TRANSIENT HEAT DIFFUSION USING THE BEM

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**Abstract**. The paper contains the short discussion of the scientific and practical results presented in the monograph (doctoral theses) 'Theoretical and practical application of the BEM in numerical modelling of unsteady heat transfer' done by S. Freus. The main subject of the work is the application of the so-called I scheme of the boundary element method for numerical simulation of unsteady heat conduction in homogeneous and non-homogeneous solid bodies with complex shape (the 2D problems are discussed). The BEM algorithm constitutes a base for professional code construction. The user of the code introduces only the shape of domain considered and its thermophysical parameters, the parameters of boundary conditions and the initial one. The simulation process is realised self-actingly.

## 1. The boundary element method

We will consider the homogeneous domain with parameters $\lambda$, $c$ (thermal conductivity and volumetric specific heat) in which the diffusion process is determined by equation

$$x \in \Omega: \quad c\frac{\partial T(x,t)}{\partial t} = \lambda \nabla^2 T(x,t) + Q(x,t) \tag{1}$$

where $Q$ is the source function.

The boundary conditions given on the outer surface of the system can be written in general form

$$x \in \Gamma: \quad \Phi\left[T(x,t), \frac{\partial T(x,t)}{\partial n}\right] = 0 \tag{2}$$

where $\partial / \partial n$ denotes a normal derivative.

The initial condition is also known $t = 0$, $T(x, 0) = T_0$.

We introduce the time grid

$$0 = t^0 < t^1 < \ldots < t^{f-1} < t^f < \ldots < \infty, \;\; \Delta t = t^f - t^{f-1} \tag{3}$$

If the 1st scheme of the BEM is taken into account [1, 2] then the transition $t^{f-1} \rightarrow t^f$ is treated as a separate task and the adequate BEM equation is of the form

$$\begin{gathered}
B(\xi)T(\xi, t^f) + \frac{1}{c}\int_{t^{f-1}}^{t^f}\int_{\Gamma} T^*(\xi, x, t^f, t) q(x, t) d\Gamma dt \\
= \frac{1}{c}\int_{t^{f-1}}^{t^f}\int_{\Gamma} q^*(\xi, x, t^f, t) T(x, t) d\Gamma dt + \iint_{\Omega} T^*(\xi, x, t^f, t^{f-1}) T(x, t^{f-1}) d\Omega \\
+ \frac{1}{c}\int_{t^{f-1}}^{t^f}\iint_{\Omega} Q(x, t) T^*(\xi, x, t^f, t) d\Omega dt
\end{gathered} \tag{4}$$

In equation (4) $T^*$ is the fundamental solution [1, 2]

$$T^*(\xi, x, t^f, t) = \frac{1}{4\pi a(t^f - t)} \exp\left[-\frac{r^2}{4a(t^f - t)}\right] \tag{5}$$

where $r$ is the distance from the point under consideration $x$ to the observation point $\xi$. In equation (4) $B(\xi)$ is the coefficient from the interval (0, 1), $q(x, t)$ is the given boundary heat flux, $T(x, t)$ is the given boundary temperature, $T(x, t^{f-1})$ is the internet temperature at the moment $t^{f-1}$, $a = \lambda/c$, while

$$q^*(\xi, x, t^f, t) = -\lambda \frac{\partial T^*(\xi, x, t^f, t)}{\partial n} \tag{6}$$

If we use the constant elements with respect to time then the boundary integral equation (4) takes a form

$$\begin{gathered}
B(\xi)T(\xi, t^f) + \int_{\Gamma} q(x, t) g(\xi, x) d\Gamma = \int_{\Gamma} T(x, t^f) h(\xi, x) d\Gamma + \\
+ \iint_{\Omega} q^*(\xi, x, t^f, t^{f-1}) T(x, t^{f-1}) d\Omega + \iint_{\Omega} Q(x, t^{f-1}) g(\xi, x) d\Omega
\end{gathered} \tag{7}$$

where

$$h(\xi, x) = \frac{1}{c}\int_{t^{f-1}}^{t^f} q^*(\xi, x, t^f, t)\, dt \tag{8}$$

and

$$g(\xi, x) = \frac{1}{c}\int_{t^{f-1}}^{t^f} T^*(\xi, x, t^f, t)\, dt \tag{9}$$

In numerical realization we consider the following discrete form of the boundary integral equation (7)

$$B(\xi^i)T(\xi^i,t^f)+\sum_{j=1}^{N}\int_{\Gamma_j} q(x,t^f)g(\xi^i,x)d\Gamma_j=\sum_{j=1}^{N}\int_{\Gamma_j} T(x,t^f)h(\xi^i,x)d\Gamma_j$$
$$+\sum_{l=1}^{L}\iint_{\Omega_l} q^*(\xi^i,x,t^f,t^{f-1})T(x,t^{f-1})d\Omega_l+\sum_{l=1}^{L}\iint_{\Omega_l} Q(x,t^{f-1})g(\xi^i,x)d\Omega_l \tag{10}$$

On the stage of numerical computations the linear boundary elements and the linear internal cells have been used. The details connected with the application of a such type of temperature approximation can be found in [3]. The equation (10) allows to find the 'missing' boundary temperatures and heat fluxes, and next to determine the internal temperature field for time $t^f$. In the monograph previously quoted one can also find the details concerning the BEM equation for the non-homogeneous domains.

## 2. The mesh generation

The first part of the code is connected with the boundary and the internal of domain discretisation. The program user introduces the shapes of external and internal boundaries (e.g. holes) using the line segments and circular arcs. For the successive parts of the boundary the type of the boundary conditions and their parameters are similarly defined. In Figure 1 the example of the 2D domain shape introduced is shown.

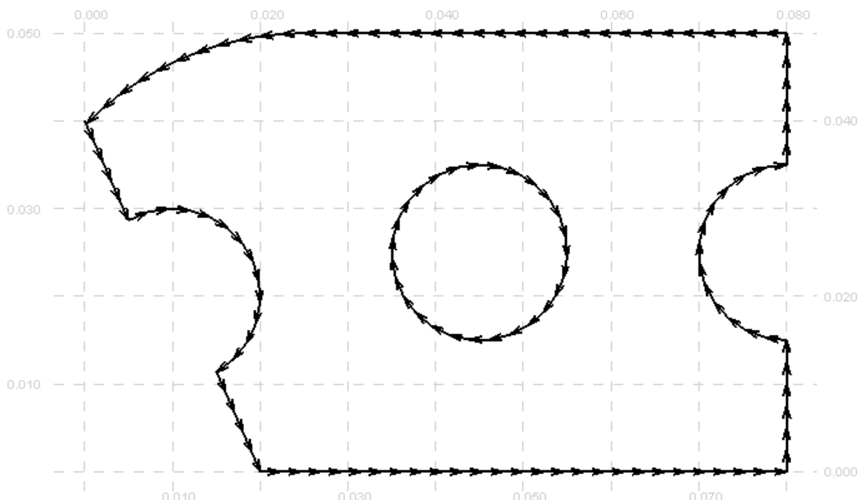


Fig. 1. Domain considered

On the basis of assumed length of boundary element the mean value of internal triangle side is found and the part of interior is covered by the set of equilateral triangles (Fig. 2).

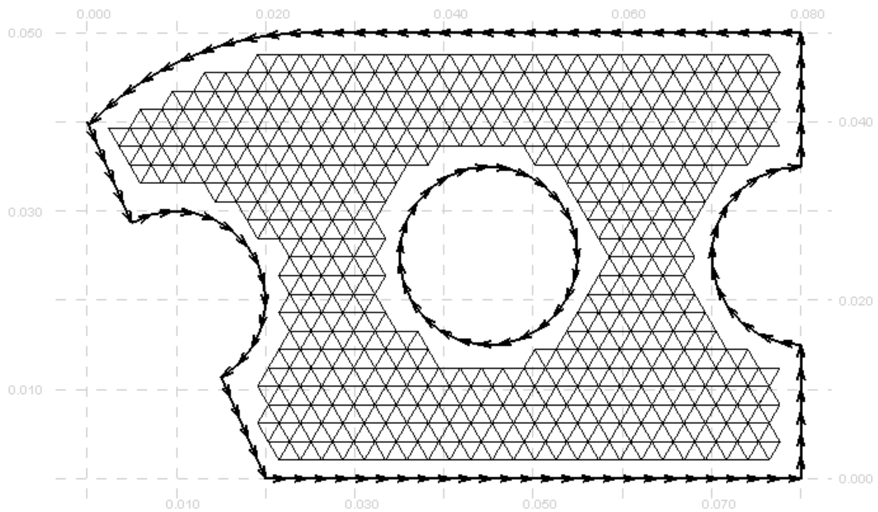


Fig. 2. The first stage of interior discretization

In the next stage of interior discretization the 'boundary triangles' are constructed using the boundary points and the vertexes of the terminal equilateral triangles, at the same time a certain optimisation procedure is taken into account. The final effect is shown in Figure 3.

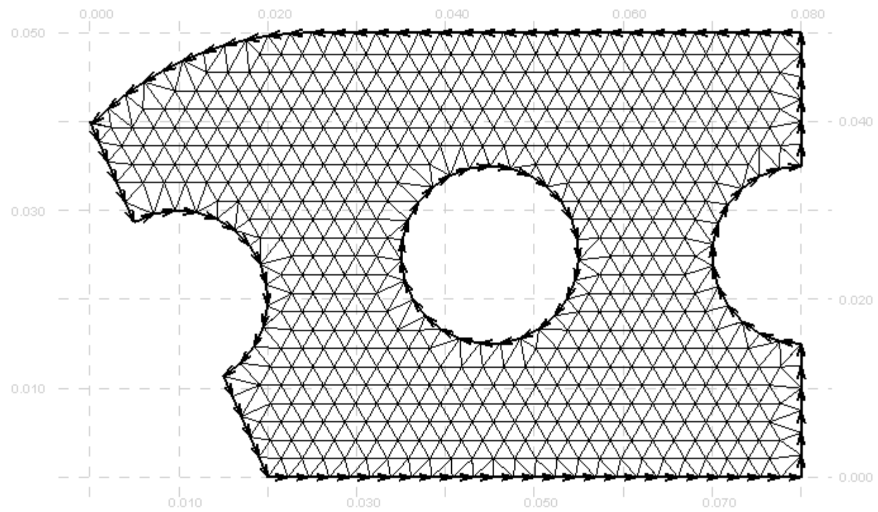


Fig. 3. The final effect of self-acting discretization

The fully automatic mesh generation is possible in the case of non-homogeneous domains, too. In Figure 4 the example of such case is shown.

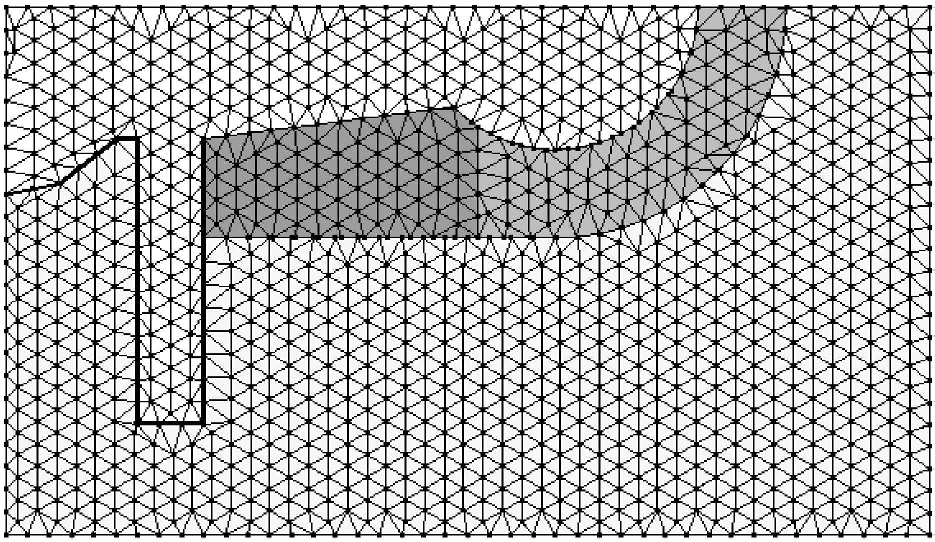

Fig. 4. Discretization of non-homogeneous domain

The mesh density can be practically optional and it results only from the assumed length of boundary element. During the test computations the meshes containing even 280 000 internal and external elements have been considered.

## 3. Computational module

The transition $t^{f-1} \to t^f$ requires the solution of liner system of equations. The number of equations corresponds to the number of boundary nodes (in the case of non-homogenous domains or domains with holes the external and internal boundary nodes must be taken into account). The main matrix of the resolving system is created automatically. The basic problem consists in the automatic construction of the successive columns of the matrix. The form of the column depends on the form of the boundary condition given on the considered boundary fragment. The algorithm discussed allows to introduce the Dirichlet, Neumann and the Robin conditions and, in the case of non-homogenous domain the continuity conditions (continuity of temperature and heat flax or continuity of heat flax in the case of thermal resistance between sub-domains). The form of right hand side of the system of equations results from the pseudo-initial condition and the form of the source function in the energy equation [1-3].

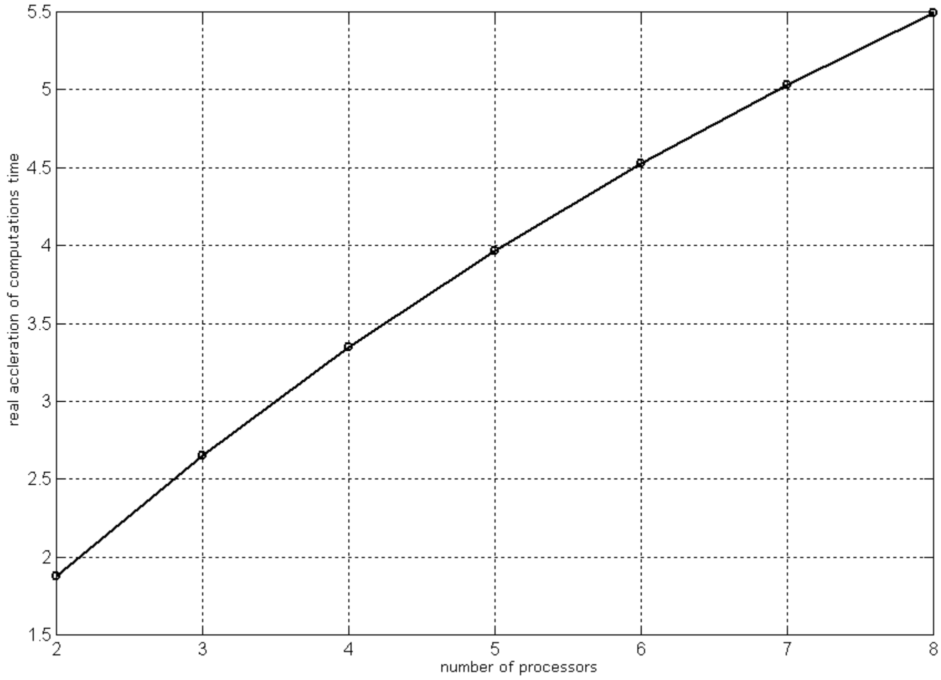


Fig. 5. The effect of distributed computations

The computational module can be applied in the version for which the distributed computations are realised. The testing computations have been done using 8 computers creating the homogeneous cluster. The numerical simulation was essentially less time-consuming. For instance, in the case of 280 000 elements we obtained the dependence between the real acceleration of computation time and the number of processors shown in Figure 5.

## 4. Examples of simulation

The heating processes proceeding in domain of continuous casting mould are analysed. The symmetrical fragment of the central lateral section of CCM is shown in Figure 1. On the outer surface of the system the Robin condition is assumed (heat transfer coefficient $\alpha = 15$ W/m$^2$K, ambient temperature $T_{ot}$ = 30$^o$C). The same type of condition is given on the surface between cooling pipes and CCM ($\alpha_w$ = 3700 W/m$^2$K, $T_w$ = 50$^o$C). On the contact surface casting - CCM the heat flax is given, namely $q = -10^6$ W/m$^2$. For the remaining parts of the boundary the adiabatic condition is taken into account. The initial condition: $T(x, 0) = 30°C$. In Figures 6 and 7 the temporary temperature fields for times 1000 s, and 3000 s are shown.

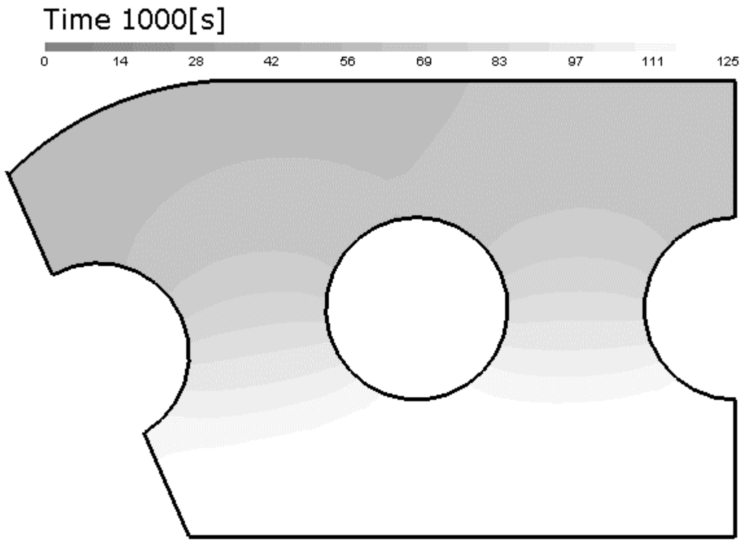


Fig. 6. Temperature field (1000 s)

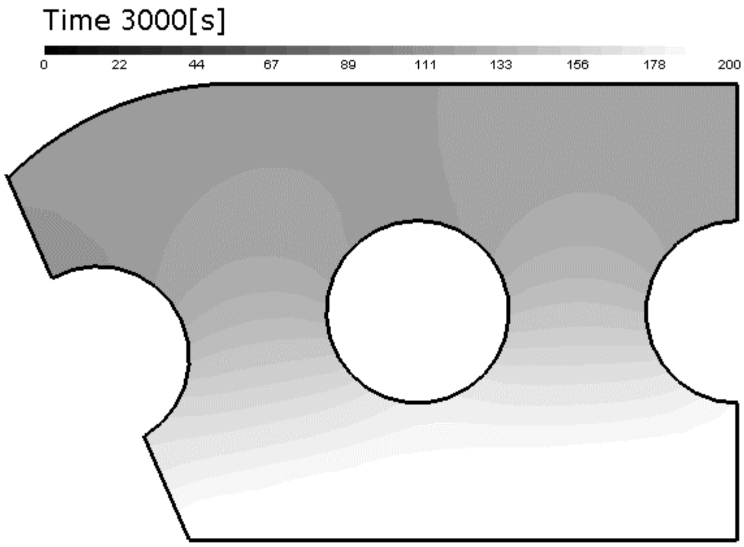


Fig. 7. Temperature field (3000 s)

Summing up, the research done in the scope of doctoral theses discussed show that the boundary element method in the variant presented in the monograph is very exact and efficient for numerical modelling of unsteady heat conduction both in the case of homogenous and non-homogenous 2D domains.

## References

[1] Majchrzak E., Boundary element method in heat transfer, Publ. of Techn. Univ. Czestochowa, Częstochowa 2001.

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[3] Freus S., Theoretical and practical aspects of BEM application for numerical modelling of transient heat transfer, Technical University of Czestochowa, Czestochowa 2003.

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