GROUPS OF TRANSFORMATIONS AS PSEUDOGROUPS OF FUNCTIONS

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Abstract. In [1] it was shown how to obtain pseudogroups of functions from quasi--algebraic spaces which were introduced by W. Waliszewski. In [2] it was shown how to obtain pseudogroups from premanifolds. In this paper we show how to obtain pseudogroups from groups.

In [1] we used the following definition of a pseudogroup.

A non-empty set Γ of functions for which domains are non-empty, will be called a pseudogroup if it satisfies the following conditions:

1° $\bigwedge_{f,g\in\Gamma} f(D_f)\cap D_g \neq \emptyset \Rightarrow g\circ f \in \Gamma$

2° $\bigwedge_{f\in\Gamma} f^{-1} \in \Gamma$ [1]

3° $\bigwedge_{\Gamma'\in\langle\Gamma\rangle} (\bigcup \Gamma' \in \Gamma)$

where

$$\langle\Gamma\rangle = \left\{\Gamma';\emptyset \neq \Gamma' \subset \Gamma \text{ and } \bigcup \Gamma' \text{ is a function and } \bigcup (\Gamma')^{-1} \text{ is a function}\right\}$$

and

$$(\Gamma')^{-1} = \left\{f^{-1}; f \in \Gamma'\right\}$$

and f^{-1} denotes an inverse relation.

It was shown in [1] that if Γ is a pseudogroup of functions, then $(\Gamma, \{D_f; f\in\Gamma\}\cup\{\emptyset\})$ is a topological space and Γ is an *Ehresmann* pseudogroup of transformations on this topological space. On the other hand, if Γ is an *Ehresmann* pseudogroup of transformations on a topological space *S*, then Γ is a pseudogroup of functions.

Let us consider the group *G* of transformations the set *S* onto *S*. We can consider every transformation which belongs to *G* as a function. So we can ask a question if the set G is a psedogroup of functions. The conditons 1^{o} and 2^{o} are satisfied in obviously way because G is a group. We will show that 3^{o} is also satisfied.

The only sets G'' which satisfy the conditon $\varnothing \neq G' \subset G$ and such that $\mathrm{Y} G'$ isa function are sets consisted of one element. In these cases $\mathrm{Y} G' = f$

where $f \in G'$. We obtain that $\mathrm{Y} G' \in G$. So we have the following theorem:

Theorem. If G is a group of transformations G is a pseudogroup of functions.

If elements of G are transformations the set S onto S it will be antidiscret topolgy on S.

References

[1] Lipińska J., Diffeomorphisms of quasi-algebraic spaces, Demonstratio Math. 1986, 19, 139-151.

[2] Lipińska J., Pseudogroups in premanifolds, Scientific Research of the Institute of Mathematics and Computer Science 2002, 1(1), 93-95.