ON PLANE CONTACT PROBLEM OF AN ELASTIC PERIODICALLY LAYERED COMPOSITE WITH BOUNDARY NORMAL TO LAYERING

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Abstract. This paper deals with some plane contact problem of an elastic laminated half-plane with boundary normal to layering. The considered problem is solved within the framework of the homogenized model with microlocal parameters given by Woźniak [6], Matysiak and Woźniak [7]. The body is assumed to be composed of two-layered, periodically repeated laminae. The perfect mechanical bonding between the layers is assumed. Moreover, the boundary condition for normal stresses on boundary normal to layering is approximated by using approach given by Perkowski et al. [8], Matysiak and Perkowski [9]. This approach allows to reduce the described problem to well-known dual integral equations and it can be solved exactly. Thus, the problem is solved by using analytical methods. The results of numerical analyses shown the distribution of contact pressure and stress distributions are presented in the form of figures.

Introduction

The construction elements, which are working in contact with another elements can be endangered to high contact pressures. The modelling of contact problem is very important from the engineering point of view and it was developed by many researchers, for example [1-5]. The contacted bodies considered in these monographs were homogenous. For the periodically layered composites, the contact problems with the boundary parallel to the layering were developed in [6-8]. In this paper, the periodically layered elastic half-plane with the boundary perpendicular to the layering is considered. Such medium is described within the framework of theory of elasticity by partial differential equations with discontinuous, oscillating coefficients. The application of this approach to solve the considered problem is rather complicated. So, the natural way is applied some approximated method, which allows to simplify the formulated problem. Some of them is homogenized model with microlocal parameters given by Woźniak [9-11] and applied to layered composites by Matysiak and Woźniak [12]. This approach was derived by using the concepts of nonstandard analysis combined with some postulated a priori physical assumptions. The equations of the homogenized model are expressed in terms by unknown macro-displacements and microlocal parameters. The microlocal parameters can be determined by derivates of macro-displacements (see for example Kaczyński and Matysiak [6-8]). In that way we obtained the partial differential equations with constant coefficient, which permit to describe of the body. The continuity conditions on interfaces, which are fulfilled within the framework of the homogenized model.

This paper considers the two-dimensional contact problem formulated within the framework of the homogenized model with microlocal parameters. The non-homogenous half-space is composed of two-layered periodically repeated sublayers. The perfect mechanical bounding is taken into account. The infinity long punch is pressured into the body on the boundary normal to the layering and boundary condition for contact pressure is averaged by using approach given by Perkowski et al. [13], Matysiak and Perkowski [14]. The obtained analytical results will be presented in the form of figures.

1. Formulation of the problem

In this paper we confined to analyse of stress distributions and contact pressures produced by long infinity rigid punch pressured with intensity P into the periodically layered composite half-plane with the boundary normal to layering, (see Fig. 1). Considerations were concerned with two cases, when the cross-section of pressured punch has a parabolic or rectangular shape. Let $2a$ denote the width of contact zone. Let (x, y, z) comprise Cartesian coordinate system such that the x axis is normal to the layering.

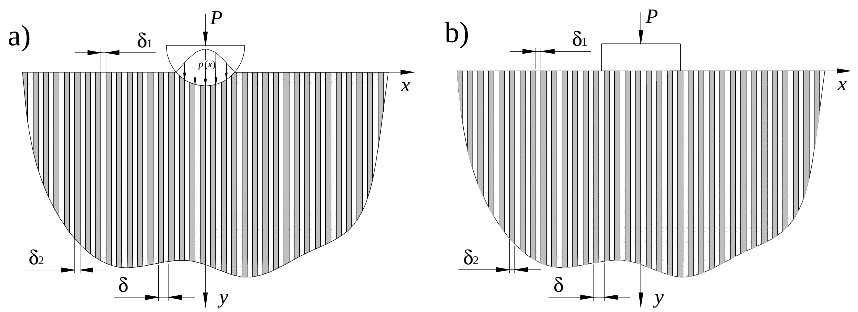

Fig. 1. A cross-section of two-layered periodically composites for two cases pressured punch: a) parabolic, b) rectangular

A representative volume element called by fundamental layer of thickness δ is composed of two homogeneous isotropic elastic layer, which thicknesses equal δ_1 and δ_2, respectively. The mechanical properties of the composite constituents are characterized by Lamé constants $\lambda_j, \mu_j, j = 1, 2$. The considered problem can be described by following boundary conditions:

– on the boundary for $y = 0$

$$\begin{aligned} &\frac{\partial V(x,0)}{\partial x} = f'(x), \quad \text{for} \quad |x| \le a \\ &\sigma_{yy}^{(j)}(x,0) = 0, \quad \text{for} \quad |x| > a \\ &\sigma_{xy}^{(j)}(x,0) = 0, \quad \text{for} \quad x \in \mathbf{R} \end{aligned} \tag{1}$$

– the regularity conditions at infinity

$$\sigma_{xx}^{(j)},\ \sigma_{xy}^{(j)},\ \sigma_{yy}^{(j)} \to 0 \quad \text{for} \quad x^2 + y^2 \to \infty \tag{2}$$

where $f(x)$ is a shape of punch cross-section. The function $f(x)$ can be written for considered cases (see Fig. 1) in the form:

– parabolic punch

$$f(x) = D - \frac{x^2}{2R}, \quad D = const., \quad R = const. \tag{3}$$

– rectangular punch

$$f(x) = D, \quad D = const. \tag{4}$$

and the constant D is called by depth of penetration.

2. The homogenized model with microlocal parameters

The displacement vector in the case of plane state of strain is postulated in the form [6, 12]:

$$\mathbf{u}(x,y) = \big(U(x,y) + h(x)q_x(x,y), V(x,y) + h(x)q_y(x,y), 0\big) \tag{5}$$

where U, V are unknown functions called the macro-displacements, and q_x, q_y are unknown the microlocal parameters. The microlocal parameters can be eliminated from the equations of the model taking into account the function $h(x)$ (called the shape function) in the form [6, 12]:

$$h(x) = \begin{cases} x - 0.5\delta_1 & \text{for} \ \ 0 \le x \le \delta_1 \\ -\eta x/(1-\eta) - 0.5\delta_1 + \delta_1/(1-\eta) & \text{for} \ \ \delta_1 \le x \le \delta \end{cases} \tag{6}$$

where

$$\eta = \delta_1 / \delta \tag{7}$$

The following approximations for displacements and derivatives of displacement are given [6, 12]:

$$u \approx U,\ v \approx V,\ \frac{\partial u}{\partial x} \approx \frac{\partial U}{\partial x} + h_j q_x,\ \frac{\partial u}{\partial y} \approx \frac{\partial U}{\partial y},\ \frac{\partial v}{\partial x} \approx \frac{\partial V}{\partial x} + h_j q_y,\ \frac{\partial v}{\partial y} \approx \frac{\partial V}{\partial y} \tag{8}$$

where h_j, $j = 1, 2$ is a derivate of shape function $h(x)$ in the j-th kind of layer being of the composite constituents:

$$h_1 = 1,\ h_2 = -\eta/(1-\eta) \tag{9}$$

The governing equations of the homogenized model with microlocal parameters are [6, 12]:

$$\begin{aligned} A_1 \frac{\partial^2 U}{\partial x^2} + C \frac{\partial^2 U}{\partial y^2} + (B+C)\frac{\partial^2 V}{\partial x \partial y} &= 0 \\ C \frac{\partial^2 V}{\partial x^2} + A_2 \frac{\partial^2 V}{\partial y^2} + (B+C)\frac{\partial^2 U}{\partial x \partial y} &= 0 \end{aligned} \tag{10}$$

and

$$\begin{gathered} \sigma_{xy}^{(j)} = C\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right),\quad \sigma_{xx}^{(j)} = A_1 \frac{\partial U}{\partial x} + B\frac{\partial V}{\partial y} \\ \sigma_{yy}^{(j)} = D_j \frac{\partial U}{\partial x} + E_j \frac{\partial V}{\partial y},\quad \sigma_{zz}^{(j)} = \frac{\lambda_j}{\lambda_j + 2\mu_j}\left(\sigma_{xx}^{(j)} + \sigma_{yy}^{(j)}\right),\quad j = 1,\ 2 \end{gathered} \tag{11}$$

where

$$\begin{aligned} \tilde{\lambda} &= \eta\lambda_1 + (1-\eta)\lambda_2, \quad [\lambda] = \eta\left(\lambda_1 - \lambda_2\right), \quad \hat{\lambda} = \eta\lambda_1 + \frac{\eta^2}{1-\eta}\lambda_2 \\ \tilde{\mu} &= \eta\mu_1 + (1-\eta)\mu_2, \quad [\mu] = \eta\left(\mu_1 - \mu_2\right), \quad \hat{\mu} = \eta\mu_1 + \frac{\eta^2}{1-\eta}\mu_2 \end{aligned} \tag{12}$$

$$\begin{gathered} A_1 = \tilde{\lambda} + 2\tilde{\mu} - \frac{\left([\lambda] + 2[\mu]\right)^2}{\hat{\lambda} + 2\hat{\mu}} > 0,\quad A_2 = \tilde{\lambda} + 2\tilde{\mu} - \frac{[\lambda]^2}{\hat{\lambda} + 2\hat{\mu}} > 0 \\ B = \tilde{\lambda} - \frac{[\lambda]\left([\lambda] + 2[\mu]\right)}{\hat{\lambda} + 2\hat{\mu}} > 0,\quad C = \tilde{\mu} - \frac{[\mu]^2}{\hat{\mu}} > 0 \\ D_j = \frac{\lambda_j}{\lambda_j + 2\mu_j} A_1,\quad E_j = \frac{4\mu_j(\lambda_j + \mu_j)}{\lambda_j + 2\mu_j} + \frac{\lambda_j}{\lambda_j + 2\mu_j} B,\ j = 1,\ 2 \end{gathered} \tag{13}$$

The system of equations (10) can be separated by introducing the potentials $\Psi_1,\ \Psi_2$ [15] as follows

$$U=\kappa_1\frac{\partial\Psi_1}{\partial x}+\kappa_2\frac{\partial\Psi_2}{\partial x}\ ,\quad V=\frac{\partial\Psi_1}{\partial y}+\frac{\partial\Psi_2}{\partial y} \tag{14}$$

where

$$\kappa_j=\frac{A_2\gamma_j^2-C}{B+C} \tag{15}$$

and $\gamma_j^2,\ j=1,\ 2,$ are the solutions of characteristic equations

$$A_2C\gamma_j^4+\left(B^2+2BC-A_1A_2\right)\gamma_j^2+A_1C=0 \tag{16}$$

Thus, we have the following separated equations

$$\gamma_j^2\frac{\partial^2\Psi_j}{\partial x^2}+\frac{\partial^2\Psi_j}{\partial y^2}=0,\ j=1,2 \tag{17}$$

The characteristic equation (16) has four real roots $\pm\gamma_1,\ \pm\gamma_2$ in the form

$$\gamma_{1,2}=\left(\frac{A_1A_2-2BC-B^2\mp\sqrt{\Delta}}{2A_2C}\right)^{\frac{1}{2}},\ \Delta=(B^2+2BC-A_1A_2)^2-4A_1A_2C^2>0 \quad (18)$$

3. Solution of the problem

Let us to consider the boundary condition (1) connected with the normal stress component $\sigma_{yy}^{(j)}$ in the form:

$$D_j\frac{\partial U}{\partial x}+E_j\frac{\partial V}{\partial y}=0,\ \text{ for }\ |x|>a,\ y=0,\quad j=1,\ 2 \tag{18}$$

The left hand side of equation (18) represents some jumps and the solution of formulated problem is rather complicated. The periodic boundary condition (18) can be replaced by averaged condition given in [13, 14]:

$$B\frac{\partial U}{\partial x}+A_2\frac{\partial V}{\partial y}=0,\ \text{ for }\ |x|>a,\ y=0 \tag{19}$$

The formulated problem will be solved by Fourier transform method. Let us denote by $\tilde{f}$ Fourier transform of function f with respect to variable x as follows

$$\tilde{f}(s,y) = F\left[f(x,y); x \to s \right] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x,y) \exp(-ixs)\, dx \tag{20}$$

By employing Eq. (20) to the Eq. (13) and (16) and using the regularity conditions (2) we obtained the transformations of the macro-displacements U and V in the form

$$\begin{aligned} \tilde{U}(s,y) &= is \sum_{k=1}^{2} \kappa_k a_k(s) \exp\left(-|s| \gamma_k y\right) \\ \tilde{V}(s,y) &= -|s| \sum_{k=1}^{2} \gamma_k a_k(s) \exp\left(-|s| \gamma_k y\right) \end{aligned} \tag{21}$$

where $a_k(s)$, $k = 1,2$ are unknown functions.

Assuming that the unknown contact pressure $p(x)$ can be represented by averaged contact pressure, we have

$$B \frac{\partial U}{\partial x} + A_2 \frac{\partial V}{\partial y} = -p(x), \quad \text{for} \quad |x| < a, \;\; y = 0 \tag{22}$$

By using Eq. (22) and $(1)_3$ we obtain that the functions $a_k(s)$, $k = 1,2$ are the solutions of system of linear algebraic equations

$$\begin{cases} \sum_{k=1}^{2} a_k(s)(A_2\gamma_k^2 - \kappa_k B) = -\dfrac{\tilde{p}(s)}{s^2} \\ \sum_{k=1}^{2} a_k(s)\gamma_k(1+\kappa_k) = 0 \end{cases} \tag{23}$$

where

$$\tilde{p}(s) = F\left[p(x); x \to s \right] = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} p(x) \exp(-ixs)\, dx \tag{24}$$

The solution to system of equations (23) takes the form:

$$a_k(s) = \frac{(-1)^{k+1}\, \tilde{p}(s)\, \gamma_{3-k} (B + C)}{s^2 (\gamma_1 - \gamma_2)\left(A_2\gamma_k^2 + B\right) C}, \quad k = 1,2 \tag{25}$$

Taking into account Eq. (25) and assuming that $p(-x)=p(x)$ we obtain the macro-displacement and stresses in j-th layer:

$$\tilde{U}(s,y)=\frac{i\tilde{p}(s)(B+C)}{s(\gamma_1-\gamma_2)C}\sum_{k=1}^{2}(-1)^{k+1}\frac{\gamma_{3-k}\kappa_k}{\left(A_2\gamma_k^2+B\right)}\exp\left(-|s|\gamma_k y\right) \tag{26}$$

$$\tilde{V}(s,y)=-\frac{\tilde{p}(s)(B+C)}{|s|(\gamma_1-\gamma_2)C}\sum_{k=1}^{2}(-1)^{k+1}\frac{\gamma_k\gamma_{3-k}}{\left(A_2\gamma_k^2+B\right)}\exp\left(-|s|\gamma_k y\right) \tag{27}$$

$$\begin{gathered}\begin{Bmatrix}\sigma_{xx}^{(j)}(x,y)\\ \sigma_{yy}^{(j)}(x,y)\end{Bmatrix}=\sqrt{\frac{2}{\pi}}\int_0^\infty\begin{Bmatrix}\tilde{\sigma}_{xx}^{p(j)}(s,y)\\ \tilde{\sigma}_{yy}^{p(j)}(s,y)\end{Bmatrix}\tilde{p}(s)\cos(xs)\,ds\\ \sigma_{xy}^{(j)}(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\tilde{\sigma}_{xy}^{p(j)}(s,y)\,\tilde{p}(s)\sin(xs)\,ds,\quad j=1,2\end{gathered} \tag{28}$$

where:

$$\begin{gathered}\tilde{\sigma}_{xx}^{p(j)}(s,y)=\frac{B+C}{(\gamma_1-\gamma_2)C}\sum_{k=1}^{2}(-1)^{k+1}\frac{\gamma_{3-k}(\gamma_k^2B-\kappa_kA_1)}{\left(A_2\gamma_k^2+B\right)}\exp\left(-|s|\gamma_k y\right)\\ \tilde{\sigma}_{yy}^{p(j)}(s,y)=\frac{B+C}{(\gamma_1-\gamma_2)C}\sum_{k=1}^{2}(-1)^{k+1}\frac{\gamma_{3-k}(\gamma_k^2E_j-\kappa_kD_j)}{\left(A_2\gamma_k^2+B\right)}\exp\left(-|s|\gamma_k y\right)\\ \tilde{\sigma}_{xy}^{p(j)}(s,y)=\frac{B+C}{\gamma_1-\gamma_2}\sum_{k=1}^{2}(-1)^{k+1}\frac{\gamma_k\gamma_{3-k}(1+\kappa_k)}{\left(A_2\gamma_k^2+B\right)}\exp\left(-|s|\gamma_k y\right)\end{gathered} \tag{29}$$

The functions $\tilde{p}(s)$ in Eq. (26-28) can be obtained from dual integrals equations by satisfying conditions $(1)_1$ and (19), which leads to the following well-known dual integral equations [16, 17]:

$$\begin{gathered}\sqrt{\frac{2}{\pi}}\int_0^\infty\tilde{p}(s)\sin(xs)\,ds=-\frac{f'(x)}{C_V}\quad\text{for}\ \ 0\le x\le a\\ \sqrt{\frac{2}{\pi}}\int_0^\infty\tilde{p}(s)\cos(xs)\,ds=0\quad\text{for}\ \ x>a\end{gathered} \tag{30}$$

where $C_V=\dfrac{\sqrt{A_1A_2}(\gamma_1+\gamma_2)}{A_1A_2-B^2}$.

After some calculations the transform of contact pressure takes the form, respectively:

- parabolic shape of punch

$$\tilde{p}(s) = \sqrt{\frac{\pi}{2}} \frac{a}{C_V R} \frac{J_1(as)}{s} \tag{31}$$

- rectangular shape of punch

$$\tilde{p}(s) = \frac{P}{\sqrt{2\pi}} J_0(as) \tag{32}$$

In equation (31) the unknown parameter a can be determined from equilibrium conditions as follows

$$\int_{-a}^{a} p(x)dx = P\,, \text{ where } p(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \tilde{p}(s)\sin(xs)\,ds = \frac{1}{C_V R}\sqrt{a^2 - x^2} \tag{33}$$

and we obtain

$$a^2 = \frac{2C_V RP}{\pi} \tag{34}$$

The mean contact pressure denoted by p_0 is given by

$$p_0 = \frac{1}{2a}\int_{-a}^{a} p(x)dx = \frac{P}{2a} \tag{35}$$

and it is the same value for the two described cases. From (35) and (31), (32) it follows that

$$\frac{\tilde{p}(s)}{p_0} = \sqrt{\frac{2}{\pi}} \int_0^a \frac{p(x)}{p_0}\cos(xs)dx = \sqrt{\frac{2}{\pi}} \frac{2J_1(as)}{s} \text{ - parabolic punch} \tag{36}$$

$$\frac{\tilde{p}(s)}{p_0} = \sqrt{\frac{2}{\pi}} \int_0^a \frac{p(x)}{p_0}\cos(xs)dx = \sqrt{\frac{2}{\pi}} aJ_0(as) \text{ - rectangular punch} \tag{37}$$

The solution of the problem given in (26)-(29) is obtained in the form of Fourier integrals and the analytical calculations are possible, but their presentation is rather too long. The integrals will be calculated numerically and distributions of stresses will be shown in the form of figures.

Remark

Let us to consider the case for $\lambda_1 = \lambda_2 = \lambda,\ \mu_1 = \mu_2 = \mu$ (then the considered body is a homogenous and isotropic). Thus, we have:

$$\begin{gathered} \tilde{\lambda} = \lambda,\ \ \tilde{\mu} = \mu,\ \ [\lambda] = [\mu] = 0 \\ A_1 = A_2 = \lambda + 2\mu,\ B = \lambda,\ C = \mu,\ D_j = \lambda,\ E_j = \lambda + 2\mu,\ \ j = 1,2 \end{gathered} \tag{38}$$

We can observe that the roots of characteristic equation (15) tend to $\gamma_1 \to \gamma_2 \to 1$ and $\kappa_1 \to \kappa_2 \to 1$. By using the d'Hospital rule in the form:

$$\lim_{\gamma_1 \to \gamma_2} \frac{L(\gamma_1)}{M(\gamma_1)} \overset{\frac{0}{0}}{=} \lim_{\gamma_1 \to \gamma_2} \frac{L'(\gamma_1)}{M'(\gamma_1)} \tag{39}$$

For example, we consider vertical macro-displacement V for the case of parabolic punch:

$$\begin{aligned} \tilde{V}(s,y) &= \frac{\tilde{p}(s)(B+C)}{|s|C} \lim_{\gamma_1 \to \gamma_2} \frac{\gamma_1\gamma_2 \sum_k^2 (-1)^k (A_2\gamma_{3-k}^2 + B)\exp(-|s|\gamma_k y)}{(\gamma_1 - \gamma_2)(A_2\gamma_1^2 + B)(A_2\gamma_2^2 + B)} = \\ &= \frac{\gamma_2^2\left(2A_2\gamma_2 + |s|y\left(B + A_2\gamma_2^2\right)\right)\exp\left(-|s|\gamma_2 y\right)}{\left(B + A_2\gamma_2^2\right)^2} \frac{\tilde{p}(s)(B+C)}{|s|C} \end{aligned} \tag{40}$$

Taking into account Eq. (38) and $\gamma_1 \to \gamma_2 \to 1$ we are obtained

$$2\mu\tilde{V}(s,y) = (2(1-\nu) + |s|y)\,\tilde{p}(s)|s|^{-1}\exp(-|s|y) \tag{41}$$

This result is adequate to solution obtained in the case of homogenous body [18]. The limit passing in the solutions of the problem can be calculated in the same way.

4. The results of numerical analysis

Let us denote the dimensionless coordinate system $\left(x^*, y^*\right)$ related to a: $x^* = x/a,\ y^* = y/a$. In all figures the stress components will be shown in the dimensionless form related to mean contact pressure p_0 described by Eq. (35). In Figure 2 the dimensionless contact pressure are presented for two considered cases of Young modulus ratio $E_1/E_2 = 4;8$ and Poisson's coefficients $\nu_1 = \nu_2 = 0.3$ for $l = \delta/a = 0.1$.

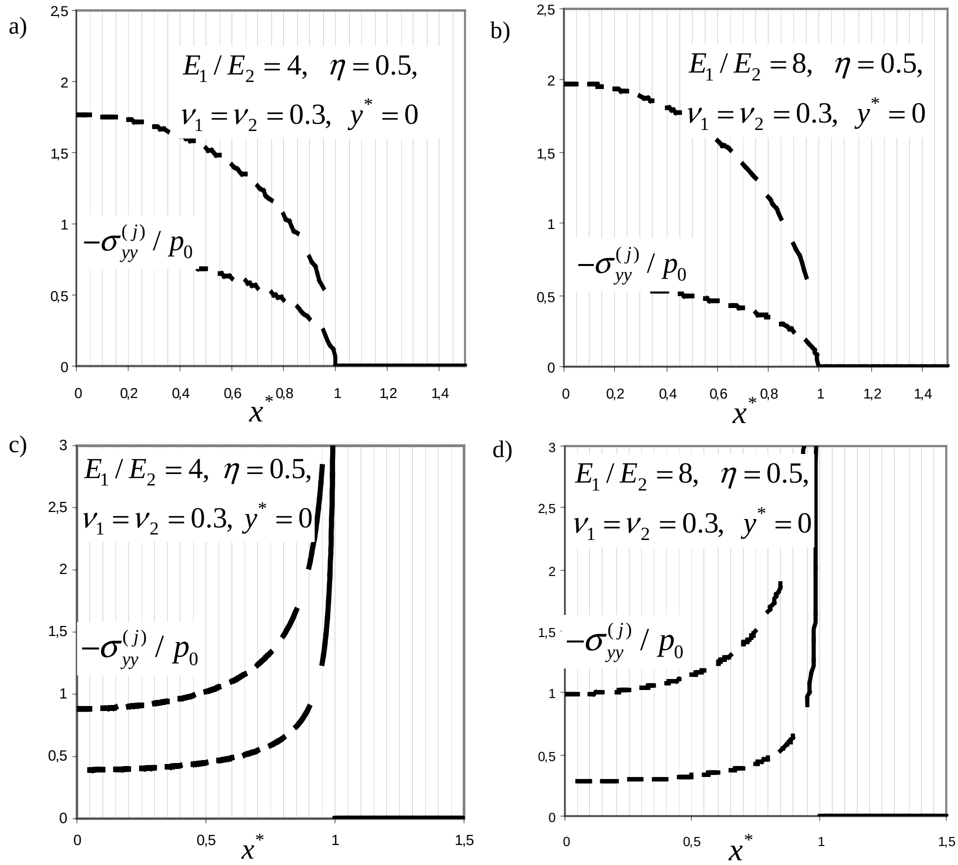

Fig. 2. The dimensionless contact pressure $-\sigma_{yy}^{(j)}(x^*,0)/p_0$: a, b) parabolic punch, c, d) rectangular punch

The inverse transform of contact pressure was calculated exactly within homogenized model with microlocal parameters and it shown in Fig. 2. Calculating the integrals in Eq. (28) we obtain:

- parabolic punch

$$\sigma_{yy}^{(j)}\left(x,0\right)/p_0=\sum_{k=1}^{2}(-1)^{k+1}(\gamma_k^2 E_j-\kappa_k D_j)G_k\,\frac{4}{\pi}\sqrt{1-(x/a)^2},\;\; j=1,2 \qquad (42)$$

- rectangular punch

$$\sigma_{yy}^{(j)}\left(x,0\right)/p_0=\sum_{k=1}^{2}(-1)^{k+1}(\gamma_k^2 E_j-\kappa_k D_j)G_k\,\frac{2}{\pi}\frac{1}{\sqrt{1-(x/a)^2}},\;\; j=1,2 \qquad (43)$$

where

$$G_1 = \frac{\gamma_2}{\gamma_1 - \gamma_2} \frac{B + C}{(A_2\gamma_1^2 + B)C}, \quad G_2 = \frac{\gamma_1}{\gamma_1 - \gamma_2} \frac{B + C}{(A_2\gamma_2^2 + B)C} \tag{44}$$

The dimensionless normal stress component $\sigma_{xx}^{(j)}$ and dimensionless shear stress component $\sigma_{xy}^{(j)}$ for parabolic punch are shown in Figure 3. We can observed that the this components are continuous on the interfaces.

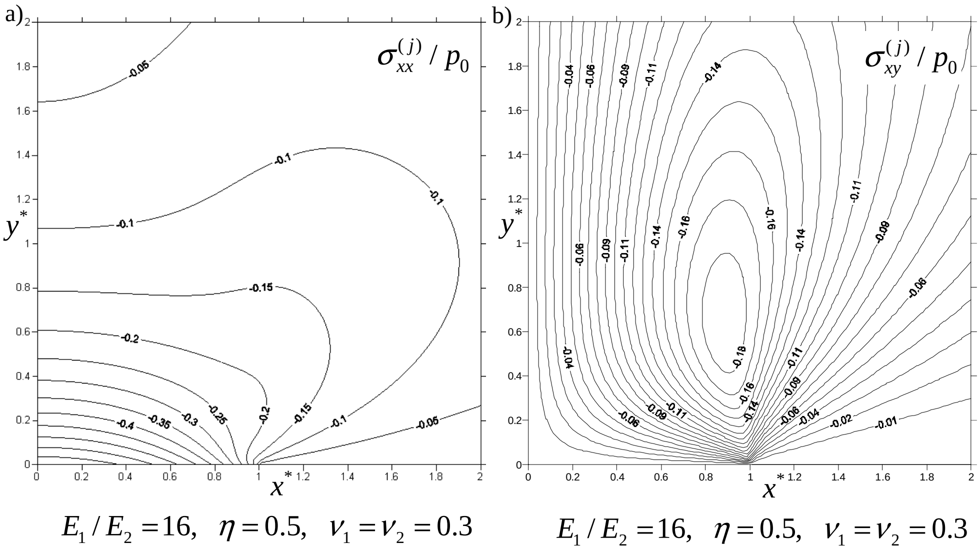

Fig. 3. The dimensionless stress distribution of lines of constant values for $\sigma_{xx}^{(j)} / p_0$ and $\sigma_{xy}^{(j)} / p_0$ for parabolic punch

These distributions of stresses are presented for Young modulus ratio $E_1 / E_2 = 16$, Poisson ratios $\nu_1 = \nu_2 = 0.3$ and $\eta = 0.5$. It is shown, that the maximal values of $\sigma_{xx}^{(j)} / p_0$ on the boundary are located on the centre of contact zone, but in the case of shear stresses, the maximal values are situated under the boundary surface located near the ends of contact zone. The next figure (Fig. 4) shows some results for rectangular punch, for $E_1 / E_2 = 8$, $\nu_1 = \nu_2 = 0.3$ and $\eta = 0.5$.

In this case, the maximal concentration of stresses is on the ends of contact zone and the stresses are fast decreases with the depth. The influence of mechanical and geometrical properties of composites on the contact pressure was presented in Figure 5.

Figure 5a shows the influence of geometrical properties represented by parameter η on the contact pressure at the centre of contact zone, $x^* = 0, \; y^* = 0$.

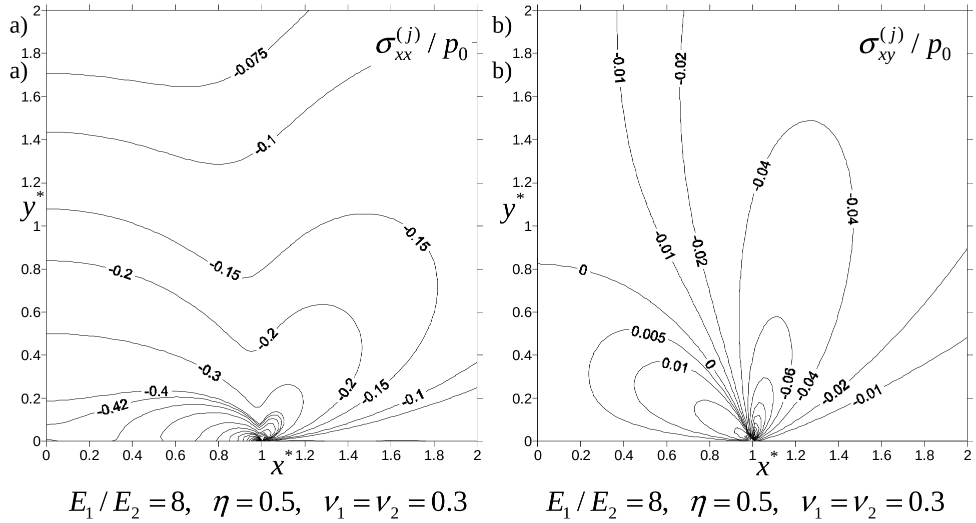

Fig. 4. The dimensionless stress distribution of lines of constant values for $\sigma_{xx}^{(j)}$ and $\sigma_{xy}^{(j)}$ for rectangular punch

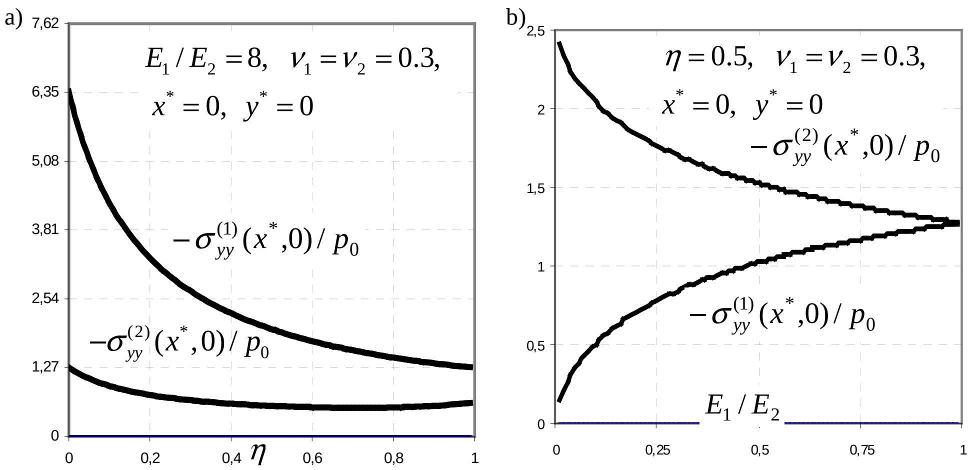

Fig. 5. The dimensionless contact pressure distribution $-\sigma_{yy}^{(j)}(x^*,0)/p_0$ for parabolic punch at the contact centre as a function of η (a) or E_1/E_2 (b)

Conclusions

In this paper it was presented the exact solution to the contact problem formulated within the homogenized model with microlocal parameters. The boundary condition connected with the stresses $\sigma_{yy}^{(j)}$ has been replaced by the averaged ones. This approach together with the application of the homogenized model was used to solve the boundary problem of laminated layer [19]. In this paper [19] the solutions within the framework of the homogenized model were compared with the results obtained by using the theory of elasticity, and good consistences of both solutions were confirmed.

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References

[1] Fabrikant V.I., Mixed boundary value problem of potential theory and their applications in engineering, Kluwer Acad. Pub., Dordrecht-Boston-London 1991.

[2] Galin L.A., Contact problems in the theory of elasticity, (in Russian), Gostekhteorizdat, Moscow 1953.

[3] Galin L.A., Contact problems in the theory of elasticity and viscoelasticity, Izd. Science, oscow 1980 (in Russian).

[4] Gladwell G.M.L., Contact problems in the classical theory of elasticity, Sijthoff and Noordhoff, Alphen An den Rijn 1980.

[5] Goryacheva I.G., Dobykhin M.N., Contact problems in trybology, Machine-Construction, Moscow 1988 (in Russian).

[6] Kaczyński A., Matysiak S.J., Plane Contact Problems for a periodic two-layered elastic composite, Ingenieur-Archiv 1988, 58, 137-147.

[7] Kaczyński A., Matysiak S.J., Rigid sliding punch on a periodic two-layered elastic half-space, J. Theor. Appl. Mech. 1993, 31, 2, 295-306.

[8] Kaczyński A., Matysiak S.J., On 3D punch problems for a periodic two-layered elastic half-epace, J. Theor. Appl. Mech. 2001, 39, 3, 523-538.

[9] Woźniak C., A nonstandard method of modeling of thermoelastic periodic composites, Int. J. Engng. Sci. 1987, 25, 483-499.

[10] Woźniak C., Homogenized thermoelasticity with microlocal parameters, Bull. Pol. Ac.: Techn. 1987, 35, 133-143.

[11] Woźniak C., On the linearized problems of thermoelasticity with microlocal parameters, Bull. Pol. Ac.: Techn. 1987, 35, 143-153.

[12] Matysiak S.J., Woźniak C., Micromorphic effects in a modeling of periodic multilayered elastic composites, Int. J., Engng. Sci. 1987, 25, 549-559.

[13] Perkowski D.M., Matysiak S.J., Kulchytsky-Zyhailo R., On contact problem of an elastic laminated half-plane with a boundary normal to layering, Compos. Sci. Technol. 2007, 67, 2683--2690.

[14] Matysiak S.J., Perkowski D.M., Singularity of stresses in a periodic laminated semi-space with a boundary normal to the layering, Journal of Theoretical and Applied Mechanics 2007, 45, 2, 325-335.

[15] Kulchytsky-Zyhailo R., Matysiak S.J., On three-dimensional problems of multilayered periodic elastic composites, Journal of Theoretical and Applied Mechanics, ?, ?, 4, 771-781.

[16] Sneddon I.N., Mixed boundary value problems in potential theory, North-Holland Publ. Co., Amsterdam 1966.

[17] Sneddon I.N., Application of integral transforms in the theory of elasticity, Springer, Wien 1975.

[18] Timoshenko G., Goodier N., Theory of elasticity, New York, McGraw-Hill 1934.

[19] Kulchytsky-Zhyhailo R., Matysiak S.J., Perkowski D.M., On displacements and stresses in a semi-infinite laminated layer. Comparative results, Meccanica 2006, 42, 117-126.