APPLICATION OF SENSITIVITY ANALYSIS IN EXPERIMENTS DESIGN

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Abstract. The system casting-mould is considered and it is assumed that the aim of experiments is to determine the course of substitute thermal capacity of casting material. The casting is made from cast iron and the austenite and eutectic latent heats should be identified. To find the optimal location of sensors the methods of sensitivity analysis are applied. In the final part of the paper the results of computations are shown.

1. Governing equations

The energy equation describing the casting solidification has the following form [1, 2]

$$x \in \Omega: \quad C(T)\frac{\partial T(x,t)}{\partial t} = \nabla\left[\lambda(T)\nabla T(x,t)\right] \tag{1}$$

where $C(T)$ is the substitute thermal capacity of cast iron, λ is the thermal conductivity, T, x, t denote the temperature, geometrical co-ordinates and time.

The equation considered is supplemented by the equation concerning a mould sub-domain

$$x \in \Omega_m: \quad c_m \frac{\partial T_m(x,t)}{\partial t} = \lambda_m \nabla^2 T_m(x,t) \tag{2}$$

where c_m is the mould volumetric specific heat, λ_m is the mould thermal conductivity.

In the case of typical sand moulds on the contact surface between casting and mould the continuity condition in the form

$$x \in \Gamma_c: \quad \begin{cases} -\lambda(T)\,\mathbf{n}\cdot\nabla T(x,t) = -\lambda_m\,\mathbf{n}\cdot\nabla T_m(x,t) \\ T(x,t) = T_m(x,t) \end{cases} \tag{3}$$

can be accepted.

On the external surface of the system the Robin condition

$$x \in \Gamma_0 : \quad -\lambda_m \mathbf{n} \cdot \nabla T_m(x, t) = \alpha\left[T_m(x, t) - T_a\right] \tag{4}$$

is given (α is the heat transfer coefficient, T_a is the ambient temperature).
For time $t = 0$ the initial condition

$$t = 0: \quad T(x, 0) = T_0(x) \ , \ \ T_m(x, 0) = T_{m0}(x) \tag{5}$$

is also known.

In the case of cast iron solidification the following approximation of substitute thermal capacity can be taken into account (Fig. 1)

$$C(T) = \begin{cases} c_L \, , & T > T_L \\ c_P + \dfrac{Q_{aus1}}{T_L - T_A} \, , & T_A < T \le T_L \\ c_P + \dfrac{Q_{aus2}}{T_A - T_E} \, , & T_E < T \le T_A \\ c_P + \dfrac{Q_{eu}}{T_E - T_S} \, , & T_S < T \le T_E \\ c_S \, , & T \le T_S \end{cases} \tag{6}$$

where T_L, T_A, T_E, T_S correspond to the border temperatures, c_L, c_S, $c_P = 0.5 \cdot (c_L + c_S)$ are the constant volumetric specific heats of molten metal, solid state and mushy zone sub-domain, $Q_{aus} = Q_{aus1} + Q_{aus2}$, Q_{eu} are the latent heats connected with the austenite and eutectic phases evolution, at the same time $Q = Q_{aus} + Q_{eu}$.

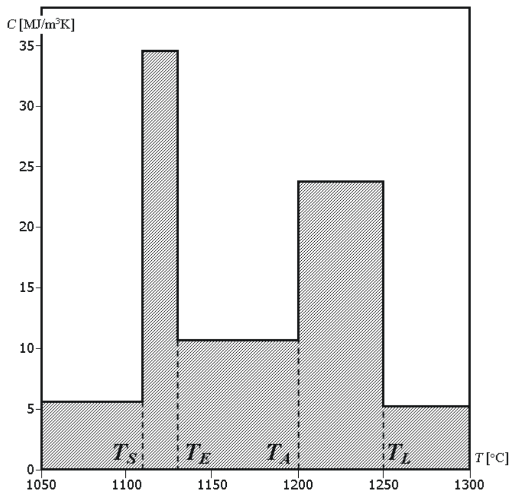

Fig. 1. Substitute thermal capacity of cast iron

The thermal conductivity is defined as follows

$$\lambda(T)=\begin{cases}\lambda_L, & T>T_L\\ \lambda_P, & T_S\le T\le T_L\\ \lambda_S, & T<T_S\end{cases} \tag{7}$$

where λ_L, λ_S, $\lambda_P = 0.5 \cdot (\lambda_L + \lambda_S)$ are the constant thermal conductivities of liquid state, solid state and mushy zone sub-domain, respectively.

2. Sensitivity analysis

It is assumed that the aim of experiments is to determine the latent heats Q_{aus1}, Q_{aus2}, Q_{eu} of casting material (an inverse problem) and in order to find the optimal location of sensors the sensitivity analysis methods [3-5] are applied.

To determine the sensitivity functions the governing equations (1)-(5) are differentiated with respect to $p_1 = Q_{aus1}$, $p_2 = Q_{aus2}$ and $p_3 = Q_{eu}$. So, the following additional problems should be solved

$$\begin{aligned}
&x\in\Omega: \frac{\partial C(T)}{\partial p_e}\frac{\partial T(x,t)}{\partial t}+C(T)\frac{\partial}{\partial p_e}\left(\frac{\partial T(x,t)}{\partial t}\right)=\\
&\qquad\frac{\partial}{\partial p_e}\left[\nabla\left[\lambda(T)\nabla T(x,t)\right]\right]\\
&x\in\Omega_m: c_m\frac{\partial}{\partial p_e}\left(\frac{\partial T_m(x,t)}{\partial t}\right)=\lambda_m\frac{\partial}{\partial p_e}\left[\nabla^2 T_m(x,t)\right]\\
&x\in\Gamma_c: \begin{cases}-\dfrac{\partial}{\partial p_e}\left[\lambda(T)\,\mathbf{n}\cdot\nabla T(x,t)\right]=-\lambda_m\,\mathbf{n}\cdot\dfrac{\partial}{\partial p_e}\left[\nabla T_m(x,t)\right]\\[2ex] \qquad\qquad\dfrac{\partial T(x,t)}{\partial p_e}=\dfrac{\partial T_m(x,t)}{\partial p_e}\end{cases}\\
&x\in\Gamma_0: -\lambda_m\,\mathbf{n}\cdot\frac{\partial}{\partial p_e}\left[\nabla T_m(x,t)\right]=0\\
&t=0: \quad \frac{\partial T(x,0)}{\partial p_e}=0\ , \quad \frac{\partial T_m(x,0)}{\partial p_e}=0
\end{aligned} \tag{8}$$

or

$$x\in\Omega:\ C(T)\frac{\partial Z_e(x,t)}{\partial t}=\nabla\left[\lambda(T)\nabla Z_e(x,t)\right]+\nabla\left[\frac{\partial\lambda(T)}{\partial p_e}\nabla T(x,t)\right]-\frac{\partial C(T)}{\partial p_e}\frac{\partial T(x,t)}{\partial t}$$

$$x\in\Omega_m:\ c_m\frac{\partial Z_{me}(x,t)}{\partial t}=\lambda_m\nabla^2 Z_{me}(x,t)$$

$$x\in\Gamma_c:\ \begin{cases}-\dfrac{\partial\lambda(T)}{\partial p_e}\mathbf{n}\cdot\nabla T(x,t)-\lambda(T)\mathbf{n}\cdot\nabla Z_e(x,t)=-\lambda_m\mathbf{n}\cdot\nabla Z_{me}(x,t)\\ Z_e(x,t)=Z_{me}(x,t)\end{cases}\tag{9}$$

$$x\in\Gamma_0:\ -\lambda_m\mathbf{n}\cdot\nabla Z_{me}(x,t)=0$$

$$t=0:\quad Z_e(x,0)=0\ ,\ \ Z_{me}(x,0)=0$$

where

$$Z_e(x,t)=\frac{\partial T(x,t)}{\partial p_e}\ ,\quad Z_{me}(x,t)=\frac{\partial T_m(x,t)}{\partial p_e}\tag{10}$$

Differentiation of substitute thermal capacity with respect to the parameters p_1, p_2, p_3 leads to the following formulas

$$\frac{\partial C(T)}{\partial p_1}=\begin{cases}0, & T>T_L\\ 1/\left(T_L-T_A\right), & T_A<T\le T_L\\ 0, & T_E<T\le T_A\\ 0, & T_S<T\le T_E\\ 0, & T\le T_S\end{cases}\tag{11}$$

$$\frac{\partial C(T)}{\partial p_2}=\begin{cases}0, & T>T_L\\ 0, & T_A<T\le T_L\\ 1/\left(T_A-T_E\right), & T_E<T\le T_A\\ 0, & T_S<T\le T_E\\ 0, & T\le T_S\end{cases}\tag{12}$$

and

$$\frac{\partial C(T)}{\partial p_3}=\begin{cases}0, & T>T_L\\ 0, & T_A<T\le T_L\\ 0, & T_E<T\le T_A\\ 1/\left(T_E-T_S\right), & T_S<T\le T_E\\ 0, & T\le T_S\end{cases} \tag{13}$$

3. Example of computations

The casting-mould system shown in Figure 2 has been considered. The basic problem and additional problems connected with the sensitivity functions have been solved using the explicit scheme of FDM [1]. The regular mesh created by 25×15 nodes with constant step h = 0.002 m has been introduced, time step Δt = 0.1 s. The following input data have been assumed: λ_L = 20 W/(mK), λ_S = = 40 W/(mK), λ_m = 1 W/(mK), c_L = 5.88 MJ/(m^3 K), c_S = 5.4 MJ/(m^3 K), Q_{aus1} = = 937.2 MJ/m^3, Q_{aus2} = 397.6 MJ/m^3, Q_{eu} = 582.2 MJ/m^3, c_m = 1.75 MJ/(m^3 K), pouring temperature T_0 = 1300°C, liquidus temperature T_L = 1250°C, border temperatures T_A = 1200°C, T_E = 1130°C, solidus temperature T_S = 1110°C, initial mould temperature T_{m0} = 20°C.

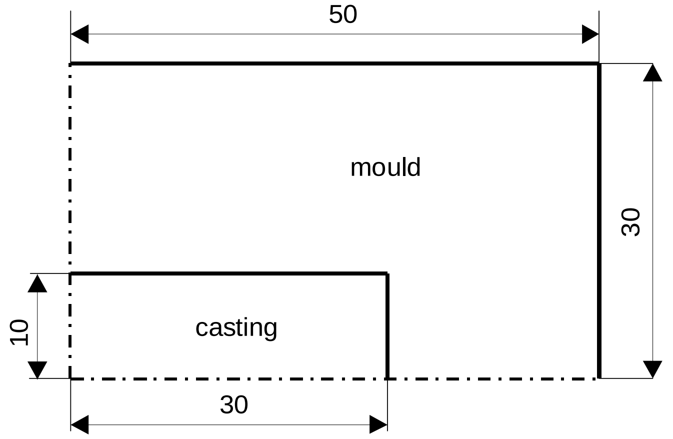

Fig. 2. Casting-mould system

In Figures 3-5 the distributions of functions $Z_1 \cdot 10^9$, $Z_2 \cdot 10^9$, $Z_3 \cdot 10^9$ in the domain of casting for different times are shown. It is visible, that maximal values of sensitivity functions for different times appear in different places, but the global maximum corresponds to the point 1 (0, 0) marked in Figure 8. Figures 6 and 7 illustrate the courses of sensitivity functions at the points 1 and 2 shown in Figure 8.

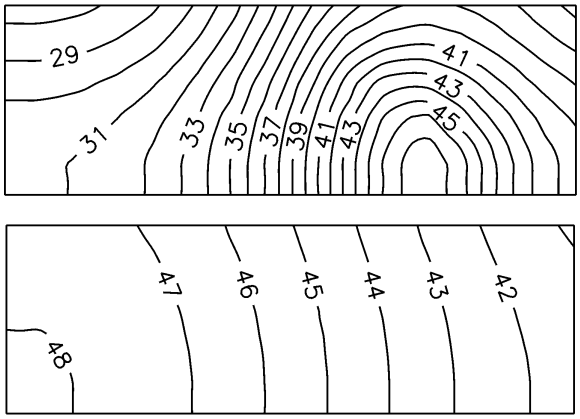

Fig. 3. Distribution of function $Z_1 \cdot 10^9$ for times 30 and 90 second

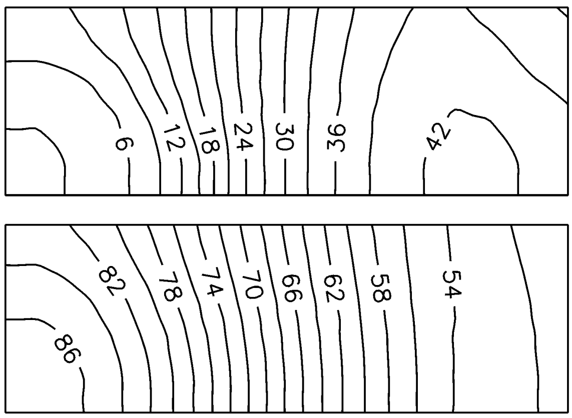

Fig. 4. Distribution of function $Z_2 \cdot 10^9$ for time 60 and 120 second

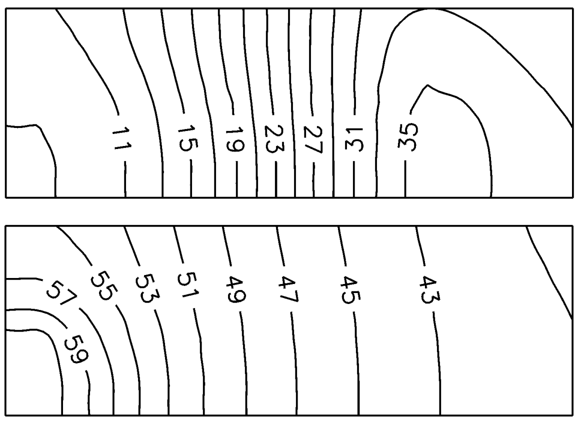

Fig. 5. Distribution of function $Z_3 \cdot 10^9$ for time 120 and 180 second

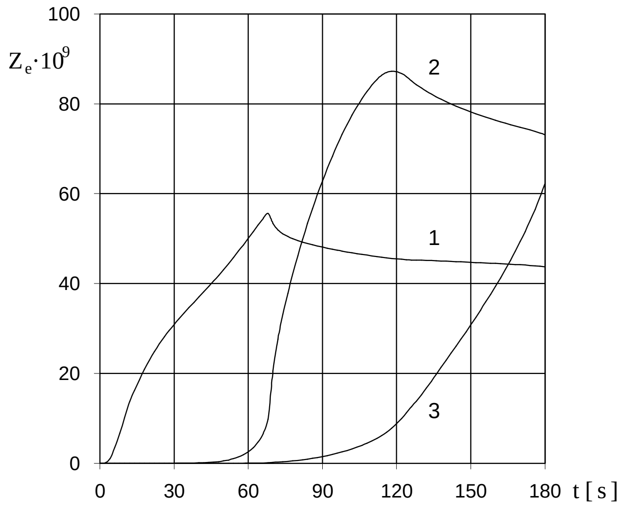

Fig. 6. Courses of sensitivity functions at the point 1 (0 , 0)

On the basis of sensitivity functions analysis the optimal location of sensors can be determined (see. Fig. 8).

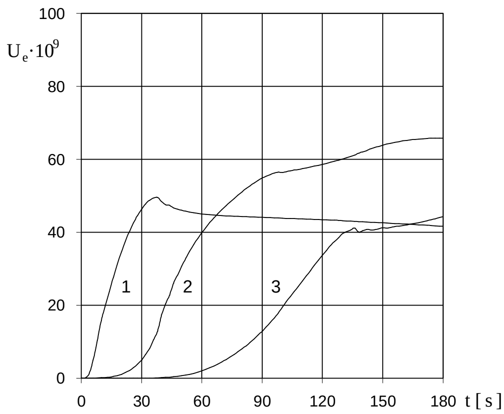

Fig. 7. Courses of sensitivity functions at the point 2 (0.02, 0)

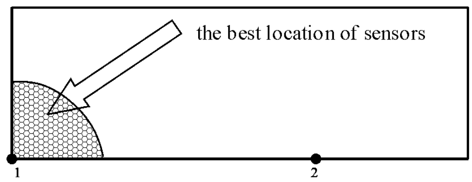

Fig. 8. Optimal position of sensors

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