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INFLUENCE OF CRYOPROBE COOLING/HEATING RATE ON THE COURSE OF FREEZING PROCESS

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Abstract. The methods of sensitivity analysis are used in order to observe the influences of changes of the cryoprobe tip cooling/heating rate on a course of biological tissue freezing process. The temperature history of tip temperature is assumed in a form of broken line. The direct variant of sensitivity analysis is here applied. At the stage of numerical modeling the control volume method is used. In the final part of the paper the examples of computations are shown.

Introduction

During biological tissue freezing in the domain considered one can select three time-dependent sub-domains, this means the tissue in a natural state, intermediate phase and frozen region. The intermediate phase corresponds to the temperature $[T_E,T_B]$, where T_B and T_E denote the border temperatures (the beginning and the end of freezing process), in particular this interval is assumed to be from −1 to −8ºC. The best and very effective mathematical model of this type of phase changes bases on the equation called 'fixed domain approach' [1-3] and then in the energy equation the parameter called a substitute thermal capacity (STC) [1-4] is introduced. For the natural state and frozen region the STC corresponds directly to the volumetric specific heats of sub-domains discussed, while for intermediate phase the formula determining this parameter contains a component resulting from the phase change thermal effect (evolution of latent heat).

1. Mathematical model

So, the following energy equation is here considered

$$C(T)\frac{\partial T}{\partial t} = \mathrm{div}\left[\lambda(T)\mathrm{grad}T\right] + q_P + q_M \tag{1}$$

where $C(T)$ is a substitute thermal capacity, $\lambda(T)$ is a thermal conductivity, q_P is a perfusion heat source and q_M is metabolic heat source.

The action of external cylindrical cryoprobe is here considered and the fragment of tissue domain analyzed should be oriented in the cylindrical co-ordinate system (axially-symmetrical task).
Then

$$C(T)\frac{\partial T}{\partial t} = \frac{1}{\mathrm{r}}\frac{\partial}{\partial r}\left[r\lambda(T)\frac{\partial T}{\partial t}\right] + \frac{\partial}{\partial z}\left[\lambda(T)\frac{\partial T}{\partial z}\right] + q_P + q_M \tag{2}$$

where $T = T(r, z, t)$.
The perfusion heat source is given by the formula

$$q_P = G_b c_b (T_b - T) \tag{3}$$

where G_b is the tissue perfusion [m^3 blood/s/m^3 tissue], c_b is the specific heat of blood per unit of volume, T_b is the arterial blood temperature. The value of metabolic heat source can be assumed as a constant value [5] from the scope 245÷24500 W/m^3.

Equation (1) (Pennes equation) is strongly non-linear because the thermophisical parameters $C(T)$ and $\lambda(T)$ are essentially temperature-dependent. The approximation of above functions is taken from literature (e.g. [6, 7]) and the courses of $C(T)$ and $\lambda(T)$ are shown in Figures 1 and 2.

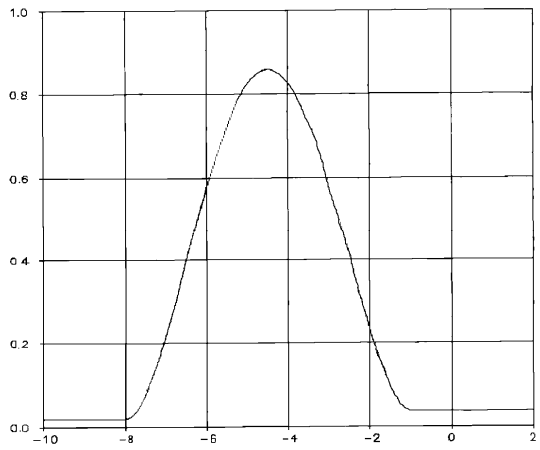

Fig 1. Function $C(T)$ [MJ/m^{3}K]

At the stage of numerical modeling the basic energy equation has been simplified. The numerical experiments show that the action of perfusion and metabolic heat sources can be neglected. The predominant factor determining the course of freezing/heating process is connected with the latent heat evolution. The course of this process is determined by the substitute thermal capacity of tissue, in others

words, the evolution of latent heat is 'hidden' in the formula describing the changes of *C(T)*.
The Pennes equation is supplemented by the following boundary conditions

$$\begin{aligned}
&(r,z)\in\Gamma_{01} \quad : \qquad q=-\lambda(T)\frac{\partial T}{\partial r}=0\\
&(r,z)\in\Gamma_{02} \quad : \qquad q=-\lambda(T)\frac{\partial T}{\partial z}=0\\
&(r,z)\in\Gamma_{1} \quad : \qquad T=T_C(t)\\
&(r,z)\in\Gamma_{2} \quad : \quad q=\lambda(T)\frac{\partial T}{\partial z}=\alpha(T-T_\alpha)
\end{aligned} \tag{4}$$

where Γ_{01}, Γ_{02} are the conventionally assumed external boundaries of tissue domain (see: Figure 3), Γ_1 is the contact surface between cryoprobe tip and skin tissue, Γ_2 is remaining part of skin upper surface, α is the heat transfer coefficient, T_α is the ambient temperature, while T_C (t) is the time-dependent tip temperature. The initial condition

$$t = 0: \; T(r, z,0) = T_0 \tag{5}$$

is also known.

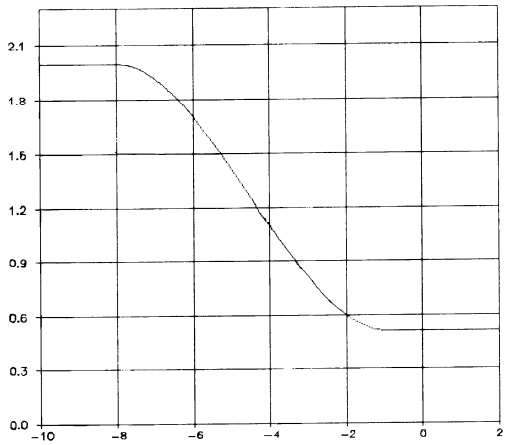

Fig. 2. Thermal conductivity $\lambda(T)$ [W/mK]

The direct problem described by equation (1) and boundary-initial conditions (4), (5) constitutes a base of sensitivity model construction. Below, the example of a such task solution will be presented (c.f. [6]).

Let us consider the cylindrical fragment of biological tissue subjected to action of external cryoprobe (diameter 15 mm). The changes of cryoprobe tip temperatures are marked in Figure 4 (line 1).

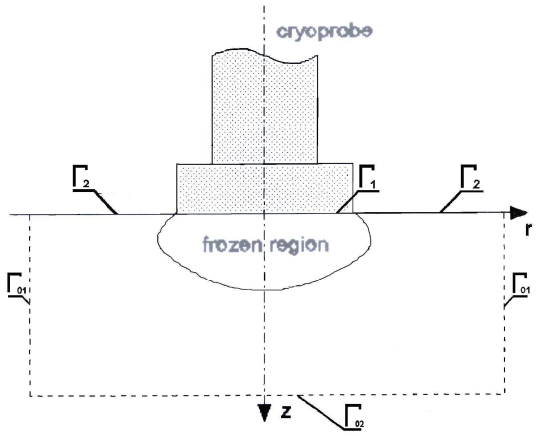

Fig. 3. Domain considered

In the same Figure the temperature histories at the points located on a skin surface are shown. The input data concerning the parameters appearing in mathematical model of thermal processes have been taken from literature (e.g. [6]). As was mentioned, at the stage of numerical computations the control volume method (CVM) [8] has been used.

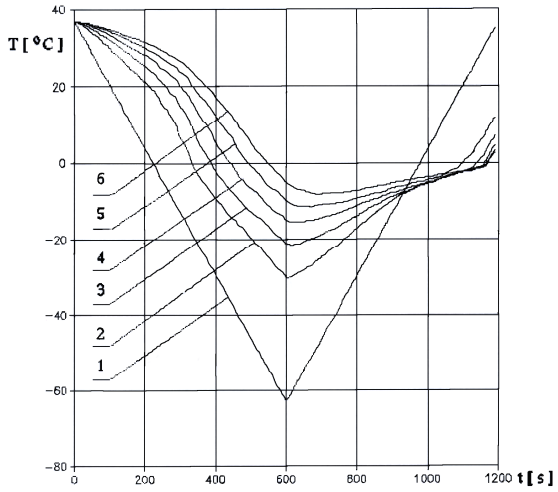

Fig. 4. Cooling/heating curves

2. Sensitivity analysis

To analyze the influence of cryoprobe tip cooling/heating rate on the course of freezing process the energy equation and boundary-initial conditions should be

differentiated with respect to parameter v, this means the slope of straight lines in equations

$$t < t^* \quad : \quad T_C(t) = T_0 - v\,t, \qquad t \geq t^*, \quad T_C(t) = T_{min} + v\,t \tag{6}$$

where T_{min} is the minimum tip temperature, t^* is the exposure time.
So

$$\frac{dC(T)}{dT} U \frac{\partial T}{\partial t} + C(T) \frac{\partial U}{\partial t} =$$
$$\frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial r} + r\lambda(T) \frac{\partial U}{\partial r} \right] + \frac{\partial}{\partial z} \left[\frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial z} + \lambda(T) \frac{\partial U}{\partial z} \right] \tag{7}$$

where $U = \partial T / \partial v$. The last equation can be written in the form

$$C(T) \frac{\partial U}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r\lambda(T) \frac{\partial U}{\partial r} \right] + \frac{\partial}{\partial z} \left[\lambda(T) \frac{\partial U}{\partial z} \right] + q_v \tag{8}$$

where q_v is the artificial source function which form results from equation (7), in particular

$$q_v = -\frac{dC(T)}{dT} U \frac{\partial T}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left[r \frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial r} \right] + \frac{\partial}{\partial z} \left[\frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial z} \right] \tag{9}$$

This form of sensitivity equation is similar to the basic equation (2) and at the stage of computations the same numerical procedures can be used.
Differentiation of Dirichlet boundary condition gives

$$t < t^*: \qquad U_C(t) = -t, \qquad t \geq t^*, \quad U_C(t) = t \tag{10}$$

while in the case of Robin condition

$$\frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial z} + \lambda(T) \frac{\partial U}{\partial z} = \alpha U \tag{11}$$

and this formula can be also written in the form similar to condition (4d).
The non-flux condition are in force, while for $t = 0$: $U_0 = 0$.

3. Example of computations

The cryoprobe of diameter 15 mm is considered. The input data concerning the thermophysical parameters are taken from [6]. The tissue domain of cylindrical shape (d = 45 mm, H = 45 mm) is subjected to the cryoprobe tip temperature, at the

same time v = 8.6 K/min, t^* = 762 s. The numerical solution of sensitivity problem has been obtained using the explicit scheme of CVM for non-linear transient problems. As an example in Figure 5 the sensitivity field in domain considered for time 1080s is shown. The solution obtained allows one to analyze the interactions between kinetics of freezing and the conditions of skin surface cooling/heating process. For example for time 1080s the maximum values of temperature field perturbations are located close to the contact surface, at the same time one can find the isoline U = 0 this means the sub-domain of very small sensitivity.

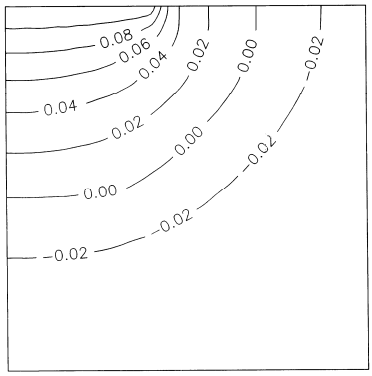

Fig. 5. Sensitivity function

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