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MATHEMATICAL MODEL AND NUMERICAL SOLUTION OF DOUBLE DIFFUSIVE NATURAL CONVECTION SYSTEM

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Abstract. In this paper a two-dimensional double diffusive natural convection system is considered. A mathematical model of heat and moisture transport driven by combined thermal- and solutal-induced buoyancy forces is described. A numerical model based on the Finite Element Method (FEM) is proposed. The results of numerical analysis are presented and discussed.

Introduction

It is obvious that natural convection appears in many situations, especially in heat and mass transfer processes. This process is driven by density changes caused by temperature and solute concentration. In nonisothermal systems with a solvent and solute, the interaction of natural heat convection with mass transport leads to a complex flow and transport phenomenon called double diffusive convection [1-4].

It can be found in many natural systems and engineering applications. Many theoretical and numerical works on the phenomenon have been performed. The analysis of similarities for natural convective flows caused by coupled buoyancy effects of thermal and mass diffusion was done by Gebhart and Pera [5]. The numerical investigation of heat and mass convections caused by both aiding and opposing buoyancy forces was performed by Mahajan and Angirasa [6]. A numerical study was also conducted by Sripada and Angirasa [7] to investigate the unsteady flows driven by heat and solute diffusion.

The paper deals with the mathematical modeling and numerical solution of double diffusive convection in a square cavity filled with fluid. The considered process is nonisothermal. Initially, a layer filled with warm salt water lies above a region containing cold fresh water. The line of division is horizontal and sharp. It divides the whole area into two identical parts. In such a system, one can observe the phenomenon called Rayleigh-Taylor instability. This is a kind of interface instability between two fluids of different densities [8].

1. Mathematical model

Let us consider a two-dimensional, square cavity filled with fluids of different properties (Fig. 1). There is cold fresh water in the lower part of the area and a warm solution containing salt in the upper one.

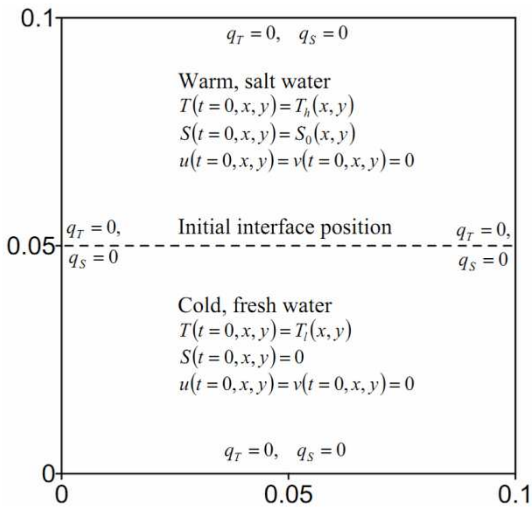

Fig. 1. Square cavity with initial and boundary conditions

The basic governing equations used in this simulation are energy conservation (1), solute concentration in a salt-water solution (2), momentum balance (3) and mass balance (4):

$$div(\lambda \nabla T) = \rho \left[u \frac{\partial (c_p T)}{\partial x} + v \frac{\partial (c_p T)}{\partial y} + \frac{\partial (c_p T)}{\partial t} \right] \tag{1}$$

$$div(D \nabla S) = u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} + \frac{\partial S}{\partial t} \tag{2}$$

$$\begin{aligned} div(\mu \nabla u) - \frac{\partial p}{\partial x} + \rho g_x \beta_T (T - T_{ref}) + \rho g_x \beta_S (S - S_{ref}) &= \rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} \right] \\ div(\mu \nabla v) - \frac{\partial p}{\partial y} + \rho g_y \beta_T (T - T_{ref}) + \rho g_y \beta_S (S - S_{ref}) &= \rho \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t} \right] \end{aligned} \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

where: T [K] is the temperature, S [-] - concentration of sodium chloride, t [s] - time, λ [W/(mK)] - coefficient of thermal conductivity, ρ [kg/m^3] - density,

c_p [J/(kgK)] - specific heat, D [m^2/s] - diffusion coefficient, u, v[m/s] - components of the velocity vector, x, y [m] - components of the position vector, p [Pa] - pressure, μ [kg/(ms)] - dynamic viscosity, g_x, g_y [m/s^2] - components of the gravitational acceleration vector, β_T [K^{-1}] - coefficient of the volumetric thermal expansion, T_{ref} [K] - reference temperature, β_S [-] - coefficient of the volumetric solutal expansion, and S_{ref} [-] - reference concentration.

The appropriate boundary and initial conditions supplement the mathematical model. They are shown in Figure 1. Heat flux q_T, solute flux q_S as well as u and v components equal zero at the cavity boundaries.

2. Numerical example

Initially the upper part of the cavity is filled with a solution of water and 1% NaCl (Fig. 2). The temperature of this fluid equals 308 K. The lower part contains fresh water with a temperature of 278 K.

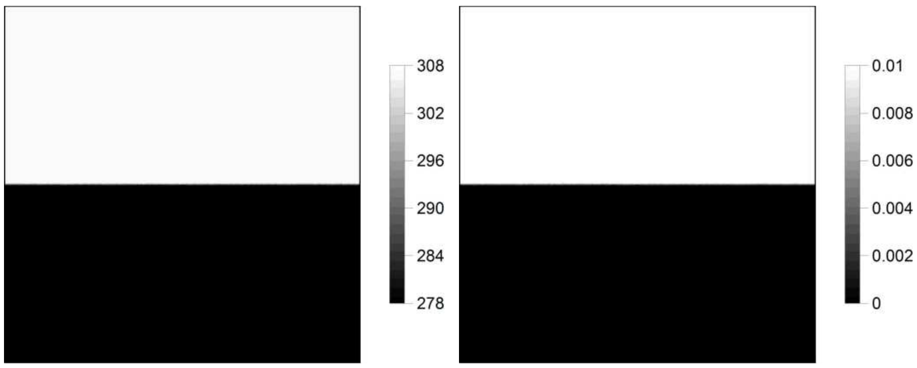

Fig. 2. Initial temperature and concentration used in calculations

The material properties such as density, specific heat, etc., are the same in fresh and salt water. Double convection is driven by appropriate values of β_T and β_S. The values of the material properties used in the calculations are shown in Table 1.

Table 1

Material properties of water

Property	Value	Property	Value
ρ	1000	g_y	–9.81
c_p	4200	β_T	$2.7\cdot10^{-4}$
λ	0.563	β_S	–0.7
D	$5.145\cdot10^{-9}$	T_{ref}	278
μ	$8.7\cdot10^{-4}$	S_{ref}	0

A computer program using the finite element method has been made on the basis of theoretical assumptions. A triangular mesh is created with the use of a GMSH generator. It contains 108291 nodes. The evolution of the Rayleigh-Taylor instability is shown in Figure 3.

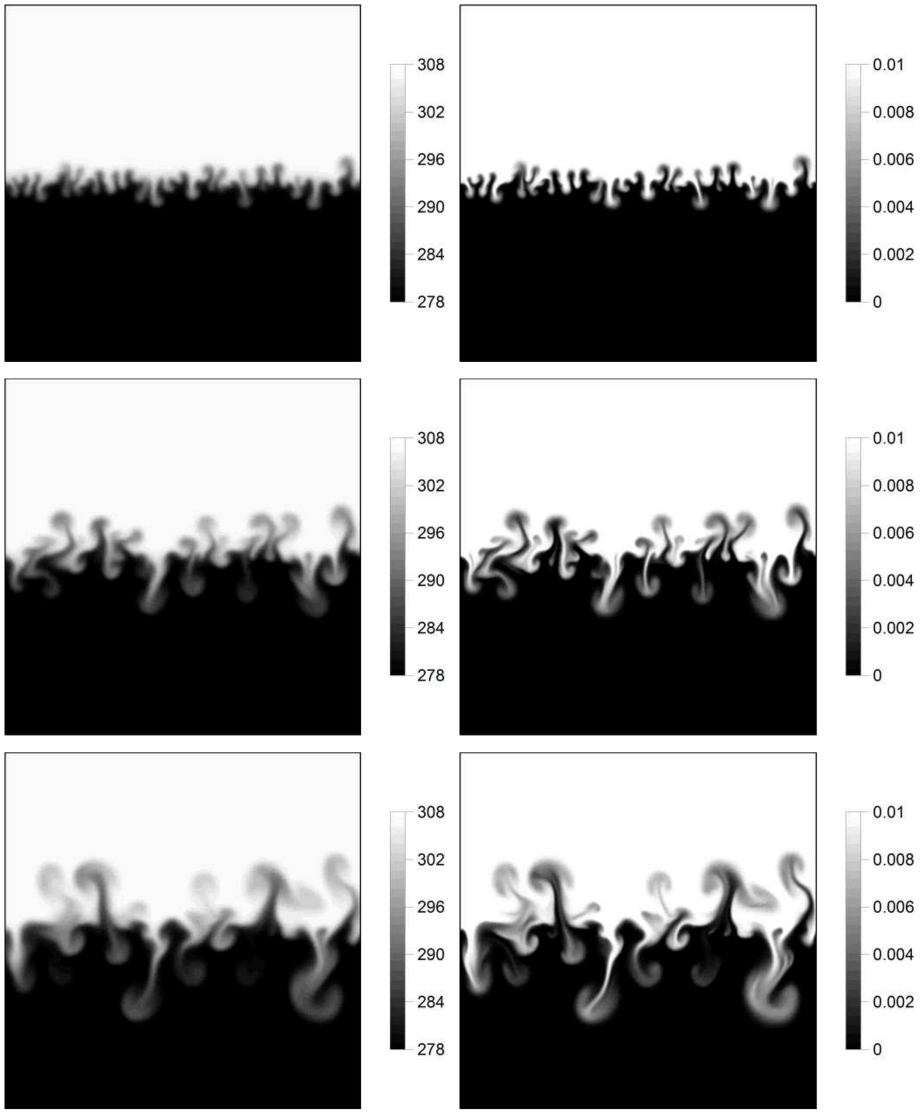

Fig. 3. Temperature and concentration of sodium chloride NaCl after t = 3.52, t = 5.27, t = 6.86 s

Interface instability can be observed immediately after starting the calculations. Small perturbances grow and destabilize the interface. Salt water forms downward-moving irregularities which magnify into sets of Rayleigh-Taylor fingers. One can compare the rate of heat and solute diffusion driven by different values of diffusivity coefficients.

Conclusions

The presented mathematical model can be used as the basis of any double-diffusive system. The computer program made with use of the finite element method is the base of further work which will focus on modelling the solidification of binary alloys with double diffusive convection.

References

[1] Ostrach S., Natural convection with combined driving forces, PhysicoChem. Hydrodyn. 1980, 1(4), 233-247.

[2] Costa V.A.F., Double diffusive natural convection in a square enclosure with heat and mass diffusive walls, Int. J. Heat Mass Transfer 1997, 40(17), 4061-4071.

[3] Costa V.A.F., Double-diffusive natural convection in parallelogrammic enclosures, Int. J. Heat Mass Transfer 2004, 47(14-16), 2913-2926.

[4] Beghein C., Haghighat F., Allard F., Numerical study of double-diffusive natural convection in a square cavity, Int. J. Heat Mass Transfer 1992, 35(4), 833-846.

[5] Gebhart B., Pera L., The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion, Int. J. Heat Mass Transfer 1971, 14(12), 2025--2050.

[6] Mahajan R.L., Angirasa D., Combined heat and mass transfer by natural convection over a horizontal surface, Int. J. Heat Mass Transfer 1993, 20(2), 279-293.

[7] Sripada R.K.L., Angirasa D., Simultaneous heat and mass transfer by natural convection above upward facing horizontal surfaces, Int. J. Heat Mass Transfer 2001, 36(7), 1019-1029.

[8] Sharp D.H., An overview of Rayleigh-Taylor instability, Physica D: Nonlinear Phenomena 1984, 12(1-3), 3-10 IN1-IN10, 11-18.