Journal of Applied Mathematics and Computational Mechanics 2019, 18(1), 5-13
www.amcm.pcz.pl p-ISSN 2299-9965
DOI: 10.17512/jamcm.2019.1.01 e-ISSN 2353-0588

NUMERICAL ANALYSIS OF FREQUENCIES AND FORMS OF OWN COLLARS OF DIFFERENT FORMS WITH FREE ZONE

***Maksym Borysenko*[1], *Andrii Zavhorodnii*[2], *Ruslan Skupskyi*[2]**

[1] *S.P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine*
Kyiv, Ukraine
[2] *Mykolayiv Inter-regional Institute of Human Development «University «Ukraine»*
Mykolayiv, Ukraine
mechanics530@gmail.com, andrew-mdu@ukr.net, skuruslan@gmail.com

Received: 8 December 2018; Accepted: 11 April 2019

Abstract. Thin plates of three different forms with different physical-mechanical characteristics and free edges are examined in this work. Modeling of geometry and numerical calculation of frequencies and forms of free oscillation of plates is accomplished by the finite element method, which is realized using the licensed computer program FEMAP with the NASTRAN solver. A comparative analysis of the calculated eigenfrequencies is carried out. The dependence of the corresponding frequencies on the physical and mechanical characteristics of the material in the form of coefficients is established.

MSC 2010: *74S05, 74K20*
Keywords: *plate, finite element method, FEMAP, eigenfrequencies, forms of proper oscillations*

1. Introduction

In many branches of modern technology, plates of various shapes are the most common elements of thin-walled structures. They are widely used in construction, engineering, electrical engineering and other fields of technology. As they are used in the designing and constructing of such structures, it is necessary to calculate their bearing elements in the event of a variety of sudden dynamic loads. The analysis of the dynamic behavior of plates of different shapes, taking into account various boundary conditions and properties of a material under dynamic load, is an actual problem. For two centuries, the fluctuations of rectangular plates were considered in a huge number of works. The review of some publications is given in work [1]. One of the first classical papers on the fluctuations of thin isotropic rectangular plates with free edges was the Treatise on Acoustic Cold [2]. Numerical calculation of frequencies and forms of free oscillations of a square plate by the variation method of Ritz is presented for the first time in [3].

The classical problem of oscillations of a plate with free edges is considered in [1]. On the basis of the superposition method, its solution is reduced to a homogeneous quasi regular infinite system of linear algebraic equations. The accuracy of the homogeneous boundary conditions is examined, the comparison of theoretical data with the experimental data is carried out.

Fluctuations of free polygonal and rounded polygonal plates with the aid of the improved Ritz method in the class of homology forms were investigated in [4]. The first five frequencies of oscillations and evolution of oscillations were presented.

The problem of oscillations of a viscoelastic plate having the form of a right triangle is considered in [5].

Oscillations occur due to the action of a uniformly distributed load according to the harmonic law. The lines of the amplitude of oscillation levels are investigated, and graphs of the amplitude distribution along the height of the triangle are given.

In comparison, with the analytical and experimental methods, numerous methods for solving the dynamics theory of plates and shells have gained wide practical application, such as the finite element method (FEM). Many modern software products for engineering calculations have been built using FEM. One of the software products is FEMAP and the NX Nastran solution. The proposed software tool is used in several studies of determine the frequency and forms of free vibrations of thin cylindrical shells [6-8]. It also tested in the calculation of a rectangular plate [9].

A comparative analysis of the calculated eigenfrequencies of a square plate is carried out in [9] using the FEMAP computer program with frequencies obtained numerically and experimentally by other authors.

The aim of this study is to determine the frequency and types of free oscillations of isotropic elastic plates of different shapes with free edges based on the finite element method and setting appropriate frequency depending on the physical and mechanical characteristics of the material in the form of coefficients.

2. Output ratios

The equation of dynamics for the FEM can be obtained by considering the equations of motion of a mechanical system with a finite number of degrees of freedom, which is described by the system of Lagrange II equations [10].

This equation of motion for a plate at its finite elemental approximation takes into account the absence of external forces $\left(F(t)=0\right)$ will take the form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\lambda}_i}\right)-\frac{\partial T}{\partial \lambda_i}=-\frac{\partial \Pi}{\partial \lambda_{i\,i}}, \quad i=1, 2, \ldots, n. \tag{1}$$

Using the discrete form of the functional of potential and kinetic energy

$$\Pi=\frac{1}{2}\{\lambda\}_i^T K_i\{\lambda\}_i, \quad T=\frac{1}{2}\{\dot{\lambda}\}_i^T M_i\{\dot{\lambda}\}_i,$$

we obtain the equation of motion of the shell in the absence of damping from the Lagrange equation (1):

$$[M]\{\ddot{\lambda}\}+[K]\{\lambda\}=\{0\}. \tag{2}$$

The solution of equation (1) can be sought in the form

$$\{\lambda\}=\{A\}\cos(\omega t+\beta), \tag{3}$$

where: $\{A\}$ is the vector of amplitude values of nodal displacements, which determine the form of proper oscillations; ω is a cyclic frequency, β is a phase of oscillations. We obtain the system of algebraic equations after the direct substitution (3) in (2) and the reduction in: $\cos(\omega t+\beta)$

$$\left(-\omega^2[M]+[K]\right)\{A\}=\{0\}. \tag{4}$$

In this system, the non-zero components $\{A\}$ of the components are possible only on condition that

$$\det\left([K]-\omega^2[M]\right)=0. \tag{5}$$

If the square matrices $[M]$ and $[K]$ are positive-defined, then equation (5) has N positive solutions - eigenfrequencies ω_k, with possible dual values (here, N is a number of unknowns in the system of algebraic equations (4)).

With the values N of eigenfrequencies ω_k, the solution of system (2) can be found in the form of a linear combination N of expressions (3):

$$\{\lambda\}=\sum_{k=1}^{N}\{A_k\}\cos\left(\omega_k t+\beta_k\right). \tag{6}$$

In order to determine the frequencies and forms of free oscillations, if dissipation and damping are not taken into account, it is advisable to use the Lanczos method, which requires fewer resources (computing time and free hard disk space) compared to other methods [3].

3. Results

For numerical solving of the tasks in this work, using the FEMAP license program, the geometry of three plates of regular shapes: a triangle, a quadrilateral, and a pentagon is constructed, in such a way that a circle of radius $R = 60$ mm can be entered each figure (Fig. 1).

Note that the thickness of all plates is the same and a circular hole with a radius $r = 6$ mm is made in their centers. Models were rigidly fixed on the surface of the hole. Three different metals were selected as the material of the plates: 40X steel

(E = 214 GPa - Young's modulus, Poisson's coefficient ν = 0.32, density ρ= = 7820 kg/m^3), aluminum (E = 110 GPa - Young's modulus, Poisson's coefficient ν = 0.34, density ρ = 2710 kg/m^3), copper (Young's modulus, Poisson coefficient ν = 0.35, density ρ = 8920 kg/m^3). The breakdown was carried out with four-node *plate* elements of constant thickness in size 2×2 mm.

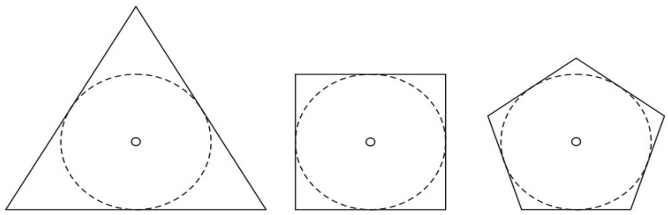

Fig. 1. The geometry of three plates of regular shapes: a triangle, a quadrilateral, and a pentagon

The reliability of the results obtained is ensured by the use of a sound mathematical model, the correctness of the problem statement, the solution of the test problem [9], and the practical convergence of the calculated frequencies in the application of the finite element method.

The spectrum of frequencies and forms of eigenvibrations of a triangular plate of constant thickness of three different metals was investigated. The frequencies of the first ten modes of their own oscillations are given in Table 1, and also in the table the frequency coefficients, which determine the dependence of the frequency of internal oscillations on the physico-mechanical characteristics of the material, namely the ratio of the Young's modulus to density. The forms of oscillation at the corresponding frequencies are presented in Figure 2, where the displacement, for better visualization, is presented in a fourfold increase, as well as in two angles of observation.

Table 1. The frequencies of the first ten modes of their own oscillations

Moda	f [Hz]			$\varphi = \frac{f_{st.}}{f_{al.}}$	$\varphi = \frac{f_{st.}}{f_{cop.}}$
	Steel	Aluminum	Copper		
1	204.01	202.46	137.98	1.01	1.48
2	204.03	202.47	137.99	1.01	1.48
3	259.47	258.03	176.55	1.01	1.47
4	512.96	516.33	349.64	0.99	1.47
5	522.02	516.40	349.69	1.01	1.49
6	804.85	796.89	540.56	1.01	1.49
7	1061.06	1048.00	707.31	1.01	1.50
8	1061.83	1052.12	714.65	1.01	1.49
9	1061.89	1052.18	714.69	1.01	1.49
10	1323.76	1305.71	878.86	1.01	1.51
$\varphi_{average}$				1.01	1.49

The spectrum of frequencies and forms of eigenvalues of a quadrangular plate was studied. The frequencies of the first ten modes of their own oscillations, together with the frequency coefficients, are given in Table 2. Forms of oscillations at corresponding frequencies are presented in Figure 3.

The spectrum of frequencies and forms of eigenvalues of a pentagonal plate was studied. The frequencies of the first ten modes of their own oscillations, together with the frequency coefficients, are given in Table 3. The forms of oscillations at the corresponding frequencies are presented in Figure 4.

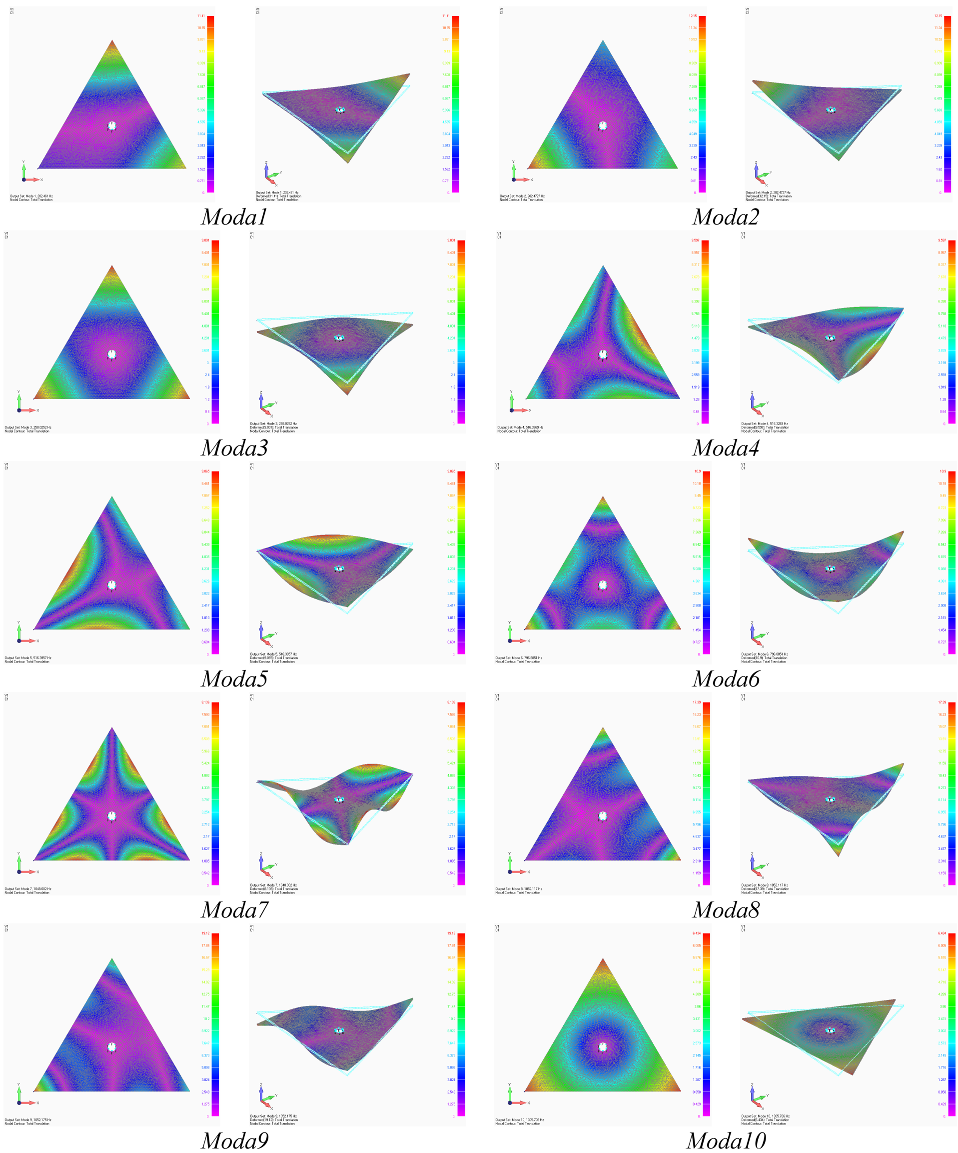

Fig. 2. The forms of oscillation

Table 2. The frequencies of the first ten modes of their own oscillations, together with the frequency coefficients

Moda	f [Hz]			$\varphi = \frac{f_{st.}}{f_{al.}}$	$\varphi = \frac{f_{st.}}{f_{cop.}}$
	Steel	Aluminum	Copper		
1	349.37	347.19	237.30	1.01	1.47
2	349.56	347.40	237.47	1.01	1.47
3	435.63	433.68	297.45	1.00	1.46
4	480.33	474.12	319.59	1.01	1.50
5	710.64	702.16	474.26	1.01	1.50
6	1273.96	1258.92	850.52	1.01	1.50
7	1274.04	1259.00	850.59	1.01	1.50
8	1722.94	1706.17	1157.54	1.01	1.49
9	1870.83	1845.26	1241.94	1.01	1.51
10	2395.55	2368.02	1600.83	1.01	1.50
$\varphi_{average}$				1.01	1.49

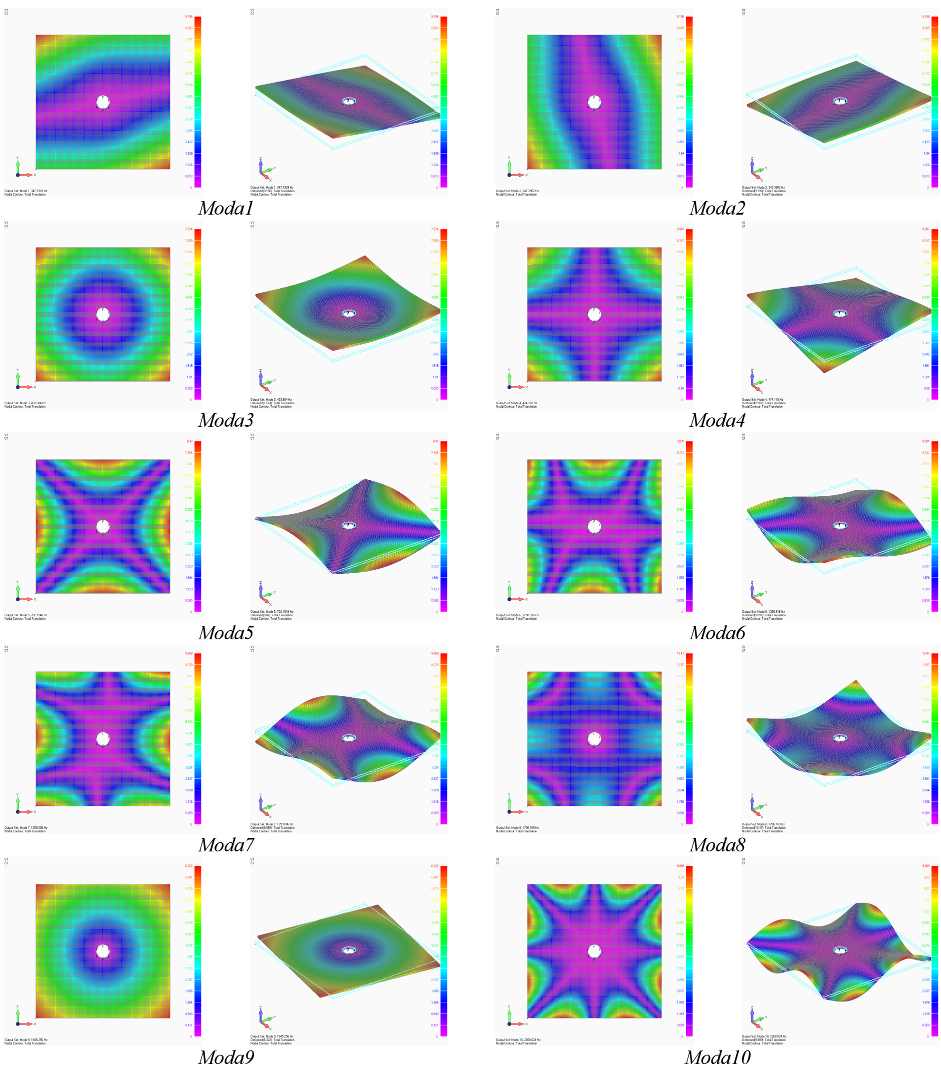

Fig. 3. Forms of oscillations

Table 3. The frequencies of the first ten modes of their own oscillations, together with the frequency coefficients

Moda	f [Hz]			$\varphi = \frac{f_{st.}}{f_{al.}}$	$\varphi = \frac{f_{st.}}{f_{cop.}}$
	Steel	Aluminum	Copper		
1	404.43	402.06	275.05	1.01	1.47
2	404.64	402.29	275.24	1.01	1.47
3	502.35	500.19	343.21	1.00	1.46
4	645.36	637.36	430.10	1.01	1.50
5	645.45	637.46	430.18	1.01	1.50
6	1472.68	1455.21	983.01	1.01	1.50
7	1472.70	1455.23	983.03	1.01	1.50
8	2077.02	2048.66	1378.91	1.01	1.51
9	2366.44	2341.73	1586.38	1.01	1.49
10	2366.68	2341.97	1586.57	1.01	1.49
$\varphi_{average}$				1.01	1.49

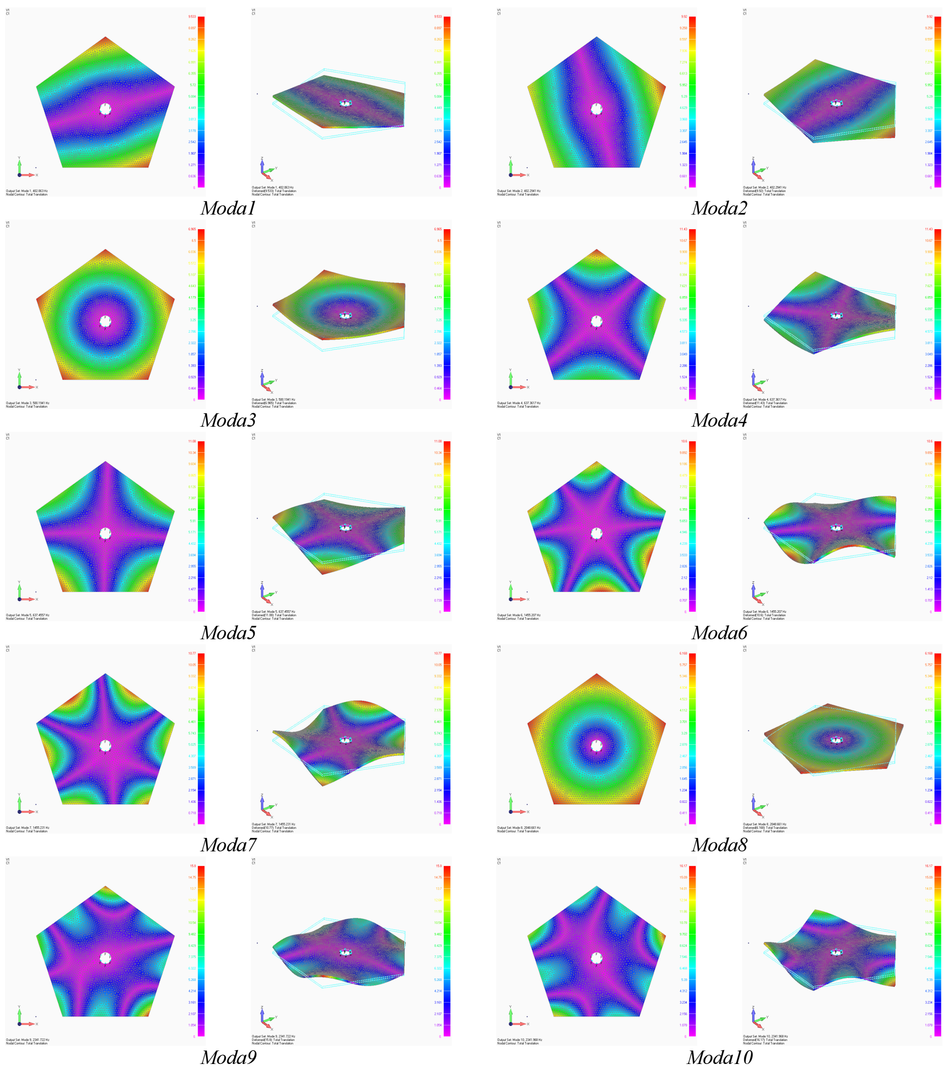

Fig. 4. The forms of oscillations

For comparison, we will present the first ten frequencies of steel plates of different shapes in the form of histograms (Fig. 5) and Table 4.

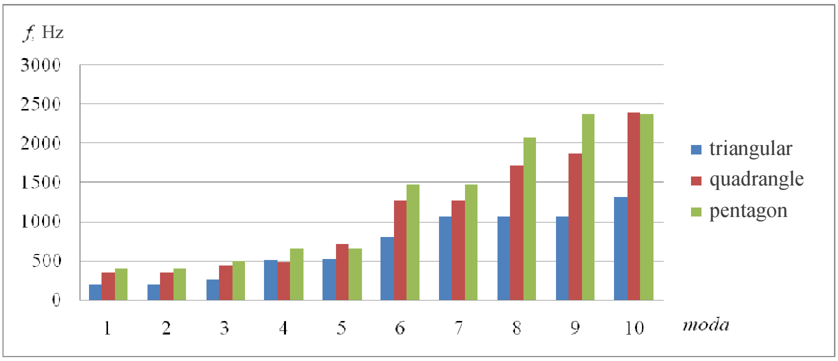

Fig. 5. The first ten frequencies of steel plates of different shapes

Table 4. The first ten frequencies of steel plates of different shapes

Moda	*f* [Hz]		
	Triangular	Quadrilateral	Pentagon
1	204.01	349.37	404.43
2	204.03	349.56	404.64
3	259.47	435.63	502.35
4	512.96	480.33	645.36
5	522.02	710.64	645.45
6	804.85	1273.96	1472.68
7	1061.06	1274.04	1472.7
8	1061.83	1722.94	2077.02
9	1061.89	1870.83	2366.44
10	1323.76	2395.55	2366.68

4. Conclusions

A numerical calculation of frequencies and forms of free oscillations of a triangular, quadrangular and pentagonal plate with free edges has been carried out. There is an increase in its own frequency with an increase in the angles of the plate, which is explained by an increase in the strength of the plate. In the future it is necessary to explore the plates with more angles.

Three materials (steel, aluminum and copper) are considered for the analysis of the influence of material characteristics on dynamic characteristics. Analyzing the obtained data, we can conclude that the frequencies of free oscillations with the same geometric parameters of the plate of steel and aluminum have a slight difference due to the small difference in the velocity of the propagation of the bulk expansion, which depends on the Young modulus and the density of the material. The frequencies for a copper plate, with an identical geometry, are, on average, 1.49 times smaller than the corresponding frequencies of the shells of steel. Note that the corresponding coefficients are obtained in [8] for other research objects of the same materials.

References

[1] Meleshko, V.V., & Papkov, S.O. (2009). Bending oscillations of elastic rectangular plates with free edges: from Chladni (1809) and Ritz (1990) to the present day. *Akustichny Visnik*, *12*, 4, 34-51.

[2] Chladni, E.-F.-F. (1809). *Trait´e d'acoustique*. Paris: Courcier.

[3] Ritz, W. (1909). Theorie der Transversalschwingungen einer quadratischen Platte mit freien R¨andern. *Ann. Physik.*, *4*, 28, 737-786.

[4] Wang, C.Y. (2015). Vibrations of completely free rounded regular polygonal plates. *International Journal of Acoustics and Vibration*, *20*, 2, 107-112.

[5] Chernyshov, N.A. (2001). Viscoelastic vibrations of a triangular plate. *Applied Mechanics and Technical Physics*, *42*, 3, 152-158.

[6] Budak, V.D., Grigorenko, A.Ya., Borisenko, M.Yu., & Boychuk, E.V. (2016). Determination of eigenfrequencies of an elliptic shell with constant thickness by the finite-element method. *Journal of Mathematical Sciences*, *212*, 2, 182-192.

[7] Budak, V.D., Grigorenko, A.Ya., Borisenko, M.Yu., & Boychuk, E.V. (2017). Natural frequencies and modes of noncircular cylindrical shells with variable thickness. *International Applied Mechanics*, *53*, 2, 164-172.

[8] Grigorenko, A.Ya., Borisenko, M.Yu., Boichuk, E.V., & Prigoda, A.P. (2018). Numerical determination of natural frequencies and modes of the vibrations of a thick-walled cylindrical shell. *International Applied Mechanics*, *54*, 1, 75-84.

[9] Borisenko, M.Yu., Boychuk, O.V., Borisenko, I.A., Rogovtsov, U.O. (2016). Computer model design of thin plates of thin plates from other materials. *Geometric Modeling and Information Technology Technologies*, *2*, 29-33.

[10] Rudakov, K.N. (2011). *FEMAP 10.2.0. Geometric and finite element modeling of structures*. K.: NTUU "KPI".